An Efficient Algorithm and Hardware Architecture for Maximum-Likelihood Based Carrier Frequency Offset Estimation in MIMO Systems

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Abstract

Carrier frequency offset (CFO), which is often caused by the mismatch between the local oscillators in transmitter and receiver, limits the performance of multiple-input multiple-output (MIMO) wireless communication systems. This paper presents a maximum-likelihood-based CFO estimation algorithm for MIMO systems and its efficient hardware design. The proposed algorithm can accurately estimate the CFO, especially at high signal-to-noise ratio. Moreover, the Cramer–Rao lower bound of the proposed method is derived as a performance guideline. In terms of hardware implementation, an efficient pipeline architecture is presented in detail, showing that the architecture only occupies minimal hardware resources so that it can be fitted into a small field-programmable gate array. The proposed architecture can be reconfigured according to different pilot lengths, making it flexible to various frame structures.

Index Terms

Carrier frequency offset, maximum-likelihood, MIMO, hardware architecture, FPGA.

I. INTRODUCTION

Multiple input multiple output (MIMO) has become one of the key technologies for modern wireless communications, especially when wireless local area network (WLAN) became a tremendous solution to indoor data transmission and IEEE 802.11n standard gained its great success. In general, during the transmission process, carrier frequency offset (CFO) occurs due to the frequency mismatch between transmitter and receiver’s local oscillators, as well as motion-induced Doppler shift [1]–[4]. However, the coherence of oscillators as well as the motionless positions of transmitters and receivers can hardly be guaranteed, indicating that carrier frequency synchronization becomes necessary. The synchronization problem has been under investigation for many years, [5]–[8] are widely used textbooks for synchronization and baseband receiver design. With an increasing number of MIMO applications, MIMO synchronization problem becomes a critical issue in wireless communication systems. Among all the synchronization impairments, CFO is the first issue to be recovered, otherwise the received signal will be drifting along time. To recover the received signal, CFO estimation and recovery techniques have been proposed in literature.

Nasraoui et al. [9] proposed a pilot aided correlation method, taking the sliding correlation of structured preamble to estimate the frequency offset in a closed form solution. The method had low complexity, however its estimation accuracy was not the best among all competitive algorithms. Pilot aided correlation method provides a tradeoff between performance and complexity. Correlation-based estimation was used to estimate the frequency offset in MIMO systems in [10]. It took the advantages of orthogonal pilots to eliminate the interference among antennas and calculate the frequency offsets between each pair of antennas. Due to this fact, the complexity of this algorithm was relatively high. The other disadvantage of this algorithm was that the mean squared error (MSE) of frequency estimation diverged from given Cramer-Rao bound when the signal-to-noise ratio (SNR)
became higher, showing that the algorithm was not optimal.

In [11], a maximum-likelihood based estimator (MLE) for frequency offset was proposed for single-input single-output (SISO) system, followed by different solutions to the MLE. Among these solutions, the linear approximation utilizes only small part of the received signal to avoid phase unwrapping, which leads to a degradation of its performance. Newton search and local grid search were also applied in [11] to refine the estimates of linear approximation method, while the Newton search frequently fails because of local maximums and the accuracy of uniform search depends heavily on resolution and search range. Recently, an high resolution CFO estimation algorithm was proposed in [12]. This algorithm uses Golay sequence as pilot signal and provides closed form solution to CFO estimation. Another advantage of this algorithm is that the MSE of estimated CFO keeps dropping along with the SNR increase while that of the method in [11] almost stops decreasing at high SNR. Unfortunately, in spite of its competitive performance, this algorithm is designed with the assumption that the communication system is SISO. On the other hand, by avoiding phase wrapping and unwrapping, this algorithm has limited CFO acquisition range.

Besides the CFO estimation algorithms for non-OFDM systems, CFO estimation methods are also proposed for OFDM systems. In [13], a ML based CFO estimation method using null subcarrier insertion scheme was proposed. The advantage of this method is that the inserted null subcarriers extend the frequency acquisition range so that the proposed method can estimate both integer and fractional CFO of an OFDM system. However, the expanded acquisition range is achieved at the expense of spectral efficiency, resulted by the inserted subcarriers.

There were also methods proposed for MIMO CFO estimation based on likelihood function in [14]–[17]. In [17], an CFO estimator for MIMO OFDM system based minimum channel residual energy (CRE) is proposed while the corresponding solution to finding the minimum CRE is not given. Grid search of the maximum was employed in [15]. Iterative search via alternating projection frequency estimator (APFE) and approximate alternating projection frequency estimator (AAPFE) were employed in [14]. The grid search approach in [15] showed a clear gap between MSE of frequency estimation and corresponding CRLB while the approaches in [14] got closer to corresponding CRLB. All these searching methods showed divergence from CRLB at high or low SNR. Besides, Mehrpouyan et al. [16] proposed a closed form data-aided least square (LS) estimator to estimate the phase change due to the frequency mismatch in MIMO system. This closed form estimator, however, still diverged from corresponding CRLB at high SNR.

Field-programmable gate arrays (FPGA) have been widely used to implement software defined radio (SDR) systems. FPGA provides a platform with lower power consumption and higher performance compared with the processor-based software radio [18]. Several hardware implementations of MIMO communication systems were proposed in [19]–[22], but these implementations focused more on decoding where frequency synchronization was not considered. Without accurate frequency synchronization, communications may suffer higher bit error rate (BER) because of the mismatch between oscillators, or the transmission could even be ruined. Some other recent works [23]–[26] proposed architectures and implementations of CFO synchronizations for orthogonal frequency division duplex (OFDM) based or non-OFDM based SISO systems.

