Dynamic relay deployment for disaster area wireless networks

Wenxuan Guo1, Xinming Huang1∗,† and Youjian Liu2
1Department of Electrical and Computer Engineering, Worcester Polytechnic Institute, Worcester, MA 01609, U.S.A.
2Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309, U.S.A.

Summary

This paper investigates the disaster area communication system using relay-assisted wireless network for first responders as mobile nodes (MNs). Firstly, a novel mobility model is proposed to describe the movement pattern of MNs within a large disaster area. Secondly, we study the relay management problem of finding a minimum number of relay nodes (RNs) and their dynamic locations to cover all the MNs within the disaster area. A square disk cover (SDC) problem is formulated and three different algorithms, including the two-vertex square covering (TVSC) algorithm, the circle covering algorithm and the binary integer programming (BIP) algorithm, are proposed to solve the SDC problem. Simulation results are presented to validate the mobility model and compare the algorithms with respect to computational complexity and worst case performance ratio. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: dynamic relay deployment; square disk cover; disaster area wireless network; mobility model

1. Introduction

Public safety organizations increasingly rely on wireless technology to provide effective communications during emergency and disaster response operations. However, since any previously installed wireless network infrastructure may be damaged or completely destroyed in a major disaster event, as occurred during Hurricane Katrina, it is necessary to develop wireless networks that can be quickly deployed to build a replacement communication system to connect all first responders. Considering the first responders as mobile nodes (MNs), the communication range of each MN is often limited by its power constraint. Mobile relay nodes (RNs) can be introduced to relay the communications between MNs and base stations. The RNs installed on wheeled vehicles can be dynamically relocated to places where the first responders are actively working in the field. We term such a dynamic communication system as disaster area wireless networks (DAWN).

When maintaining the functionality of DAWN, understanding the mobility model of the MNs is an important task because the network topology highly depends on the model used. Traditional assumptions are: MNs are allowed to move over the whole disaster area; MNs are connected to the backbone network all the time. However, such assumptions do not always
apply in reality. First of all, the assumption that MNs can choose any destination within the disaster area is not valid. For instance, a firefighter walking across a burning forest would endanger his life. Then, does it always apply that the MNs can connect to the backbone network? Again, the answer would be negative in most occasions because MNs have limited transmission range. As rescue teams move further away from the base station, they risk losing connections with the backbone network.

Connectivity is of the greatest importance to MNs in DAWN. Frequently lost connection to the backbone network could decrease disaster relief efficiency, disorder rescue efforts, and even jeopardize first responders’ lives. As we can picture, first responders cannot start their work deep within the disaster area, but at certain places on the boundary and then proceed into the disaster area. Since the transmission range of MNs is often small, to facilitate their communication with the outside of the disaster area, we ought to place RNs near the MNs to establish the wireless network. Note that because the locations of first responders are different at times, a new network topology can be formed by different placement of RNs. To justify this assumption, we can exemplify the movable RNs as vehicles whose communications are always available.

Obviously, many application scenarios fit into our disaster area constraints. For instance, firefighters working within a large ablaze forest together with some RNs would make up a disaster area wireless network. As another example, rescue teams searching for survivors in an earthquake or tsunami area would also be categorized as an application of disaster area wireless network. In general, it is believed that scenarios that include first responders and a large-sized disaster area without any unreachable spots or obstacles inside would abide by our assumptions and thus the moving pattern of first responders can be described by our mobility model.

In this paper, we mainly fulfill two tasks: (1) proposing a novel and practical mobility model for MNs in disaster area and (2) placing minimum number of RNs such that the each MN following the mobility model can connect to at least one RN. For the first task, we describe typical movement pattern of first responders in a disaster area. Since the movement pattern for all the first responders is not random within the disaster area as the random waypoint model [17], and they basically are heading deep into the disaster area from the boundary, we put forward our mobility model for MNs by combining these two traits together. The disaster area are divided into many small square regions (we call them squares in this paper later), each square with a catastrophic intensity (CI) value to show how severe the disaster situation is in that area. The larger the CI value is, the more time and first responders are required to relieve that square. At the beginning, first responders are disseminated into several arbitrarily chosen starting squares on the boundary of the disaster area, and then each time after they finish relieving one square, the first responders in that square are divided into three groups, entering the three adjacent squares based on their CI values. Therefore, we actually observe the mobility pattern of MNs as a square-based moving behavior. Then what is the moving pattern for MNs within each square? We model it by using the waypoint model that gives MNs freedom to displace themselves within the square, which can be justified by our ignorance of the situation in each square. As for the second task, how can we keep all the MNs connected with the backbone network as they move randomly within many squares? Then we need to place a minimum number of mobile RNs, such that every active square can be fully covered by at least one RN, which is called square disk cover (SDC) problem. We bring forward three algorithms in this paper, the two-vertex square covering (TVSC) algorithm, the circle covering algorithm and the binary integer programming (BIP) algorithm. An example of DAWN is described in Figure 1.

