Kinematic Redundancy

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Quiz (10 points)

• (6 pts) Explain what are the “regularity” and “variability” in human motion.

• (4 pts) What is Jacobian matrix augmentation?
What we already know ...

Regularity in human arm motions

Robot manipulator redundancy resolutions
Robot manipulator redundancy resolution

- **Resolutions at Different Levels**
  - Position, velocity, acceleration

- **General Resolution to Inverse Kinematics**
  - Pseudo-inverse, general IK solver, ...

- **Task-based Resolutions**
  - Jacobian matrix augmentation

- **Performance-based Resolutions**
  - Various performance indices, global vs local optimization, ...
Human arm motion control

- **Motion Regularity and Variability**
  - Donders’ law [Donders:1848], Fitts’ law [Fitts:54], 2/3-power law [Terzuola, Viviana:80], motion variability [Bernstein:67], uncontrolled manifold [Scholz, Schoner:99]

- **Arm Motion Control Criteria**

- **Criterion Synthesis**
  - Spatial+temporal [Biess, Flash:07]
Human arm model

Anatomical arm model.

Swivel-angle arm model.
How to compute the swivel angle?

\[ \mathbf{f}' = \mathbf{Pe} - \mathbf{Pc} \]

\[ f = \frac{\mathbf{Pe} - \mathbf{Ps}}{||\mathbf{Pe} - \mathbf{Ps}||} \]

\[ \mathbf{P}_{w} = \mathbf{P}_{w} - \mathbf{Ps} \]

\[ \mathbf{n} = \frac{\mathbf{P}_{w} - \mathbf{Ps}}{||\mathbf{P}_{w} - \mathbf{Ps}||} \]

\[ \mathbf{a} = \begin{bmatrix} 0, 0, -1 \end{bmatrix}^T \]

\[ \mathbf{u} = \frac{\mathbf{a} - (\mathbf{a} \cdot \mathbf{n}) \mathbf{n}}{||\mathbf{a} - (\mathbf{a} \cdot \mathbf{n}) \mathbf{n}||} \]

\[ \mathbf{v} = \text{Atan2} \left( \frac{f' \times u}{f' \cdot u} \right) \]

\[ \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||} \]

\[ \sin \theta = \frac{||\mathbf{a} \times \mathbf{b}||}{||\mathbf{a}|| \cdot ||\mathbf{b}||} \]
How to compute the swivel angle?

\[ \vec{f}' = \vec{P_e} - \vec{P_c} \]

\[ \vec{f} = \vec{P_e} - \vec{P_s} \]

\[ \vec{f}' = \vec{f} - (\vec{f} \cdot \vec{n}) \cdot \vec{n} \]

Reference direction \( \vec{a} \):

\[ \vec{a} = [0, 0, -1]^T \]

\( \vec{u} \) is the projection of \( \vec{a} \) on Plane \( S \):

\[ \vec{u} = \frac{\vec{a} - (\vec{a} \cdot \vec{n})\vec{n}}{||\vec{a} - (\vec{a} \cdot \vec{n})\vec{n}||} \]

Swivel angle \( \phi \):

\[ \phi = \arctan(\vec{n} \cdot (\vec{f}' \times \vec{u}), \vec{f}' \cdot \vec{u}) \]
Performance to Optimize

Performance indices | Formula | Comments
--- | --- | ---
Determinant of Jacobian (1984) | $\det(J) \neq 0$ | Uniformity of the torque-velocity gain
Condition number (1982) | $\kappa = \frac{\sigma_{\max}}{\sigma_{\min}}$ | Variance in velocity/force transmission
Isotropy (1987) | $\delta_{so} = \frac{\sigma_{\min}}{\sigma_{\max}}$ | same as condition number
Min eigen-value of Jacobian (1987) | $\lambda_{so} = \frac{\sigma_{\min}}{\sigma_{\max}}$ | Efficiency of force/velocity transmission
Dynamic Manipulability (1985) | $G = \sqrt{MJ^{-1}}$ | Uniformity of this torque-acceleration gain
Distance from singularities (1987) | $H = \left(\frac{\lambda_{min}}{\lambda_{max}}\right)^{1/p}$ | Related to manipulability by $w_n = \sqrt{\sum_j \lambda_j}$
Acceleration ratio (1988) | $\gamma = M(\theta) + C(\theta, \dot{\theta})\ddot{\theta}$ | acceleration capability of the end-effector
Force transmission ratio (1988) | $\alpha = \left|\frac{u^T(J^TJ)u}{\sigma_{\min}}\right|^{1/2}$ | Force gain along task-compatibility direction
Velocity transmission ratio (1988) | $\beta = \left|\frac{u^T(J^TJ)^{-1}u}{\sigma_{\min}}\right|^{1/2}$ | Velocity along task-compatibility direction
Min Jerk model (1984) | $\min\left|\frac{\ddot{x}}{\Delta} \right|$ | Motion smoothness
Min (commanded) torque-change (1985,1989) | $\min\left|\frac{\ddot{u}}{\Delta} \right|$ | Motion smoothness
Min work model (1983) | $\min(W)$ | Energy
Min variance model (1989) | $\min\left[\text{var}(x-x_d)\right]$ | Task accuracy
Real-time control, Unplanned Task
Our goals