At present, there is a lack of highly accurate and hardware efficient solutions to ML based CFO estimation for non-OFDM MIMO systems. Therefore, in this paper, we propose an asymptotically optimal algorithm that can accurately estimate the carrier frequency offset as well as an efficient hardware implementation for proposed algorithm. More practically, since the proposed algorithm assumes one sample per symbol where timing synchronization is a prerequisite, we consider the case that the timing synchronization is completed so that our algorithm can be directly applied without further treatment. The main contributions of this paper are as follows:

1) We propose a maximum likelihood based frequency offset estimation algorithm for MIMO system, with an asymptotically optimal closed form solution to the estimator, giving the result that the MSE of proposed method approaches corresponding lower bound as SNR increases. The proposed method has large acquisition range up to 25% of bandwidth.

2) We present an efficient hardware architecture for our ML based CFO estimation algorithm. By avoiding matrix inversion and searching for maximum which are common in the existing solutions to CFO estimation, hardware implementation of our algorithm is efficient and therefore can be easily fitted in a small FPGA.

3) The proposed hardware architecture is pipelined so that the processing delay is minimal. Meanwhile, the architecture is reconfigurable to different pilot lengths, which makes it possible to accommodate various frame structures.

The rest of this paper is organized as follows: In Section II, the general system model and corresponding signal format are presented. In Section III, the frequency synchronization problem, corresponding maximum likelihood based solution for MIMO system, and CRLB for proposed algorithm as a performance guideline are proposed. In Section IV, the architecture of proposed method for hardware implementation is revealed in details, the throughput is also discussed. In Section V, to show the performance of the proposed algorithm and compare with that of other methods, simulations are carried to analyze the performance over a wide range of SNRs and frequency offsets, and even different pilot lengths. Simulation results of MSE versus SNR also show that our method is asymptotically optimal. In addition, the implementation results in regard of hardware resource consumption, achievable circuit frequency are reported and
II. SYSTEM MODEL

The notations that are used for system model description and algorithm derivation are summarized in Table 1. Besides, the operators in Section II and III are as follows: \( \tilde{a}_t \) denotes a tall vector whose \( x \)-th row of vector is \( \tilde{a}_x \), \( [A]_{x_1,x_2} \) denotes a block matrix whose block at \( x_1 \)-th row and \( x_2 \)-th column is \( A \), \( \Re[\cdot] \) and \( \Im[\cdot] \) stand for real and imaginary part, \( E[\cdot] \), \((-)^{\dagger} \) and \( \text{Tr}(\cdot) \) denote expectation, conjugate transpose and trace, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Transmit Antennas</td>
<td>( l_t )</td>
</tr>
<tr>
<td># of Receiver Antennas</td>
<td>( l_r )</td>
</tr>
<tr>
<td>Sample Period</td>
<td>( t_b )</td>
</tr>
<tr>
<td>Symbol Energy</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Channel Variance</td>
<td>( \sigma_h^2 )</td>
</tr>
<tr>
<td>Noise Variance</td>
<td>( \sigma_n^2 )</td>
</tr>
<tr>
<td>Identity Matrix</td>
<td>( I )</td>
</tr>
<tr>
<td>Fisher Information Matrix</td>
<td>( Z )</td>
</tr>
<tr>
<td>CFO Estimate</td>
<td>( \delta_f )</td>
</tr>
<tr>
<td>Symbol Energy</td>
<td>( P_s )</td>
</tr>
<tr>
<td>Kronecker Delta</td>
<td>( \delta[r_1-r_2] )</td>
</tr>
</tbody>
</table>

To investigate the frequency synchronization of a MIMO systems, this section considers the system model as follows. Suppose the transmitter has \( l_t \) antennas, the receiver has \( l_r \) antennas and the MIMO system is located where the channel has flat-fading or slow fading that the channel state can be assumed to be constant during the transmission of pilot [27]. Let \( \tilde{h}_{r,t} \) be the channel coefficient from the \( r \)-th transmit antenna to the \( t \)-th receive antenna and define \( \tilde{h}_{r,t} = [\tilde{h}_{r,t}]_{l_r=1:l_r} \in \mathbb{C}^{l_t \times l_r} \) as the vector of channel states at the \( r \)-th receive antenna. The channel state matrix \( H = \left[ (\tilde{h}_{r,t})_{l_r=1:l_r} \right] \) comprises circularly symmetric complex Gaussian random variables with joint distribution \( \mathcal{CN}\left( \tilde{0}, \mathbb{E}\left[ (\tilde{h}_{r,t})_{l_r=1:l_r} (\tilde{h}_{r,t})_{l_r=1:l_r}^{\dagger} \right] \right) \). Define \( N = \left[ (\tilde{n}_{r,t})_{l_r=1:l_r} \right] \) as the noise at receiver with distribution \( \mathcal{CN}\left( \tilde{0}, \mathbb{E}\left[ (\tilde{n}_{r,t})_{l_r=1:l_r} (\tilde{n}_{r,t})_{l_r=1:l_r}^{\dagger} \right] \right) \); \( H \) and \( N \) are independent. We send length \( n \) pilot signals \( S^T = \left[ s_{r,t}\right]_{l_r=1:l_r} \in \mathbb{C}^{l_t \times n} \) through the channel, with sample period \( t_b \). The pilot signals, known to the receivers, for different transmit antennas are orthogonal, i.e., \( \tilde{s}_{t_1} \perp \parallel \tilde{s}_{t_2} \), \( \forall t_1 \neq t_2 \); \( S^T = \frac{n_2}{\rho} I_{l_t \times l_t} \), where \( \rho \) is the symbol energy and \( S^T \)'s energy satisfies \( \text{Tr}(S^T S) = \sum_{l=1}^{l_t} \sum_{k=1}^{n} |s_{l,k}|^2 = \rho n > 0 \). At the receiver, assuming perfect symbol timing recovery is applied so that with a frequency offset \( f_b \) and additive white Gaussian noise (AWGN) with double sided power spectral density \( 2N_0 \), the received signal can be written as:

\[
Y^T = HS^T F + N^T = \left[ \tilde{h}_r^{\dagger} \right] s_r^{\dagger} F + \left[ \tilde{n}_r^{\dagger} \right]_r
\]

(1)

where the frequency offset matrix

\[
F = \begin{bmatrix}
\sigma_0^{2l_2f_b} & 0 & \cdots & 0 \\
0 & \sigma_0^{2l_2f_b} & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \sigma_0^{2l_2f_b(n-1)}
\end{bmatrix}
\]

(2)

is a diagonal matrix, which rotates the pilot signal according to the frequency offset \( f_b \).