The rest of this paper is organized as follows. In Section 2, related works on disaster area networks, mobility model, and coverage in wireless networks are presented. In Section 3, we describe the mobility model of MNs. Section 4 formulates the SDC problem, followed by the algorithms presented in Section 5. Complexity
2. Related Work

2.1. Disaster Area Networks

Research efforts focusing on disaster area networks begin to increase in recent years. In [9], Doumi describes the use of radio frequency spectrum dedicated for public safety communications in the United States. Regarding research efforts focused on first responder networks, many works [3, 8, 25] have discussed incorporating public safety communication wireless technologies into commercial wireless networks, which provides public safety communication terminals with unified network resources to satisfy tactical requirements. In [22], Lu et al. examine two hierarchical network solutions which allow the delivery of mission-critical multimedia data between rescue teams and headquarters over extremely long distances using a combination of wireless network technologies and multimedia software applications to meet the requirements of disaster rescue communication scenarios. However, the mobility model assumes that MNs move randomly within a large disaster area that is supposed to be covered, which does not apply practically. Ad hoc network has also been applied to the scenarios such as disaster area [27] and battle field [29]. However, the topology of ad hoc network is static or very slow in motion, otherwise it would be difficult to maintain the connectivity of all network nodes. According to DAWN, all the MNs (we will use our terminology to describe related works) are randomly moving within busy squares. In [24], Pogkas et al. introduce a sensor network designed for disaster area relief operations. However, the sensors distributed over the disaster area are fixed and are used by survivors to contact first responders. Such kind of network cannot be adopted either, because first responders can not use fixed sensors to contact each other as they are moving all over the disaster area.

2.2. Mobility Model

In recent years, several different mobility models have been proposed and used for performance evaluation of networks. Models like the abstract random waypoint mobility model [17] or Gauss–Markov mobility model [21] describe random-based movement and distribute the nodes over the entire communication area. However, the random movement of the nodes over the entire simulation area does not fit into the characteristics of DAWN. To study more realistic non-equally distributed movement, in [4], Bettstetter and Wagner introduce a mobility model in which the probability that a node selects an attraction point or a point in an attraction area as next destination is larger than the choice of other points. Thus, some nodes visit several spots within the simulation area more frequently than others. This mobility model does not apply to widely spreading disaster area in which disaster intensity is measured in large areas. In [19], Kraaier and Killat divide the simulation area into pixels similar to the squares in our proposed scenario. However, the mobility pattern of MNs within the area follows moving from pixel to pixel according to probabilities. Such a moving pattern do not exactly match the needs for relieving a disaster area as our mobility model does. The reference point group mobility model [15] considers the movement in groups and relative movement inside each group. Disaster area relief might favor such a group-based movement pattern, since first responders work in groups identified by each square. However, the concepts of splitting up and reassembling fail to be introduced to help cover the whole simulation area. Thus this model does not apply to the large scale disaster area. Other works [5, 11] divide the whole simulation area into small subareas and a different mobility model is utilized in each subarea. Therefore, nodes moving between different subareas are subject to changes of moving pattern. Since in a large scale disaster area, usually different small subareas are similar except CI values, thus various kinds of mobility models would only increase the complexity of the model. Furthermore, there are two approaches [2, 23] that describe two event-driven role-based mobility model for disaster area relief applications. However, these two models only apply in small area with specific disaster sites instead of large-sized disaster area.

2.3. Coverage in Wireless Networks

There have been many research efforts dedicated to covering targets using RNs. In [7], a similar problem named as connected dominating set is studied. However, it abides by that the transmission range of MNs and RNs are the same and RNs can only be deployed at the positions of the MNs. In DAWN, the placement of RNs does not have this constraint. In [12], the connected sensor cover problem is studied, which involves placing the minimum number of RNs such that they...
form a connected network, while still covering a specified area. According to DAWN, as MNs are randomly moving within each busy square, we aim to place RNs such that every busy square has to be fully covered by at least one RN to ensure all MNs can access the backbone network. Therefore, the target covering area is divided into several small squares that need to be covered by RNs respectively. There are plenty of work done to study covering a set of specified target points [10,13,16,26,28]. All of these research efforts differ with ours in the goal that is pursued: given the transmission range of MNs, their aim is to cover those specified target points, while RNs are deployed to cover several squares in our work. Furthermore, different from many other research works on the RN placement in wireless networks, the connection between RNs is not an issue in our scenario, since the transmission range of RNs are very large compared to the simulation area.

3. The Disaster Area Mobility Model

In this section, we first describe the characteristics of movements in a large disaster area. Secondly, we propose a disaster area mobility model which represents these characteristics. The notation utilized in this section are provided as follows. $t$ denotes the time counter that records the current time; $CL_t$ denotes the CI values of squares within the disaster area at time $t$; $MN_t$ denotes the distribution of MNs over the disaster area at time $t$; $\Delta$ denotes the change in the distribution of MNs over the disaster area at the next time unit; $si,j$ denotes the number of MNs in $si,j$ at time $t$; $CI_{i,j,t}$ denotes the CI value of $si,j$ at time $t$; $\xi$ denotes the time needed for one MN to reduce one unit of CI; $MN_{i,j,t}$ denotes the number of MNs moving downward at time $t$ from $si,j$; $MN_{i,j,t}^d$ denotes the number of MNs moving leftward at time $t$ from $si,j$; $MN_{i,j,t}^l$ denotes the number of MNs moving rightward at time $t$ from $si,j$; $T$ denotes the time required to clear three adjacent squares. Table I lists the notation used in this paper.

