Learning Low-level Motion Trajectory

Jane Li

Assistant Professor
Mechanical Engineering Department, Robotic Engineering Program
Worcester Polytechnic Institute
Quiz (10 point)

- Examine the DMP Transformation system

$$\tau \ddot{y} = \alpha_z (\beta_z (g - y) - \dot{y}) + f$$

- (2 pts) How does the system behave if $f=0$?
- (2 pts) Goal of the system?
- (2 pts) How does the system behave if $f \neq 0$?
- (2 pts) How to set the forcing term such that the system will eventually stabilize at the goal?
- (2 pts) What can the forcing term encode?
A damped spring model

\[ \tau \ddot{y} = \alpha_z (\beta_z (g - y) - \dot{y}) + f. \]

• How does the system behave if \( f = 0 \) ?
• Goal of the system?
• How does this system behave if it is critically damped?
The forcing term

\[ \tau \ddot{y} = \alpha_x (\beta_x (g - y) - \dot{y}) + f \]

• How does the system behave if \( f \neq 0 \) ?

• How to set the forcing term such that the system will eventually stabilize at the goal?

• What can the forcing term encode?
The forcing term preserves the shape of a trajectory

- Fixed basis function
- Adjustable weight
- How to get rid of the explicit time dependence?
Change basis by introducing a canonical system

\[ \tau \dot{x} = -\alpha_x x, \]

\[ f(t) \rightarrow f(x) \]

- \( t = 0 \) when motion starts
- \( t = \infty \) when motion ends
Reformulation of Basis

\[ f(t) = \frac{\sum_{i=1}^{N} \Psi_i(t)w_i}{\sum_{i=1}^{N} \Psi_i(t)} \quad \rightarrow \quad f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x)w_i}{\sum_{i=1}^{N} \Psi_i(x)}x(g - y_0) \]

- Where

\[ \Psi_i(x) = \exp\left(-\frac{1}{2\sigma_i^2}(x - c_i)^2\right) \]
Basis and weights

Psi activations in the x-domain and t-domain.
How to use DMP to learn from a demonstration

- Transformation system:
  \[ \tau \ddot{y} = \alpha_z \beta_z (g - y) - \dot{y} + f \]

- Forcing term:
  \[ f_{\text{target}} = \tau^2 \ddot{y}_{\text{demo}} - \alpha_z \beta_z (g - y_{\text{demo}}) - \tau \dot{y}_{\text{demo}} \]
How to use DMP to learn from a demonstration

- Estimate the weights using locally weighted regression (LWR)

\[
 w_i = \frac{s^T \Gamma_i f_{\text{target}}}{s^T \Gamma_i s}
\]

- Where

\[
 s = \begin{pmatrix}
 \xi(1) \\
 \xi(2) \\
 \vdots \\
 \xi(P)
\end{pmatrix}, \quad
 \Gamma_i = \begin{pmatrix}
 \Psi_i(1) & 0 \\
 \Psi_i(2) & \ddots \\
 0 & \ddots & \Psi_i(P)
\end{pmatrix}, \quad
 f_{\text{target}} = \begin{pmatrix}
 f_{\text{target}}(1) \\
 f_{\text{target}}(2) \\
 \vdots \\
 f_{\text{target}}(P)
\end{pmatrix}
\]
Apply DMP on a single DOF
Use DMP to imitate trajectory pattern

DMP imitate path

- System trajectory vs. time (ms)
- DMP 1, DMP 2, desired paths

[Graph showing DMP trajectories and desired paths]
# of basis

![Graph 1](Image)

**DMP imitate path**

- - - desired path

![Graph 2](Image)