We consider the simplest case that channels are independent where \( \mathbb{E}\left[ \tilde{h}_{r_1} \tilde{h}_{r_2}^{\dagger} \right] = \sigma_h^2 \delta[r_1 - r_2] \) and i.i.d. noise \( \mathbb{E}\left[ \tilde{n}_{r_1} \tilde{n}_{r_2}^{\dagger} \right] = \sigma_n^2 I \delta[r_1 - r_2] \). It is the case of choice when we have little knowledge of the channel. To write the model in the familiar linear transformation form, we define tall vectors \( \tilde{y} = (\tilde{y})_r \), \( \tilde{h} = (\tilde{h})_r \), \( \tilde{n} = (\tilde{n})_r \), and block matrix

\[
\tilde{X} = \begin{bmatrix}
X & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & X
\end{bmatrix} = [X]_{r,t=1:l_r}
\]

(3)

where \( I_{l_t} \) is a \( l_t \times l_t \) identity matrix and \( \otimes \) is the Kronecker product. With these notation, the received signal can be written as

\[
\tilde{y} = \tilde{X} \tilde{h} + \tilde{n}.
\]

(4)

For \( r \)-th receive antenna, the received signal can be written as:

\[
y_r(k) = x_r(k) + n_r(k) = \sum_k h_{r,t} s_{l,t} e^{j2\pi f_b t_b k} + n_r
\]

(5)

III. FREQUENCY OFFSET ESTIMATION

A. PROPOSED CFO ESTIMATION ALGORITHM

To perform ML estimation of frequency offset, we need to obtain the estimate from

\[
\hat{f}_b = \arg \max_{f_b} \log p(y|f_b)
\]

(6)

We observe that \( y \) is a summation of Gaussian random variables and has distribution \( \mathcal{CN}\left( \tilde{0}, \Sigma_f(f_b) \right) \), where the covariance matrix of \( y \) can be written as

\[
\Sigma_f(f_b) = \sigma_n^2 \tilde{X} \tilde{X}^{\dagger} + \sigma_h^2 I
\]

\[
= \sigma_h^2 I_{l_t} \otimes (FS^T S^\dagger) + \sigma_n^2 I
\]

(7)
Therefore,

\[ \hat{f}_b = \arg \max_{f_b} f_b(\tilde{y}(f_b)) \]
\[ \propto \arg \max_{f_b} e^{-\frac{1}{2} \Sigma y(f_b)^{-1} y} \]
\[ = \arg \min_{f_b} y^\dagger \Sigma y(f_b)^{-1} y \]
\[ = \arg \min_{f_b} \sum_r \tilde{y}_r^\dagger \frac{1}{\sigma_n^2} \left( \frac{\sigma_h^2}{\sigma_n^2} F S^\dagger F^\dagger + I \right)^{-1} \tilde{y}_r \]  
(8)

Apply Woodbury identity [28] to (8) and bear in mind that \( F^\dagger F = I, S^\dagger S = \frac{n}{K} I \), we can obtain

\[ \hat{f}_b = \arg \min_{f_b} \sum_r \tilde{y}_r^\dagger \]
\[ = \left( I - FS \left( S^\dagger F^\dagger FS + \frac{\sigma_h^2}{\sigma_n^2} I \right) S^\dagger F^\dagger \right)^{-1} \tilde{y}_r \]
\[ = \arg \max_{f_b} \sum_r \tilde{y}_r^\dagger FS \left( \frac{n}{K} I + \frac{\sigma_h^2}{\sigma_n^2} I \right) S^\dagger F^\dagger \tilde{y}_r \]
\[ = \arg \max_{f_b} \sum_r \tilde{y}_r^\dagger F S S^\dagger F^\dagger \tilde{y}_r \]  
(9)

To find \( \frac{\partial g(\tilde{y}, f_b)}{\partial f_b} = 0 \), we calculate the differential first:

\[ g(\tilde{y}, f_b + df_b) - g(\tilde{y}, f_b) \]
\[ = \sum_r \tilde{y}_r^\dagger F(f_b + df_b) S S^\dagger F(f_b + df_b)^\dagger \tilde{y}_r \]
\[ - \sum_r \tilde{y}_r^\dagger F(f_b) S S^\dagger F(f_b)^\dagger \tilde{y}_r \]
\[ = 2 \pi \delta f_b \left( \sum_r \tilde{y}_r^\dagger F(f_b) S S^\dagger F(f_b)^\dagger \tilde{y}_r \right) \]
\[ + o(df_b) \]
\[ = 4 \pi \delta f_b \left( \sum_r \tilde{y}_r^\dagger F(f_b) S S^\dagger F(f_b)^\dagger \tilde{y}_r \right) \]  
(10)

where we have used

\[ F(f_b + df_b) = F(f_b) F(df_b) \]
\[ = F(f_b) \left( I + j 2 \pi df_b t_0 J + o(df_b) \right) \]
\[ F(df_b) = [1 + j 2 \pi df_b t_0 (i - 1) + o(df_b)]_{i=1}^n \]
\[ \approx I + j 2 \pi df_b t_0 J \]  
(11)

and

\[ J \approx \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & \vdots \\ \vdots & \ddots & \cdots & 0 \\ 0 & 0 & \cdots & (n - 1) \end{bmatrix} \]  
(12)