### 3.1. Movements Within a Large Disaster Area

In catastrophic situations, the users that need reliable communication are civil protection forces, such as troops, fire brigades, rescue teams, etc. Faced with a mission of relieving a large scale disaster area, first responders ought to start from a few squares on the boundary of the disaster area. There are two reasons for not to begin relieving in the middle of the disaster area. Firstly, it seems unnecessary and practically difficult to build a passage for first responders to move to the middle of the disaster area. Secondly, it is dangerous for first responders to be encompassed by severe unrelieved surroundings, such as fire and toxicant dissemination.

After justifying the initialization of first responders, herein we describe how they proceed exploring the disaster area. We first divide the whole disaster area into
small squares, each square with a CI value to show how severe the disaster situation is in it. The squares that have never been relieved are called raw squares. \( \xi \) denotes how much CI value one MN can reduce in a unit time. Then \( \xi MN_{i,j,t} \) denotes how much CI value can be reduced for the square \( s_{i,j} \) from time \( t \) to \( t+1 \). However, if the current CI value of the square is less than \( \xi \) times the number of MNs in that square at time \( t \), the CI value will be 0 at time \( t+1 \), as shown in (1). Such squares are named as busy squares. Obviously, raw squares and busy squares are uncleared squares. A square is said to be cleared if the CI value is reduced to 0. First responders in the square will not stop working until the square is cleared. When first responders finish clearing one square, they split up and enter the adjacent uncleared squares. Specifically, the larger number of first responders working in the square and less the CI value of the square is, the fewer first responders are entering that square. In this way, from several initial squares, the first responders can finally clear the whole disaster area as they go deeper and wider.

\[
CI_{i,j,t+1} = \begin{cases} 
CI_{i,j,t} - \xi MN_{i,j,t} & : CI_{i,j,t} > \xi MN_{i,j,t} \\
0 & : CI_{i,j,t} \leq \xi MN_{i,j,t}
\end{cases}
\]

(1)

Now we have described the square-based movement pattern for the MNs, then what about their mobility pattern within each square? As the actual situation in each square is unknown, we presume first responders are moving according to the waypoint model [17] for simplicity concerns: each picks up a random destination within the square and then heads for it. The proposition of MNs’ random movement within a square would render this mobility model easily extended to other kinds of disaster area scenarios.

3.2. Mobility Model for First Responders

In Section 3.1, we discussed the movements of first responders to relieve a large scale disaster area and provide intuition behind. In this section we formalize the mobility model of MNs as in Table II.

From Table II, the function \textit{regroup} is adopted to compute how the MNs split up and enter adjacent squares after they clear a square. The procedure of the function \textit{regroup} goes as follows: first obtain the CI values and number of MNs of the 3 squares adjacent to \( s_{i,j} \), which are \( CI_{i,j-1,t}, CI_{i,j+1,t}, CI_{i+1,j,t} \), and \( MN_{i,j-1,t}, MN_{i,j+1,t}, MN_{i+1,j,t} \) (without loss of generality, we assume that the MNs enter the disaster area from the top boundary and explore downward, leftward and rightward). The number of MNs moving toward an square plus the number of MNs in that destination square should be inversely proportional to the CI value of the square, such that the three adjacent neighboring squares can be cleared at the same time, illustrated as:

\[
\frac{CI_{i,j+1,t}}{MN_{i,j+1,t} + MN^2_{i,j,t}} = \frac{CI_{i,j-1,t}}{MN_{i,j-1,t} + MN^2_{i,j,t}} = \frac{CI_{i+1,j,t}}{MN_{i+1,j,t} + MN^2_{i,j,t}}
\]

(2)

In this case, the time required to clear the three adjacent squares would be:

\[
T = \frac{CI_{i,j+1,t} + CI_{i,j-1,t} + CI_{i+1,j,t}}{\xi \times (MN_{i,j+1,t} + MN_{i,j-1,t} + MN_{i+1,j,t} + MN^2_{i,j,t})}
\]

(3)

Then we compute the change of number of MNs in these squares after the \textit{regroup} function is executed at \( s_{i,j} \) and add the change into the record matrix \( \Delta \).

Note that \( MN^2_{i,j} = MN^2_{i,j} + MN^2_{i,j} + MN^2_{i,j} = MN_{i,j} \).

However, there are several exceptions when MNs do not move as prescribed above:

- **Ignore zero-CI squares**: when an adjacent square is cleared or is about to be cleared at the next time, the first responders will not enter it.
- **Small CI or many MNs**: there are already many MNs working in an adjacent square or the CI value of the adjacent square is small, then this square actually lose some MNs based on (2). Such a condition would

<table>
<thead>
<tr>
<th>Table II. Mobility model for MNs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Divide disaster area into squares</td>
</tr>
<tr>
<td>2 while ( CI_i \neq 0 ) /* uncleared square(s) still exists*/</td>
</tr>
<tr>
<td>3 ( \Delta = 0 )</td>
</tr>
<tr>
<td>4 for each busy square ( s_{i,j} )</td>
</tr>
<tr>
<td>5 if ( CI_{i,j,t} &gt; \xi MN_{i,j,t} ) /* ( s_{i,j} ) cannot be cleared now*/</td>
</tr>
<tr>
<td>6 ( CI_{i,j,t+1} = CI_{i,j,t} - \xi MN_{i,j,t} )</td>
</tr>
<tr>
<td>7 else</td>
</tr>
<tr>
<td>8 ( \Delta = \text{regroup}(\Delta, MN_{i,j}, s_{i,j}, CI_{i,j}) ) /* ( MN_{i,j} ) split up and enter into neighbors of ( s_{i,j} )*/</td>
</tr>
<tr>
<td>9 ( CI_{i,j,t+1} = 0 )</td>
</tr>
<tr>
<td>10 end if</td>
</tr>
<tr>
<td>11 end for</td>
</tr>
<tr>
<td>12 ( MN_{i+1} = MN_i + \Delta )</td>
</tr>
<tr>
<td>13 ( t = t + 1 )</td>
</tr>
<tr>
<td>14 end while</td>
</tr>
</tbody>
</table>
be illustrated as (4). We assume the square considered is the neighbor on the right without loss of generality.