- Real-time control, unplanned motion
- Natural arm posture
- Related biological functions to behavior
  - New bio-inspired motion control criteria
Criterion 1 – Maximize Motion Efficiency to Head

\[ P_e \text{ lies on the plane of } P_s, P_w \text{ and } P_m: \]
Criterion 2 – Close to Equilibrium Arm Posture

\[ \mathbf{P}_e \text{ lies on the plane of } \mathbf{P}_w \text{ and } \mathbf{v}_e. \]
Criterion 3 – Minimize Joint Angle Change

Given \( \phi(k), \phi(k + 1) \in [\phi(k) - 0.5^\circ, \phi(k) + 0.5^\circ] \) such that:

\[
\phi(k + 1) = \arg \min_{\phi'(k+1)} |\tilde{\theta}(k) - \tilde{\theta}'(k + 1)| = \arg \min_{\phi'(k+1)} \sqrt{\sum_{i=1}^{4} (\theta_i(k) - \theta_i'(k + 1))^2}
\]
Criterion 4 – Minimize Kinetic Energy

Given $\phi(k)$, $\phi(k + 1) \in [\phi(k) - 0.5^\circ, \phi(k) + 0.5^\circ]$ such that:

\[
\phi(k + 1) = \arg\min_{\phi'(k+1)} |Ke(k) - Ke'(k + 1)|
\]
Criterion 5 – Minimize Work in Joint Space

Given \( \phi(k), \phi(k + 1) \in [\phi(k) - 0.5^\circ, \phi(k) + 0.5^\circ] \) such that:

\[
\phi(k + 1) = \arg\min_{\phi'(k+1)} \left| W_i \right|_{t_k, t_{k+1}} = \arg\min_{\phi'(k+1)} \sum_{i=1}^{4} \left| W_i \right|_{t_k, t_{k+1}}
\]
Further Questions

- We have so many optimization criteria
  - Which is the best?
  - Individual or blending?
  - How to combine?
  - What can we infer from the algorithm that can accurately predict arm posture?
A Framework for Comparison
Propose methods for synthesize multiple control criteria
  - Exponential method
  - Least squares method

Validate prediction algorithm using reaching motion data
Exponential method

At time step $k$, estimation error $\varepsilon_i$:

$$\varepsilon_i(k) = |\phi_{\text{exp}}(k) - \phi_i(k)|$$

The inferred contribution for Criterion $i$:

$$C_i(k + 1) = \exp\left[-\frac{\varepsilon_i^2(k)}{\hat{\sigma}^2(k)}\right]$$

Coefficient normalization:

$$c_i(k + 1) = \frac{C_i(k + 1)}{\sum_{i=1}^{5} C_i(k + 1)}$$

According to the principle of maximum entropy, the probability of the criterion $i$ can be expressed as:

$$p_i = c \cdot \exp(-\lambda \varepsilon_i^2)$$
Least Squares Method

At time step $k$:

$$\phi(k) = \sum_{i=1}^{5} c_i(k) \phi_i(k)$$

Linear regression:

$$C(k+1)_{5 \times 1} = A^{-1} \cdot b$$

Coefficient normalization:

$$c_i(k+1) = \frac{C_i(k+1)}{\sum_{i=1}^{5} C_i(k+1)}$$

$$A = \begin{bmatrix} \phi_{c1}(k-19) & \cdots & \phi_{c5}(k-19) \\ \vdots & \ddots & \vdots \\ \phi_{c1}(k) & \cdots & \phi_{c5}(k) \end{bmatrix}_{20 \times 5}$$

$$b = \begin{bmatrix} \phi_{exp}(k-19) \\ \vdots \\ \phi_{exp}(k) \end{bmatrix}_{20 \times 1}$$
Experiment

Shoulder \((P_s)\), elbow \((P_e)\), wrist \((P_w)\) position at 100 Hz
8 start points × 7 end points × 5 repeats × 10 subjects = 2800 trials
Prediction Error Distribution

Exponential method (EXP).

Least squares method (LSQ).
Estimated Coefficient

Exponential method (EXP).

Least squares method (LSQ).
Assignment 5

• Given the arm motion data
  • Time sequence of shoulder, elbow, wrist position
  • [Link](https://drive.google.com/open?id=0B7SwE0PHMbzbc1Z4TjcoWEhWYoU)

• Compute the time sequence of swivel angle
  • Implement your algorithm using Matlab function