From the differential, the first order derivative can be expressed as

\[ \frac{\partial \delta g(\tilde{y}, f_b)}{\partial f_b} = 4 \pi \delta f_b \left( \sum_r \tilde{y}_r^\dagger F(f_b) S S^\dagger F(f_b)^\dagger \tilde{y}_r \right) \]  
(13)

and therefore to find the frequency offset, we solve

\[ 0 = \exists \left( \sum_r \tilde{y}_r^\dagger F(f_b) S S^\dagger F(f_b)^\dagger \tilde{y}_r \right) \]  
(14)

It is obvious that directly solve (16) requires matrix operations which is not easy. Therefore, we transform these matrix operations into scalar form. Define \( d \triangleq d^{2 \pi \delta f_b t_0} \left[ J_{n-1} \right]_{i=1}^n = \left[ d^0 \cdots d^{m-1} \right] \), and let \( \tilde{y}_{c,r} = C_i^\dagger \tilde{y}_r \) to be the unscrambled received signal which is partitioned into length \( l_i \) vectors \( \tilde{y}_{c,r}(k) \in C_i^\dagger \) such that \( \tilde{y}_{c,r} = \left[ \tilde{y}_{c,r}(k) \right]_{k=1}^{l_i} \). Define the sample correlation \( \hat{\beta}_{k,i} \triangleq \sum_{r=1}^{l_i} < \tilde{y}_{c,r}(i), \tilde{y}_{c,r}(k) > \) with the property \( \hat{\beta}_{k,i} = \beta_{k,i}^* \). Also bearing in mind that for a matrix \( A, \exists [A] = 0 \) is equivalent to
\[ A - A^\dagger = 0. \text{ Then (16) is equivalent to} \]
\[
0 = \left[ \left( \sum_r \tilde{y}_r F(f_b) S S^\dagger F(f_b) \right)^\dagger J \tilde{y}_r \right] - \left( \sum_r \tilde{y}_r F(f_b) S S^\dagger F(f_b) \right) \]
\[
= \rho \sum_r \left[ d^{k-1} \right]_{k=1:m} \times \left[ \tilde{y}_{c,r} (k_1) (k_2 - k_1) I_i \tilde{y}_{c,r} (k_2) \right]_{k_1=1:m, k_2=1:m} \times \left[ \left( d^i \right)^{k-1} \right]_{k=1:m} \]
\[
= - \rho l_i \sum_{k_1=1}^m \sum_{k_2=1}^m d^{k_1-k_2} (k_2 - k_1) \beta_{k_1, k_2} (17) \]

Take a further step simplification of (17), let \( r_i e^{-j \theta_i} = \sum_{k=i+1}^{m} \beta_{k-k-i} \), we have
\[
0 = -2 j \rho l_i \sum_{k=2}^{m} i \cdot d^i \beta_{k-k-i} \]
\[
-2 j \rho l_i \sum_{i=1}^{m} e^{j 2 \pi f_i t_b l_i} (i \cdot r_i e^{-j \theta_i}) \]
\[
-2 j \rho l_i \sum_{i=1}^{m} i \cdot r_i \sin (2 \pi \hat{f} t_b l_i - \theta_i) (18) \]

Consequently, if we denote \( \alpha = 2 \pi t_b l_i \hat{f}_b \), the solution to (16) is
\[
0 = \sum_{i=1}^{m-1} i \cdot r_i \sin (i \alpha - \theta_i) (19) \]

For high SNR, \( i \alpha - \theta_i \) approaches 0 for all possible \( i \), under which condition \( \sin(x) \approx x \) can be applied to (19) obtain (20)
\[
0 = \sum_{i=1}^{m-1} i \cdot r_i (2 \pi \hat{f} t_b l_i - \theta_i) (20) \]

and we can calculate the estimate of frequency offset easily from (20).

However the approximation \( i \alpha - \theta_i \approx 0 \) may not be suitable for lower SNR case. We consider the Taylor series \( \sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \) which stands under all conditions, (20) may be expanded as
\[
0 = \sum_{i=1}^{m-1} i \cdot r_i \left( \sum_{k=0}^{\infty} \frac{(-1)^k (i \alpha - \theta_i)^{2k+1}}{(2k+1)!} \right) (21) \]

Since there is no general solution to a quintic or higher order equation, a third order Taylor series approximation is applied to (20) and we obtain (23)
\[
0 = \sum_{i=1}^{m-1} i \cdot r_i \left( \sum_{k=0}^{1} \frac{(-1)^k (i \alpha - \theta_i)^{2k+1}}{(2k+1)!} \right) (22) \]

and therefore we have
\[
\alpha^3 \sum_{i=1}^{m-1} - \frac{i^4 r_i}{6} + \alpha^2 \sum_{i=1}^{m-1} \frac{i^3 r_i \theta_i}{2} + \alpha \sum_{i=1}^{m-1} \frac{i^2 r_i (1 - \frac{\theta_i^2}{2}) + \sum_{i=1}^{m-1} i r_i \left( \frac{\theta_i^3}{6} - \theta_i \right)}{2} (23) \]

Define \( a = \sum_{i=1}^{m-1} \frac{i^4 r_i}{6}, b = \sum_{i=1}^{m-1} \frac{i^3 r_i \theta_i}{2}, c = \sum_{i=1}^{m-1} i^2 r_i (1 - \frac{\theta_i^2}{2}) \) and \( d = \sum_{i=1}^{m-1} \frac{i r_i \left( \frac{\theta_i^3}{6} - \theta_i \right)}{2} (23) \) now is
\[
\alpha a^3 + ba^2 + ca + d = 0 (24) \]

Since the frequency offset is a real number and there should exist only one solution, the real valued solution to (24) is \( \alpha = -b \), plug in \( \alpha = 2 \pi t_b l_i \hat{f}_b \), we have
\[
\hat{f}_b = \frac{1}{2 \pi t_b l_i} \frac{- \sum_{i=1}^{m-1} \frac{i^4 r_i \theta_i}{6}}{3 \sum_{i=1}^{m-1} \frac{i^3 r_i}{2}} \approx \frac{1}{2 \pi t_b l_i} \frac{\sum_{i=1}^{m-1} \frac{i^3 r_i \theta_i}{2}}{3 \sum_{i=1}^{m-1} \frac{i^4 r_i}{6}} (25) \]

The entire procedure of proposed method is shown in Algorithm 1.