\[ \frac{CI_{i,j+1,t}}{\xi} + CI_{i,j-1,t} + CI_{i+1,j,t} \]

\[ > \frac{CI_{i,j+1,t}}{\xi} \times (MN_{i,j+1,t} + MN_{i,j-1,t} + MN_{i+1,j,t} + MN_{i,j,t}) \]

then only the neighboring squares on the left and beneath are considered in the regroup function.

- **Boundary case:** when the square that has been just cleared is on the boundary of the disaster area, then the number of adjacent squares would be less than 3.

## 4. Problem Formulation

We consider a set of MNs moving within the disaster area following the mobility model mentioned above and assume that a set of RNs has to be deployed near MNs to keep all the MNs connected with the backbone network. We assume that all the MNs have small transmission range \( r \), while the transmission range of RNs is large and RNs can communicate with each other regardless of their positions. The transmission circle of an MN is defined as the boundary of an area in which points are distanced less than \( r \) from the MN. \( MN_i \) can communicate bi-directionally with \( RN_j \) if the distance between them \( d(MN_i, RN_j) \leq r \). In other words, \( MN_i \) is said to be covered by \( RN_j \) if \( RN_j \) is within the transmission range of \( MN_i \).

We also assume that the number of RNs is not bounded, which means more RNs will be deployed if required. However, we will try to minimize the number of RNs to be deployed at different times to cover different distribution of busy squares. At last, it is also assumed that some command center, which controls the placement of RNs, has full knowledge of the distribution of busy squares at each time. We shall define our RN placement problem within the disaster area as follows.

**RN placement problem:** given a set of MNs moving according to the proposed mobility model, place the minimum number of RNs such that

- At all times, \( \forall \) MN \( MN_i \), \( \exists \) RN \( RN_j \) such that \( d(MN_i, RN_j) \leq r \). (Problem SDC)

## 5. Placing RNs for SDC Problem

In this section, we introduced three algorithms, TVSC algorithm, circle covering algorithm and the BIP algorithm, to place minimum number of RNs such that for every MN \( MN_i \), there exists at least one RN \( RN_j \) satisfying \( d(MN_i, RN_j) \leq r \). The first two algorithms are based on greedy strategy [6].

We first provide some notation utilized in this section. We let \( V_{i,j} \) denote the set of four vertices of \( s_{i,j} \). \( C_k \) denotes the circle centered at \( RN_k \) with radius \( r \). A spot \( p \) is said to be covered by \( C_k \) if \( d(p, RN_k) \leq r \), denoted as \( p \in C_k \). A polygon \( G \) is said to be covered by \( RN_k \) if \( \forall p \) within \( G \), \( d(p, RN_k) \leq r \), denoted as \( G \subset C_k \).

Before introducing the algorithms, we first claim Theorem 1 as the primary prerequisite for the algorithms.

**Theorem 1.** Assume a polygon \( G = (V, E) \), where \( V \) and \( E \) denote the set of vertices and edges, respectively. If \( \forall \) a vertex \( v_i \in V \), \( d(v_i, RN_k) \leq r \), then \( G \subset C_k \).

**Proof.** First of all, we need to acknowledge the fact that if \( d(v_i, RN_k) \leq r \), \( d(v_j, RN_k) \leq r \), then \( \forall p \in e(v_i, v_j) \) \( \in v_i, v_j \) denotes the edge connecting \( v_i \) and \( v_j \), \( d(p, RN_k) \leq r \). (It is obviously true since the edge is fully contained in one circle if the two terminals is within the same circle). \( \forall p \in G, \) we have \( p \in e(p_1, p_2) \), where \( p_1 \in e(v_{m1}, v_{n1}) \) and \( p_2 \in e(v_{m2}, v_{n2}) \). Since \( \forall v \in V, d(v, RN_k) \leq r \), then \( d(p_1, RN_k) \leq r \) and \( d(p_2, RN_k) \leq r \). As a result, \( d(p, RN_k) \leq r \), and \( G \subset C_k \).

We now claim that the SDC problem is computationally NP-complete. Apparently, the task of deploying the minimum number of RNs to cover the target squares can be decomposed into two steps: acquiring the set containing all possible positions of RNs; choosing the minimum number of RNs from the set obtained in step 1 to cover the target squares. Step 2 can be more formally restated as: given a universe \( \mathcal{U} \) containing all the squares, a family \( \mathcal{S} \) of subsets of \( \mathcal{U} \), find a subfamily \( \mathcal{C} \subset \mathcal{S} \), such that the union of \( \mathcal{C} \) is \( \mathcal{U} \) and the cardinality of \( \mathcal{C} \) is minimized. The restatement of step 2 is equal to the set covering problem in computer science and complexity theory, which was one of Karp’s 21 problems shown to be NP-complete [18].