**DMP imitate path**

- 10 BF$\text{s}$
- 30 BF$\text{s}$
- 50 BF$\text{s}$
- 100 BF$\text{s}$

**System trajectory**

- time (ms)
- system trajectory

0 20 40 60 80 100

0 -0.5

0 0.5 1.0 1.5 2.0 2.5

0 20 40 60 80 100

0 -0.5
Spatial scaling – change the target

\[ f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x)w_i}{\sum_{i=1}^{N} \Psi_i(x)} x(g - y_0) \]
Spatial scaling – change the target

\[ f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x)w_i}{\sum_{i=1}^{N} \Psi_i(x)} (g - y_0) \]

DMP imitate path

- DMP 1
- DMP 2
- desired paths
Temporal scaling

\[ \ddot{y} = \alpha_y (\beta_y (g - y) - \dot{y}) + f \]
\[ \dot{x} = -a_x x \]

\[ \ddot{y} = \tau^2 (\alpha_y (\beta_y (g - y) - \dot{y}) + f) \]
\[ \dot{x} = \tau (-a_x x) \]
DMP - Obstacle avoidance \([1]\)

\[ \tau \ddot{y} = \alpha_z (\beta_z (g - y) - \dot{y}) + f + C_t \]

\[ C_t = \gamma R\dot{y} \theta \exp(-\beta \theta), \]

where

\[ \theta = \arccos \left( \frac{(o - y)^T \dot{y}}{|o - y||\dot{y}|} \right) \]

\[ r = (o - y) \times \dot{y}. \]
DMP – Moving Obstacle [1]

\[ t = 0.76 \text{ s} \]

\[ \begin{array}{ccc}
\text{Start} & \text{Goal} \\
\text{Goal} & \text{Start} \\
\end{array} \]

\[ t = 1.34 \text{ s} \]

\[ \begin{array}{ccc}
\text{Start} & \text{Goal} \\
\text{Goal} & \text{Start} \\
\end{array} \]

\[ t = 1.05 \text{ s} \]

\[ \begin{array}{ccc}
\text{Start} & \text{Goal} \\
\text{Goal} & \text{Start} \\
\end{array} \]

\[ t = 4.2 \text{ s} \]

\[ \begin{array}{ccc}
\text{Start} & \text{Goal} \\
\text{Goal} & \text{Start} \\
\end{array} \]
DMP – obstacle avoidance demonstration
DMP Application – Motion recognition
DMP Application – Motion generation
DMP Application – Motion coordination

Side View  Isometric View
DMP Variants

- ProMP [2]
  - Time-domain basis
  - Application on learning human-robot interaction

- Probabilistic MP [3]
  - Integrate motion and sensing uncertainty to system transformation
  - Motion reproduction becomes a feedback control loop
  - Online motion tracking becomes robust given sensing uncertainty
Motivation

- Probabilistic modeling of motion primitive given multiple demonstration

When learning human-robot interaction, robot needs to

- Observes human partner’s motion
- Predict human partner’s motion
- Generate motions that match human partner’s motion
Example of Human-robot Interaction
Challenges

• How to represent averaged behavior?

• How to represent variability?

• How to align fast/slow trajectory?

• How to match phase between robot and human partner?
• Observes human partner’s motion
  • Sparse

• Predict end user’s motion
  • Prediction may vary by fitting sparse data to variants of a model that differ by temporal scaling

• Generate motions that match
  • Wrong prediction leads to mismatch between human and robot motions
Related work - Trajectory alignment

- Dynamic time warping (DTW)
Related work – GMM/GMR [4, 6]
Limitation of DTW

- Represent averaged behavior?
  - No. Pairwise matching

- How to represent variability?
  - No. Pairwise matching

- How to align trajectory?
  - Yes. Align fast and slow trajectory (deterministic, need to choose a unique reference)

- How to match phase in the learning of human-robot interaction?
  - Yes.
Limitation of GMM/GMR

• Represent averaged behavior?
  • Yes

• How to represent variability?
  • Yes

• How to align trajectory?
  • No. Cannot align fast or slow motions

• How to match phase in the learning of human-robot interaction?
  • No.
Pro-MP

Training
Multiple demonstrations
- Human trajectories
- Robot trajectories

Phase normalization
- $\alpha \sim \mathcal{N}(\mu_\alpha, \sigma^2_\alpha)$
- $\alpha_i = T_i / T_{nom}$

Correlated model
- $p(w; \theta) = \mathcal{N}(w | \mu, \Sigma)$

Inference
Human observations
- $y^o$

Searching
- $\alpha^*$

Conditioning
- $p(y_{1:T}; \theta^{new}) = \int p(y_{1:T}|w)p(w; \theta^{new})dw$. 