**Algorithm 1 ML Based CFO Estimation Algorithm for MIMO System**

1) **Input:** \( \tilde{y}_r \in C^{n \times 1}, r = 1, ..., l_r \) one sample per symbol
2) Unscramble \( \tilde{y}_r \) and perform a partition so that \( \tilde{y}_{c,r} = \left[ \tilde{y}_{c,r} (1) : \tilde{y}_{c,r} (m) \right] \), \( \tilde{y}_{c,r} (k) \in C^{l_1 \times 1}, r = 1, ..., l_r \), \( ml_r = n \)
3) **For** \( i = 1, ..., m - 1 \)
   a) \( \beta_{\text{sum}} = 0 \)
   b) For \( k = i + 1, ..., m \)
      i) \( \beta_{k-k-i} = \sum_{r=1}^{l_r} < \tilde{y}_{c,r} (k - i), \tilde{y}_{c,r} (k) > \)
      ii) \( \beta_{\text{sum}} = \beta_{\text{sum}} + \beta_{k-k-i} \)
   c) \( r_i = | \beta_{\text{sum}} |, \theta_i = \angle \beta_{\text{sum}} \)
4) \( \theta_i = \text{unwrap} (\theta_i), i = 1, ..., m - 1 \)
5) \( \alpha = \sum_{i=1}^{m-1} i r_i \theta_i \)
6) \( \hat{f}_b = \frac{2 \pi t_b l_i}{\alpha} \)
7) **Output:** \( \hat{f}_b \).

**B. CRLB: PERFORMANCE GUIDELINE**

The CRLB of a random variable \( \lambda \) is
\[
\sigma_{\lambda}^2 \geq \frac{1}{I_{\hat{\lambda}}(\lambda)} (26) \]
where \( \hat{\lambda} \) is the received signal and \( I_{\hat{\lambda}}(\lambda) \) is the Fisher information matrix with \( \lambda \) to be estimated. \( I_{\hat{\lambda}}(\lambda) = \mathbb{E} \left[ \frac{d^2 \ln p(\hat{\lambda})}{\lambda^2} \right] = \mathbb{E} \left[ \sigma_{\hat{\lambda}}^2 \right] \) where \( f \) is the log likelihood function. Consider a channel with circularly symmetric complex
Gaussian random noise, the received signal observed during an interval $nt_b$ can be expressed as

$$\hat{y}(k) = \hat{x}(k) + \hat{n}(k)$$  \hspace{1cm} (27)

In (27), $\hat{x}(k)$ is the information-bearing signal with frequency offset and $\hat{n}(k)$ is the additive white Gaussian noise with double sided power spectral density $2N_0$. Then the Fisher information matrix can be written as

$$\mathcal{I}_{\hat{y}(k)\hat{\lambda}} = -E \left[ \frac{2}{N_0} \sum_k \left( \hat{y}(k) - \hat{x}(k) \right) \frac{\partial^2 \hat{x}}{\partial \lambda^2} - \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right]$$  \hspace{1cm} (28)

Consider that the noise is AWGN, we have $E \left[ \hat{y}(k) - \bar{y}(k) \right] = 0$, therefore the Fisher information matrix in (28) is

$$\mathcal{I}_{\hat{y}(k)\hat{\lambda}} = \frac{2}{N_0} E \left[ \sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right]$$  \hspace{1cm} (29)

Since we are seeking the CRLB of frequency offset and consider a MIMO case in which an has $l_t$ transmitter antennas and $l_r$ receiver antennas, let $\lambda$ to be the frequency offset then we have [29]

$$\sigma_f^2 \geq \frac{N_0}{2 \sum_{r=1}^{l_r} E \left[ \sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right]}$$  \hspace{1cm} (30)

Under MIMO case, suppose the observation and estimation process starts at $t_0$, the received signal at the $r$-th receiver antenna with frequency offset is (as also shown in 5, Section II)

$$y_r(k) = x_r(k) + n_r(k) = \sum_{k=1}^{l_t} h_{r,t,k} s_{t,k} e^{i(2\pi f_k t_b - t_0)} + n_r$$  \hspace{1cm} (31)

Assume that the channel on a transmission link is invariant during the observation time $nt_b$ and has a Gaussian distribution with variance $\sigma_{h_r,t}^2$, we have

$$x_r(k) = \sum_{t=1}^{l_t} h_{r,t} s_{t,k} e^{i(2\pi f_k t_b - t_0)}$$  \hspace{1cm} (32)

Using (32) it is found that

$$\sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 = 4\pi^2 \sum_s \sum_k h_{r,t} s_{t,k} \left( k t_b - t_0 \right)^2$$  \hspace{1cm} (33)

Since the channel is independent from pilot symbols, we have

$$E \left[ \sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right] = E \left[ h_{r,t}^2 \right] E \left[ \left( \sum_k s_{t,k} (k t_b - t_0)^2 \right)^2 \right]$$  \hspace{1cm} (34)

Suppose that the pilot for fine frequency estimation has $n$ symbols and is uniformly distributed on the constellation points, also according to Section II and (34), we can easily know that

$$E \left[ h_{r,t}^2 \right] = \sigma_{h_r,t}^2$$  \hspace{1cm} (35)

$$E \left[ \left( \sum_k s_{t,k} (k t_b - t_0)^2 \right)^2 \right] = E \left[ \left( \sum_k (k t_b - t_0)^2 \right)^2 \right]$$  \hspace{1cm} (36)