## 5.1. TVSC Algorithm

In this section, we will introduce the TVSC algorithm for the SDC problem. We first give the intuition behind the algorithm. Then we will present the TVSC
algorithm in details, followed by the analysis of the algorithm.

5.1.1. Intuition

We first provide the intuition behind the proposed TVSC algorithm. Considering that all the MNs are moving randomly within each square, it is impossible to predict the exact position of each MN at specified time. Thus, we need to cover all the busy squares using the minimum number of ‘disks’, each with radius \( r \). According to Theorem 1, if the four vertices of one square are able to connect to the same RN, then all the MNs within that square are able to communicate with that RN. In other words, to guarantee that \( s_{i,j} \subset C_k \), we just need to make sure that \( V_{i,j} \) are covered by RN \( k \). Therefore, we disregard \( s_{i,j} \) being covered if \( V_{i,j} \) cannot be covered by one RN \( RN_k \) (there might be cases that different parts of \( s_{i,j} \) are contained by different disks, while \( \forall p \in s_{i,j}, \exists C_k, p \in C_k \), shown in Figure 2. In our simulation, such situation is not taken into account. In a situation where a square is partially covered by multiple RNs but no single RN can cover the entire square, handoffs will happen too frequently since MNs travels randomly within the square. It is technically less desirable as it will increase the design complexity of both MNs and RNs).

Hochbaum and Maass introduced a method of covering a set of nodes with minimum number of disks [14]. Their method is based on the idea that each disk should have at least two nodes on its border. This intuition is justified by the fact that two points on the border can determine the position of a disk (there should be two disks determined, one can choose the disk that better serve his interests) given a fixed radius, while having the most probability to cover other nodes. As shown in Figure 3, the circle with dashed line can move right and do not lose any node until two left-most points are on its border, as the circle with the solid line displays. The solid-line circle clearly covers one more node than the dashed line circle does. Inspired by this intuition, we propose the TVSC algorithm below.

5.1.2. Two-vertex square covering algorithm

The TVSC algorithm aims to solve the SDC problem through two steps: acquiring the set of all the possible positions for placing RNs \( A \) and choosing a subset \( S \subset A \) to place RNs such that all the busy squares \( U \) are covered. The TVSC algorithm is presented in Table III. We denote the two centers of the two circles with radius \( r \) and nodes \( v_i \) and \( v_j \) on its border as \( C_{v_i,v_j} \), \( |U| \) denotes the cardinality of the set \( U \).

The algorithm works as follows. The set \( U \) contains, at each stage, the set of remaining uncovered busy squares. The set \( S \) contains the already selected positions for placing RNs. Line 7 is the greedy decision-making step. A position \( C \) is chosen as the placement for one RN that covers as many uncovered squares as possible. \( UC \) denotes the set of squares that are covered by the RN placed at \( C \). After \( C \) is selected, the corresponding covered squares are removed from \( U \), and \( C \) is added into the subset \( S \). When the algorithm terminates, each of the busy squares is covered by at least one RN.

Table III. TVSC algorithm for SDC problem.

<table>
<thead>
<tr>
<th>TVSC((\mathcal{U}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( A \leftarrow \emptyset )</td>
</tr>
<tr>
<td>2 ( S \leftarrow \emptyset )</td>
</tr>
<tr>
<td>3 ( \forall v_i \in V_{r-p}, v_j \in V_{r-q}, 1 \leq p, q \leq</td>
</tr>
<tr>
<td>4 ( \text{if } d(v_i, v_j) \leq 2r )</td>
</tr>
<tr>
<td>5 ( A \leftarrow A \cup C_{v_i,v_j} )</td>
</tr>
<tr>
<td>6 ( \text{while } \mathcal{U} \neq \emptyset )</td>
</tr>
<tr>
<td>7 ( \text{Do select } C \in A \text{ that maximizes }</td>
</tr>
<tr>
<td>8 ( U \leftarrow U - S_C )</td>
</tr>
<tr>
<td>9 ( S \leftarrow S \cup C )</td>
</tr>
<tr>
<td>10 ( \text{return } S )</td>
</tr>
</tbody>
</table>
5.2. Circle Covering Algorithm

5.2.1. Intuition

To begin with, we give the intuition behind the proposed circle covering algorithm. According to Theorem 1, to cover a square with a disk, one ought to place an RN that is within the transmission range of its four vertices. Then we try to figure out the feasible area to place the RN so that it can cover the target square. As can be seen from Figure 4, such a feasible area of one busy square is demarcated by four arcs as part of transmission circles of the four vertices, which approximates to a circle with radius \( r - \sqrt{2} \) (the side length of a square is 1). The inscribed circle is named as the feasible circle. There are four approximate regions between the feasible area and the feasible circle. We then approximately use the feasible circle as the area for placing the RN to cover the square for simplicity concern. Therefore, to satisfy the requirement imposed by the SDC problem, it is necessary to deploy at least one RN in the feasible circle of each busy square (note that there are cases when one RN can be deployed in the overlap parts of feasible circles, then one RN could serve the communications for multiple busy squares). As one can picture, the feasible circles of neighboring squares would intersect, resulting in circle regions demarcated by arcs from several feasible circles corresponding to different squares. An example is shown in Figure 5. Therefore, the solution for the SDC problem when using the circle covering algorithm is a set of such circle regions, each determined by a set of feasible circles.

