Configuration Space

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Quiz (10 pts)

• (5 pts) Describe one challenge that novice user faces in the teleoperation of TRINA?

• (5 pts) Explain one method to help with this problem
Challenges

• Motion
  • Many DOFs to control
  • Coordinated dexterous manipulation
  • User interface are not intuitive

• Perception
  • Hard to perceive spatial relationship through multiple 2D images
  • Lack of tactile sensing
Configuration space
Recap

- Plan paths for a point in 2D → simple
- Real-world robots are **complex**, often **articulated** bodies

A space where the robots could be treated as points?
• **Configuration q**
  - A specification of the position of **every** point on the object.
  - Expressed as a vector of the **DOF** of the robot

\[ q = (q_1, q_2, \ldots, q_n) \]

• **Configuration space C**
  - The set of all possible configurations

**A configuration q is a point in C**
The minimum number of DOF needed to specify the configuration of the object completely.

\[ q = (q_1, q_2, \ldots, q_n) \]
Example – A Rigid 2D Mobile Robot

- 3-parameters: $q = (x, y, \theta)$ with $\theta \in [0, 2\pi)$.

- C-space dimension = 3

- Topology?
  - SE(2) = $\mathbb{R}^2 \times S^1$

- Shape of C-space?
  - Cylinder
Example – Rigid Robot in 3D workspace

• $q = (\text{position, rotation}) = (x, y, z, ???)$

• Representations for rotation?
  • Euler Angles – yaw, pitch roll
  • 3X3 Transform Matrices
  • Unit quaternion

• Regardless of the representation, rotation in 3D is 3 DOF
Example – Rigid Robot in 3D workspace

- C-space dimension = 6

- Topology?
  - $\text{SE}(3) = \mathbb{R}^3 \times \text{SO}(3)$
Configuration Space for Articulated Objects

- Articulated object
  - A set of rigid bodies connected by joints

- For articulated robots (arms, humanoids, etc.), the DOF are usually the joints of the robot
  - Exceptions?
Configuration Space for Articulated Objects

- Topology of two-link manipulator?

With joint limits?
Path and Trajectory in C-Space

- **Path**
  - A continuous curve connecting two configurations $q_{\text{start}}$ and $q_{\text{goal}}$

$$\tau : s \in [0,1] \rightarrow \tau(s) \in C$$

- **Trajectory**
  - A path parameterized by time

$$\tau : t \in [0,T] \rightarrow \tau(t) \in C$$
Obstacles in C-space

Workspace

Configuration space

Initial

Goal
(Collision)-free configuration – $q$
- Robot placed at $q$ has no intersection with any obstacle in the workspace

Free Space – $C_{\text{free}}$
- A subset of $C$ that contains all free configurations

Configuration space obstacle – $C_{\text{obs}}$
- A subset of $C$ that contains all configurations where the robot collides with workspace obstacles or with itself
How to compute $C_{obs}$?

Robot Geometry → Compute → $C_{obs}$

Obstacle Geometry → Compute → $C_{obs}$
Example – 2D Robot without Rotation

• A simple setup
  • Disc in 2D space $\rightarrow$ not a point anymore
  • Polygonal obstacle in task space
Example – 2D Robot without Rotation

Workspace (2D)  configuration space (2D)
Minkowski Sum

\[ A \oplus B = \{ a + b \mid a \in A, b \in B \} \]
Minkowski Sum

- Dip B into paint
- Put B’s origin on A’s border
- Translate it along A’s edge
- Sum = the painted area
Example – 2D Robot with Rotation

- C-space?
- Minkowski Sum?
Example – 2D Robot with Rotation
High-dimensional space
**Discussion**

- Do we need to have an explicit representation of C-obstacles to do path planning?
  - Exact method?
  - Approximate method?
  - Sampling-based method?
Why topology matters?

Because coffee mug is indeed a donut!
Topological properties are very useful
Homotopic paths

- Two paths with the same endpoints is homotopic if one path can be deformed into continuously deformed into the other.

![Homotopic paths diagram](image)
Homotopic class of paths

- On a cylinder surface without ends

Which paths are homotopic?
Topology and homeomorphism
• **C is connected**
  • If every two configurations can be connected by a path.

• **C is simply-connected**
  • if any two paths connecting the same endpoints are **homotopic**.
  • Examples: $\mathbb{R}^2$ or $\mathbb{R}^3$

• Otherwise $C$ is multiply-connected.
  • Example?
A distance function $d$ in configuration space $C$ is a function

$$d : (q, q') \in C^2 \rightarrow d(q, q') \geq 0$$

- $d(q, q') = 0$ if and only if $q = q'$,
- $d(q, q') = d(q', q)$,
- $d(q, q') \leq d(q, q'') + d(q'', q')$
Discussion

• Do we need a specialized distance metric in C-space to do path planning?

• Metrics for distance?
  • Euclidian distance
  • Other metrics?
Distance in C-space
Distance metrics

- **L₁-norm (Manhattan distance)**
  \[ d₁(p, q) = ||p - q||₁ = \sum_{i=1}^{n} |p_i - q_i|, \]

- **L₂-norm (Euclidian distance)**
  \[ d(p, q) = d(q, p) = \sqrt{(q₁ - p₁)^2 + (q₂ - p₂)^2 + \cdots + (qₙ - pₙ)^2} \]

- **Lₘₙ-norm (chessboard distance)**
  \[ D_{Chebyshev}(p, q) := \max_i (|p_i - q_i|). \]
Once specified a C-space and its obstacle, we should be able to **discretize** it and **search** for a path.
How about the C-space of a self-driving car?

- Discretization
  - Exact method, approximate method?

- Search
  - Can we get from one cell to another, directly? Why?

- How does the C-space look like?
State lattice

How to handle moving obstacles?
Assignment – individual paper review

• Paper

• Topic
  • Spatiotemporal state lattice → Due on Friday (Feb 2) at noon
  • Present student talk on Friday? → Submit by Thursday (Feb 1) by mid-night
Student talk – James Kuszmaul
Complexity of sweeping-line algorithm
End