Note that the symbol is fixed where the expectation of its square is symbol energy. Substituting (36) and (35) into (33) yields

$$E \left[ \sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right] = 4\pi^2 \sum_t \sigma_{h_r,t}^2 E \left[ \left( \sum_k (k t_b - t_0)^2 \right)^2 \right]$$  \hspace{1cm} (37)

Since CRLB is a lower bound, we are interested in the maximum achievable value of the bound, which can be obtained by choosing $t_0$ as the midpoint of observation time where we have $t_0 = \frac{n t_b}{2}$. Then with the summation in (37), we have

$$E \left[ \sum_k \left( \frac{\partial \hat{x}}{\partial \lambda} \right)^2 \right] = 4\pi^2 \sum_t \sigma_{h_r,t}^2 E f \left[ \sum_k (k t_b - t_0)^2 \right]$$  \hspace{1cm} (38)

Plug (38) into (30), we eventually obtain

$$\sigma_f^2 \geq \frac{N_0}{2 \sum_r \pi^2 \left( \pi^2 \sum_t \sigma_{h_r,t}^2 \right) \left( \sum_k (k t_b - t_0)^2 \right)^2}$$  \hspace{1cm} (39)

where $\sigma_{h_r,t}^2 = \sigma_{h_r,t}^2$, $\forall r, t$ and $\frac{E}{N_0} = \frac{\sigma_s^2}{\sigma_n^2}$ (see also Section II).

### IV. ARCHITECTURE FOR PROPOSED ALGORITHM

Figure 1(a) shows the architecture of proposed method, and Figure 1(b) shows the architecture of method in [8] which is a commonly adopted architecture for searching algorithms. From the figures we can observe that for the architecture of method in [8], each potential CFO candidate requires an extra processing unit while a large number of candidates could consume significant resources. Furthermore, multiple antennas require multiple times of resources for a single antenna architecture. Therefore, it is hard to find the balance between hardware consumption and searching accuracy for the method in [8]. To limit the resource consumption, the proposed method avoids searching techniques so that the resource consumption is only related to the number of receivers. From 1(a) it is clear that when pilot length changes, the overall architecture keeps the same as long as the memories that store unscrambled received signal are not overflowed. In the proposed architecture, the pilot length is an coefficient for control logic that tells what to do at specific states. Therefore, the architecture is reconfigurable to different pilot lengths. To accomodate frames with various structures, frame lengths and pilot lengths, only the pilot length and corresponding valid signal are required for proposed architecture to reconfigure the FSM.
A. MEMORY BASED SAMPLE CORRELATION

Take a glance at (17) and (25), we find that sample correlation $\beta$ is an important step when approaching the final solution. Considering that each unscrambled received pilot symbol/sample may appear more than once during the process, we allocate a dual port block memory or two single port block memories for each antenna to store the received pilot. After receiving the pilot signal, multiplication with the conjugate pilot signal is performed to acquire the unscrambled received signal $\vec{y}_{c,r}$ for each antenna, and then the unscrambled received signal is stored in the memory bank, waiting to be read for further operations. After all pilot symbols are received, a finite state machine (FSM) controlled accumulator works on preparing the sample correlation for next steps. Figure 2 shows the architecture of memory based sample correlation. The control signals consists of valid signal for incoming data and memory read and write address indices along with the valid signal. Once the incoming data is valid, it is unscrambled with pilot symbols and then stored in specific entries of the BRAMs. Once the unscrambled pilot signals are stored, the FSM controlled accumulator accumulates the sample correlation. The states transitions of FSM during the accumulation process is controlled by the indices from control logic.

B. SAMPLE BASED PHASE UNWRAPPING

From Figure 1(a), it is easy to observe that the output of memory based sample correlation is then turned into magnitude and angle. The constantly increasing or decreasing angle reflects the impact of frequency offset, but the complex-to-angle module output always stays within the range $(-\pi, +\pi)$. To solve this issue, we design a state controller that tracks the phase change and compensates for the phase increment. Every time the angles of two consecutive output cross $\pi$ or $-\pi$, the coefficient for compensation increase or decrease by 1. Therefore, we can always know how much we need to compensate to the angle. After the phase unwrapping, parameters are delivered for CFO calculation, which contains only arithmetic operations, as shown exactly the same in (25). The architecture of implemented phase unwrapping is shown in Figure 3. Different from frame based phase unwrapping which takes a whole frame of symbols as input and unwraps the phase, sample based phase unwrapping is uses delay-and-compare to find out whether the phase exceeds detection range, performs phase unwrapping accordingly, and therefore has shorter processing latency.

C. CFO CALCULATION WITH ACCUMULATORS

For the CFO estimate shown in (25), it is a weighted average of $\theta_i$. However, instead of keeping all calculated weights and doing average which may causes longer processing delay
higher resource consumption, two self-defined accumulators are implemented to realize the weighted average of $\theta_i$. And also, noticing that both the numerator and denominator in (25) have the specific term $i^3 r_i$, the resources can be further optimized. Figure 4 shows the architecture of CFO calculation with the accumulators. When $\beta_{sum}$ (definition given in Algorithm 1) and corresponding index $i$ are provided, the elements of numerator and denominator are calculated and accumulated, until the index value increases to a certain point. The coefficient $\frac{1}{\pi \times 100}$ is a fixed constant as long as the properties of the communication system do not change.