5.2.2. Circle covering algorithm

With all the feasible circles established, the algorithm to provide the circle covering algorithm is presented in Table IV.

The circle covering algorithm works also based on a greedy strategy. The set \( U \) contains, at each stage, the set of remaining uncovered busy squares. The set \( S \) contains the already selected circle regions for placing RNs. Line 5 is the greedy decision-making step. A circle region \( CR_i \) is chosen for placing one RN that covers as many uncovered squares as possible. After \( CR_i \) is selected, the corresponding covered squares are removed from \( U \), and \( CR_i \) is added into the subset \( S \). When the algorithm terminates, each of the busy squares is covered by at least one RN.

5.3. BIP Algorithm

The BIP algorithm is designed for obtaining the optimal solution to the SDC problem. Firstly, it is required that all possible positions for placing RNs should be

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Table IV. Circle covering algorithm.

<table>
<thead>
<tr>
<th>Circle-covering((U))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( S \leftarrow \emptyset )</td>
</tr>
<tr>
<td>2. Draw feasible circles for all the squares in ( U )</td>
</tr>
<tr>
<td>3. Obtain the set of all circle regions ( CR )</td>
</tr>
<tr>
<td>4. while ( U \neq \emptyset )</td>
</tr>
<tr>
<td>5. Do select ( CR_i ) from ( CR ) that maximize (</td>
</tr>
<tr>
<td>6. ( U \leftarrow U - S CR_i )</td>
</tr>
<tr>
<td>7. ( S \leftarrow R \cup CR_i )</td>
</tr>
<tr>
<td>8. return ( S )</td>
</tr>
</tbody>
</table>

---
obtained, which means no approximation method is allowed. To this end, we follow the first step of TVSC algorithm instead of the circle covering algorithm to obtain all the possible solutions. Then we reformulate the SDC problem as a BIP problem as follows. Let \( \mathcal{P} = \{p_1, p_2, \ldots, p_N\} \) denote the set of all possible positions obtained through executing step 1 of the TVSC algorithm. Let \( x_n \) denote the binary assignment variable with \( x_n = 1 \) indicating that the selection of the \( n \)th position as the placement for one RN and 0 otherwise. Let \( \mathcal{S} = \{s_1, s_2, \ldots, s_M\} \) denote the set of all the busy squares. Construct a \( M \times N \) binary coefficient matrix \( B \) with its element \( b_{m,n} = 1 \) if the RN placed at \( p_n \) can cover the square \( s_m \) and 0 otherwise. Then the SDC problem is restated as (5) to minimize the number of positions selected while keeping all the busy squares covered.

\[
\begin{align*}
\min & \quad \sum_{n=1}^{N} x_n \\
\text{s.t.} & \quad x_n \in \{0, 1\}, 1 \leq n \leq N \\
& \quad \sum_{n=1}^{N} b_{m,n} x_n \geq 1, 1 \leq m \leq M
\end{align*}
\]

The second constraint is to guarantee that each square is covered by at least one RN in the optimal solution.

We can use a linear programming (LP)-based branch-and-bound algorithm to solve the SDC problem. The algorithm searches for an optimal solution to the BIP problem as stated in (5) by solving a series of LP-relaxation problems, in which the binary integer requirement on the variables is replaced by the weaker constraint \( 0 \leq x \leq 1 \). More details on BIP solutions can be referred to [30].

6. Complexity Analysis

We first discuss the complexity as well as the worst case performance ratios of the TVSC algorithm. Assume \( N \) denotes the number of busy squares. Since the number of iterations of the loop on lines 3–5 in Table III is at most \( 2N(N-1) \), and the iterations of the loop on lines 6–9 in Table III would be run \( N \) times. Thus the total computational complexity is \( 8N^2 - N \). Since step 2 of TVSC algorithm is basically a greedy methodology, the TVSC algorithm yields a solution with number of RNs mostly \( H(\max|S_{CR}|, 1 \leq i \leq |S_{CR}|) \) times larger than the optimal one [6], where \( H(d) \) denotes the \( d \)th harmonic number \( H_d = \sum_{i=1}^{d} 1/i \).

Secondly, we will analyze the circle covering algorithm in terms of computational complexity as well as its worst case performance ratios. Assume \( N \) denotes the number of busy squares. Since drawing the feasible circle for each busy square should be done \( N \) times, and the iterations of the loop on lines 5–7 in Table IV would be run at most \( N \) times, the total computational complexity is \( 2N \). When the circle covering algorithm is employed, we lose four approximate regions for placing RNs since the area of the feasible circle is less than that of the feasible area. Then we use \( \beta \) to denote the approximate ratio, which is defined as the ratio of the area of feasible regions plus the area of feasible circle to the area of feasible circle, shown as Equation (6).

\[
\beta = \frac{\pi (r - \frac{\sqrt{3}}{2})^2 + 4\left(\frac{\alpha}{2\pi} \pi r^2 - \frac{\pi (r - \frac{\sqrt{3}}{2})^2}{4} - \frac{1}{2} \sqrt{3} r \sin \frac{\alpha}{2} \times 2\right)}{\pi (r - \frac{\sqrt{3}}{2})^2}
\]

where \( \alpha = \arccos \frac{2\sqrt{2}r - 1}{2r} \) represents the radian of the arc, which is served as one out of four boundaries of the corresponding feasible area for a busy square. \( r \) is the transmission range of MNs. Since the circle covering algorithm is a greedy strategy-based algorithm, which has an upper ratio bound \( H(\max|S_{CR}|, 1 \leq i \leq |S_{CR}|) \) [6], the circle covering algorithm yields a solution with number of RNs that at most \( \beta H(\max|S_{CR}|, 1 \leq i \leq |S_{CR}|) \) times larger than the optimal, where \( \beta \) is defined as the approximate ratio in (6).