RBE 595 – Synergy of Human and Robotic Systems – Instructor: Jane Li, Mechanical Engineering Department & Robotic Engineering Program - WPI 11/20/2017
Pro-MP – Training

Training

Multiple demonstrations

Human trajectories

Robot trajectories

Phase normalization

$\alpha \sim N(\mu, \sigma^2)$

$\alpha_i = T_i / T_{nom}$

Correlated model

$p(w; \theta) = N(w | \mu, \Sigma)$

Correlated model

Human

Robot

Inference

Human observations

$y^0$

Searching

$\alpha^*$

$\alpha_1$

$\alpha_2$

$\ldots$

$\alpha_N$

Conditioning

$\alpha^*T_{nom}$

$\alpha^*T_{nom}$

Trajectory tracking

$p(y_{1:T} | \theta^{new}) = \int p(y_{1:T} | w)p(w | \theta^{new})dw.$
Pro-MP – Inference

Training

Multiple demonstrations

Human trajectories

Robot trajectories

Phase normalization

$\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$

$\alpha_i = T_i / T_{nom}$

Correlated model

$p(w; \theta) = \mathcal{N}(w | \mu, \Sigma)$

Inference

Human observations

$y^o$

Searching $\alpha^*$

$\alpha_1$

$\alpha_2$

$\ldots$

$\alpha_N$

Conditioning

$\alpha^* T_{nom}$

Trajectory tracking

$p(y_{1:T} | \theta^{new}) = \int p(y_{1:T} | w) p(w; \theta^{new}) dw.$
Pro-MP – Posterior distribution of phase

\[ p(\alpha | y_{t-t'}^0, \theta) \propto p(y_{t-t'}^0 | \alpha, \theta) p(\alpha) \]

\[ p(y_{t-t'}^0 | \alpha, \theta) = \int p(y_{t-t'}^0 | \tilde{w}, \alpha) p(\tilde{w}) d\tilde{w} \]

\[ = \mathcal{N}(y_{t-t'}^0 | A(z_{t-t'})^T \mu_w, A(z_{t-t'})^T \Sigma_w A(z_{t-t'}) + \Sigma_y^0) \]

\[ \alpha^* = \arg \max_\alpha p(\alpha | y_{t-t'}^0, \theta) \]

\[ A(z_{t-t'}) = \begin{bmatrix} \psi(z_{t-t'})^T & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \psi(z_{t-t'})^T \end{bmatrix} \]

Inference

Human observations

Searching \( \alpha^* \)

\( y^0 \)

\( \alpha_1 \)

\( \alpha_2 \)

\( \ldots \)

\( \alpha_N \)

\( \alpha^* T_{nom} \)

Conditioning

Trajectory tracking

\[ p(y_{1:T}; \theta^{new}) = \int p(y_{1:T} | w) p(w; \theta^{new}) dw. \]
• Represent averaged behavior?
  • Yes

• How to represent variability?
  • Yes

• How to align trajectory?
  • Yes. Align motions by phase matching

• How to match phase in the learning of human-robot interaction?
  • Yes.
• Read

• Understand how to integrate sensing uncertainty into the transformation system
Assignment 13 – Due Nov 27

• Prepare 4-6 presentation slides on (30 pts)
  • Make sure you either write a paper review digest, or integrate notes to your slides to explain your understanding
Reference