D. PIPELINE AND THROUGHPUT

In spite of unscrambled pilot read and write, the architecture of proposed method is pipelined. Figure 5 sketches the timing series and processing latency of proposed architecture. The latency of an entire estimation process consists of 6 parts: unscrambled pilot write and read ($T_W$ and $T_R$), sample correlation ($T_S$), coordinate rotation digital computer (CORDIC) based complex to magnitude and angle conversion ($T_{CO}$), phase unwrapping ($T_P$) and CFO computation ($T_C$). In Figure 5, the read process starts after all unscrambled pilot symbols are written in the BRAMs, while other modules start immediately when the data comes out of memories, taking the advantage of pipeline architecture. Therefore, the processing latency of an estimation process is $T_{single} = T_W + T_R + T_S + T_{CO} + T_P + T_C$. The number of cycles for each stage in the estimation process is shown in Table 2. Note that $n$ and $l_i$ are length of pilot and number of transmit antennas, as described in Section II. Consider a burst mode transmission, where the burst data is first loaded to buffer after reception, and the estimation process starts with full speed, as shown in Figure 5. Therefore, suppose there are $N$ frames that require CFO estimations, the achievable throughput $S$ of the proposed architecture can be expressed with processing latency:

$$S = \frac{1}{N(T_W + T_R + T_S + T_{CO} + T_P + T_C)}$$

V. SIMULATION AND IMPLEMENTATION RESULTS

In this section, simulations are conducted to analyze the performance of proposed method. The relationships between normalized MSE and SNR, as well as pilot length, are discussed. Besides, the acquisition range is also tested. Simulations of other synchronization algorithms are also conducted under same conditions to compare with. This section also quantifies the hardware resource consumption for an FPGA-based implementation, showing that the low hardware resource utilization of proposed architecture makes it accessible for a number of target devices, especially for FPGA-based software defined radio (SDR) where the vacant hardware resource could be distributed to other essential parts in an entire communication system.

A. SIMULATION RESULTS

We consider a general 2$\times$2 MIMO system with random complex Gaussian channel with unit variance and i.i.d noise-plus-interference with unit variance as the scenario. To acquire accurate simulation results, we conduct Monte-Carlo simulation and gather the mean square error (MSE) over the simulations. With these given parameters, CRLBs with different SNRs are calculated to compare with simulation results as the performance guideline. For convenience, we normalize the CFO, where the normalized CFO is set to be a proportion to baseband bandwidth $\frac{f}{f_0}$. Simulations of our algorithm and those in [8], [11], and [16] are conducted and compared under same conditions.

1) SNR AND PILOT LENGTH

Figure 6 shows the relationship between SNR and normalized MSE of CFO estimation with different pilot lengths. To emphasize the influence of SNR, for each given pilot length, simulations are conducted with the normalized CFO held at 1% of the bandwidth, in a 2$\times$2 MIMO system. As the SNR increases, the MSE for each pilot length drops fast between $-5$dB and 0 dB, and then keeps approaching corresponding CRLB. At the same time, Figure 6 reveals that at each time the pilot length is doubled, both the MSE and corresponding CRLB move around 1 dB drops, which matches the derivation in (39).

![Figure 4. Architecture of CFO calculation.](image)

![Figure 5. Process of pipeline architecture.](image)

<p>| TABLE 2. Number of cycles for stages in estimation process. |
|-------------|-------------|-------------|-------------|-------------|-------------|</p>
<table>
<thead>
<tr>
<th>Cycles</th>
<th>$T_W$</th>
<th>$T_R$</th>
<th>$T_S$</th>
<th>$T_{CO}$</th>
<th>$T_P$</th>
<th>$T_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n + 2$</td>
<td>$\frac{n^2-m n l_i}{2 l_i}$</td>
<td>$4$</td>
<td>$7$</td>
<td>$2$</td>
<td>$4$</td>
<td></td>
</tr>
</tbody>
</table>
2) ACQUISITION RANGE

Figure 7 shows the acquisition range of different CFO estimation methods. Since the acquisition range of searching methods is also confined by the searching range, we exclude estimators with searching techniques from this comparison and compare the proposed method with those in [11], [12], and [16] which are also closed form estimators. For all four methods, we conduct simulations under the same conditions: pilot with 32 symbols, 20 dB SNR, and a $2 \times 2$ MIMO system for the proposed method and the method in [16], SISO system for the methods in [11] and [12] which are dedicated for SISO. From Figure 7 we notice that by avoiding phase unwrapping, the acquisition range of CFO estimation techniques in [11], [12], and [16] are limited, while the MSE of the method in [12] is stable within the acquisition range. If we recognize normalized MSE at $10^{-4}$ as a threshold for successful CFO estimations, the maximum detectable normalized CFOs are 25%, 3%, 3%, and 2%, respectively for the proposed method, Chen’s method, Mehrpouyan’s method, and Kuo’s method. On the other hand, MIMO with different number of antennas using proposed method are also tested. From Figure 7 we can observe that due to the scrambled periodic matrix $\mathbf{O}_{1,m}$ in (10), the acquisition range is closely related to the number of transmit antennas where every time the number of transmit antennas is doubled, the acquisition range reduced by one half.

3) MEAN SQUARE ERROR

To examine the performance, we conduct simulations of different CFO estimation methods over a wide range of SNRs and obtain their normalized MSE to compare proposed method with others. To make the comparison more realistic, we extend the CFO estimator for SISO system in [8] to an estimator for MIMO system, so that the comparison could be made under same conditions. However, the methods in [11] and [12] are designed for SISO systems. For the test cases, pilot length is 32 symbols, normalized CFO is 1% and all simulations run with a $2 \times 2$ MIMO system except for the methods in [11] and [12] which runs in a SISO system where all system settings are the same except for the number of antennas. For the MLE in [8] which searches among a number of potential CFO candidates, the searching range lays between $-30\%$ and $30\%$, and the interval between adjacent candidates is $0.3\%$. The performance comparison of proposed method with the methods in [8], [11], [12], and [16] is shown in Figure 8. It can be seen that the proposed method holds a competitive performance when the SNR is low, and the best performance when SNR goes higher. The linear approximation employed in [11] results in a degradation in the performance where the quantization error plays a vital role. At low SNRs ($\leq 10$ dB), the performance of proposed method is competitive with the method in [16]. But the normalized MSE of methods in [11] and [16] almost stop decreasing when SNR goes higher than 20 dB. Besides, the normalized MSE of the method in [8] is way higher than that of other methods, which is led by the fact that the searching grid is not fine enough. Besides $2 \times 2$ MIMO, the proposed method is also tested with $4 \times 2$ and $4 \times 4$ MIMO systems and the results verifies that the number of receive antennas has influence on the MSE (more receive antennas, lower MSE) while the number of transmit antennas does not, as shown in (39).
4) BIT ERROR RATE