Furthermore, the analysis of the BIP algorithm is presented as follows. For the SDC problem, the solution space contains all the combinations of \( N \) variables, each with two values 1 and 0, showing whether the position is selected to place one RN or not. Thus the BIP algorithm could potentially search all \( 2^N \) binary integer vectors, and the execution time for BIP is \( O(2^N) \). \( N \) is the number of variables that need to be specified. In [20], it has been shown that such a BIP problem can be solved using a graph theoretical approach by transforming it into a linear optimal distribution problem in a directed graph, which has a computational complexity of \( O(N^3) \).

Although the BIP algorithm yields the least number of RNs, then is it always beneficial to resort to the BIP algorithm? According to Table V, the computational complexity of the BIP algorithm is much higher than the TVSC algorithm and circle covering algorithm. In
Table V. Comparison of complexity and approximation ratio for three relay coverage algorithms.

<table>
<thead>
<tr>
<th>Algorithms/metrics</th>
<th>Complexity</th>
<th>(\delta)-Approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TVSC</td>
<td>(O(N^2))</td>
<td>(H(\max</td>
</tr>
<tr>
<td>Circle Covering</td>
<td>(O(N))</td>
<td>(\beta H(\max</td>
</tr>
<tr>
<td>BIP</td>
<td>(O(N^3))</td>
<td>1</td>
</tr>
</tbody>
</table>

contrast, the TVSC algorithm and circle covering algorithm yield much less computational complexity, with only a tiny disadvantage in terms of the number of RNs deployed compared with the BIP algorithm. Therefore, in real scenarios when the computational resource is precious and timing is critical, the circle covering algorithm might be the best approach among all three, followed by the TVSC algorithm.

7. Simulation Results

In this section, we present our simulation results. By adopting the BIP algorithm, we first give the RN placement results in a simple 4 × 4 square disaster area at all time periods when the set of busy squares changes, from the beginning until all the squares are cleared. Then in a larger 10 × 10 square disaster area, we compare the performance between these three algorithms, the TVSC algorithm, the circle covering algorithm and the BIP algorithm in terms of number of RNs deployed. All the simulation results are obtained through MATLAB 2007b [1].

7.1. Simulation Example: A Small 4 × 4 Square Disaster Area

We establish a small square disaster area and divide it into 16 squares. The matrix of the CI values imposed on the disaster area is shown as Figure 6. We assume \(\xi = 1\). There are totally 20 MNs starting from \(s_{2,1}\). Suppose the transmission range of MNs is 1.3 unit length.

Figure 7 denotes the solution to the RN placement problem in this 4 × 4 square disaster area, when the BIP algorithm is employed. Each subfigure corresponds to a time period when the set of busy squares remain the same.

7.2. Simulation Example: A Large 10 × 10 Square Disaster Area

7.2.1. Simulation setup

We establish a large 10 × 10 square disaster area. There are totally 100 first responders. We propose two initial placement for first responders: all at \(s_{1,1}\) and evenly distributed at four corners \(s_{1,1}, s_{1,10}, s_{10,1}, s_{10,10}\). Besides, we also provide simulation results when adopting two kinds of distribution of CI values of squares in the disaster area: the CI values of all square are equal to 10; integer CI values are randomly chosen from the interval \([5, 15]\). We also assume \(\xi = 1\).

7.2.2. Results on mobility model

In this section, we present some simulation results on the proposed mobility model. We also study the cases of using different number of first responders and initial placement and their impact on the time required to clear a 10 × 10 disaster area and the average number of busy squares over the entire relief process. In Figure 8, we show how the number of MNs can influence the length of disaster area relief period with different initial placement of MNs and distribution of CI values, based on the proposed mobility model. In Figure 9, we present simulation results in terms of average number of busy squares over the disaster area relief period, when the number of first responders changes from 40 to 400.

From Figure 8, it is clear that as the number of MNs increases from 40 to 280, the length of the disaster area relief period is reduced. However, the curve in Figure 8 goes down more slowly as the number of MNs becomes larger, until it remains unchanged. This phenomenon is due to the mobility model, which renders many of the added first responders useless in speeding the relief process, as the number of first responders might be larger than the CI values in each square. When the number of MNs exceed a large value, the added first responders would not accelerate the relief process, because they are always in busy squares that can be cleared in one unit time. Besides, we can tell that the initial place of...

![Fig. 6. The distribution of CI value over the disaster area.](image)
Fig. 7. Simulation results of the BIP algorithm at different time periods when the busy squares are different. The box-shape objects represent RNs. The squares with heads inside are busy squares. The shaded squares represent the raw squares. The rest squares are cleared ones. (a) From time 1 to time 2 (b) from time 2 to time 6 (c) from time 6 to time 7 (d) from time 7 to time 9 (e) from time 9 to time 10 (f) from time 10 to time 11 (g) from time 11 to time 12 (h) from time 12 to time 13 (i) from time 13 to time 15 (j) from time 15 to time 16 (k) from time 16 to time 17.