With the CFO estimates, the communication systems compensate the CFOs and recover the distorted signals so that the transmitted data can be correctly revealed. Therefore, we simulate an entire communication system with other impairments corrected, add the CFO synchronization methods and perform CFO synchronization to examine the BER performance. The BER comparison of different CFO estimation methods is shown in Figure 9. The setup of communication system is the same as that described in Section V-A.3. Each transmitted frame consists of 32 pilot symbols and 288 payload symbols where the symbols are QPSK modulated. From Figure 9 we can observe the same trend shown in Figure 8. There is a noticeable case that the BER performance of the method in [8] stays on the same level throughout the entire SNR range. This is caused by the quantization error of grid search and therefore we conclude that the grid search without high resolution may not be suitable for fine frequency estimation.

B. IMPLEMENTATION RESULTS

The circuits are synthesized and implemented using Xilinx Vivado 2017.2, targeting one of the popular FPGA development boards Xilinx Zynq XC7Z020 ZedBoard. The implementation results in terms of flip-flops (FFs), lookup tables (LUTs), DSP blocks, block RAMs (BRAMS) and achievable frequency are reported in Table 3.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>FFs</th>
<th>LUTs</th>
<th>DSPs</th>
<th>BRAMs</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop_2rec</td>
<td>1640</td>
<td>2638</td>
<td>13</td>
<td>2</td>
<td>125.8MHz</td>
</tr>
<tr>
<td>Prop_2recS</td>
<td>1640</td>
<td>2638</td>
<td>13</td>
<td>2</td>
<td>90.9MHz</td>
</tr>
<tr>
<td>Prop_4rec</td>
<td>1492</td>
<td>2793</td>
<td>16</td>
<td>4</td>
<td>125MHz</td>
</tr>
<tr>
<td>Search_10can</td>
<td>35280</td>
<td>18550</td>
<td>70</td>
<td>10</td>
<td>83.3MHz</td>
</tr>
</tbody>
</table>

Due to the fact that there is a lack of published ML based frequency synchronization implementation techniques, we extend the method in [8] to MIMO case and implement the corresponding architecture shown in Figure 1(b), in the case of a 2 × 2 MIMO system, for the purpose of comparison.

Search_10can refers to the architecture we implemented for the method in [8] with 10 potential CFO candidates. As is shown in Table 3, Search_10can shows significant resource utilization compared with proposed architectures Prop_2rec and Prop_4rec. Unfortunately, it is hard to find a balance between hardware resource consumption and estimation accuracy for Search_10can. Even though additional CFO estimation candidates result in smaller searching interval (assume same searching range) which leads to smaller quantization error, these additional candidates bring about higher hardware consumption, potentially exceeding the available amount.

Prop_2rec and Prop_4rec refer to the architecture of proposed method, with 2 receiver antennas and 4 receiver antennas respectively. According to Figure 4, there is extra hardware consumption brought by additional receiver antennas. However, the extra hardware resource consumption mainly consists of BRAMS for pilot storage and DSPs for received signal aggregation. The proposed architecture consumes only a trivial portion of available resources, and the rest can be allocated for other parts of an entire communication system. Or, on the other hand, the proposed architecture can be easily fitted in small FPGAs such as Xilinx Spartan-7 XC7S50CS (Prop_2recS). It is also worth noticing that the achievable frequencies of proposed architectures are around 40MHz higher than that of Search_10can. When CFO estimation unit is connected to other units in a communication system, lower achievable frequency may becomes the bottleneck of achieving high processing speed.

The maximum circuit frequency, reported as post-route, is 125.8MHz for proposed architecture with 2 receiver antennas implemented on ZedBoard. During our tests, pilot-only frames are employed and each pilot-only frame has 32 QPSK symbols. Assume that the frames are received in burst mode, as discussed in Section IV-D, the proposed architecture can accommodate symbol rate up to 53.08Mbps, comfortably exceeds the requirements for MIMO communication systems under most conditions.

VI. CONCLUSION

This paper has investigated CFO estimation in MIMO communication systems. An ML based CFO estimation method is proposed with closed form MLE and close-to-bound performance. Since CFO estimation has significant influences on communication quality as well as the complexity of a MIMO synchronizer design, this paper is important for MIMO systems where CFO estimation accuracy and time consumption of CFO estimation are sensitive. Accurate CFO estimation can also allow for a relaxation of analog RF constraints at the radio front end, potentially resulting in a reduced implementation cost. An efficient architecture of our algorithm
is also proposed in this paper. The proposed architecture is reconfigurable to different pilot lengths, and therefore can accommodate various frame structures without changing the architecture. Meanwhile, by avoiding searching techniques, the hardware resource consumption of proposed architecture is reduced hugely and is only related to the number of receiver antennas.

The proposed algorithm and architecture have been evaluated, both in terms of simulation and post-place-and-route implementation. The simulations for performance analysis show that the proposed algorithm is asymptotically optimal, where the MSE of proposed method approaches its CRLB especially when SNR is high. On the other hand, implementation results show that the proposed architecture consumes only a trivial amount of hardware resources, and therefore can be easily fitted into small FPGAs. The achievable frequency, at the same time, allows the system to approach high data rate.

REFERENCES

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