First responders have a tremendous impact on the length of disaster area relief period. When the first responders are evenly divided and assembled at four corners of the disaster area, the mobility model yields much less time than the case that first responders are assembled at one corner initially. The reason also lies in the mobility model of first responders: the wider distribution of MNs can lessen the chance that the number of MNs in each busy square is larger than its CI value, thus rendering relief efforts of first responders more efficient. At last, it should be noted that whether the CI values are uniformly or randomly distributed does not matter much. In Figure 9, we present the simulation results in terms of average number of busy squares over the whole disaster relief period. It can be seen that the average number of busy squares is much less when the first responders are placed at the four corners initially compared with the case that they all start from a single square, which is very straightforward. It is also worth pointing out that the average number of busy squares mainly decreases as the number of MNs increases. This is because when the number of MNs is smaller, fewer MNs stay in each busy square during the relief process. Then First responders would spend more time in clearing the squares, and the number of busy squares would keep large for a longer time especially when MNs are widely
Fig. 8. Number of time units needed to ease a 10 × 10 square disaster area with the mobility model proposed, as the number of first responders changes from 40 to 400.

Fig. 9. Average number of busy squares over the disaster area relief period with the mobility model proposed, as the number of first responders changes from 40 to 400.

7.2.3. Deployment results

The total number of first responders is 100. When first responders are assembled at $s_{1,1}$ at initial stage, and
the CI values of all squares are equal to 10, simulation results are presented in Figure 10(a) in terms of the number of RNs deployed with the TVSC algorithm, the circle covering algorithm and the BIP algorithm at each time from the start to the end, when $r = 1.7$; another set of results under the same scenario is presented in Figure 11(a) in terms of the average number of RNs deployed over the whole disaster relief period with respect to different transmission range of MNs changing from 1.5 to 2.5. Similarly, when first responders are assembled at $s_{1,1}$ at initial stage, and the CI values of all squares are randomly chosen between 5 and 15, simulation results are presented in Figures 10(b) and 11(b); when the first responders are initially evenly divided and distributed in $s_{1,1}, s_{1,10}, s_{10,1}, s_{10,10}$, and the CI values of all squares are set to 10, simulation results are presented in Figures 10(c) and 11(c); at last when the first responders are initially evenly divided and assembled in $s_{1,1}, s_{1,10}, s_{10,1}, s_{10,10}$, and the CI values of all squares are randomly chosen between 5 and 15, simulation results are presented in Figures 10(d) and 11(d). From Figure 10(a–d), with all four initializations, we can see that the number of busy squares increases during initial periods, then it decreases after reaching its peak value. It is very straightforward in explaining this trend since during the preliminary periods, first responders expand into adjacent raw squares, and thus the number of busy squares can increase. However, during posterior periods when first responders have cleared approximately half of the squares in disaster area, they keep entering a small number of adjacent raw squares from a larger set of cleared squares. Because all the
busy squares are next to each other, the movement of MNs from one cleared square can turn at most one raw square into a busy square. Therefore, it is obvious that the number of busy squares tends to be decreased within posterior periods. At each time, the number of RNs deployed to cover the busy squares follows the same trend. But it can be seen clearly at each time, the BIP algorithm, yields the least number of RNs to be deployed, coinciding with the fact that it is the optimal approach; the TVSC algorithm yields more number of RNs to be deployed than the BIP algorithm, as it utilizes the greedy strategy. Furthermore, the circle covering algorithm also yields more number of RNs than the optimal solution, because it employs both the greedy strategy and approximation methods.

In Figure 11(a–d), we show how the number of RNs changes as the transmission range of MNs increases with respect to different algorithms and different initial conditions utilized. It is very clear that for all three algorithms, the number of RNs deployed decreases as $r$ increases. Such a result is straightforward in that the enlarged transmission range of MNs will probably render more squares covered by each RN. Thus, one surely needs less number of RNs with a larger $r$ to cover the same set of busy squares. Again, the BIP algorithm is the best among the three in terms of the number of RNs deployed when the transmission range of MNs varies, followed by the TVSC algorithm and the circle covering algorithm.

8. Conclusion

In this paper, we study the topology control of DAWN to facilitate MNs’ communication by deploying a minimum number of RNs dynamically. Firstly, we put forward a novel mobility model that describes the movement of first responders within a large disaster area. Secondly, we formulate the SDC problem and propose three algorithms to solve it, including the TVSC algorithm, the circle covering algorithm and the BIP algorithm. Simulation results demonstrate that based on our proposed mobility model, first responders can eventually clear the disaster area after certain time periods. At each time, RNs only have to cover a number of busy squares. We also investigate carefully into the performance comparison between the TVSC algorithm, the circle covering algorithm, and the BIP algorithm. As the optimal approach, the BIP algorithm yields the deployment of the least number of RNs, while having the highest computational complexity $O(N^3)$; the TVSC algorithm yields the deployment of the second least number of RNs, and consumes much less computational resources in $O(N^2)$; the circle covering algorithm yields the deployment of the most number of RNs, but consuming the least computational resources only in $O(N)$. In practice, the TVSC algorithm and circle covering algorithm might be more preferable because they require much less computational complexity, but yield only a slightly more RNs deployed than the BIP algorithm does. Future work would be dedicated to the state-of-art configuration of RNs to better facilitate communication between MNs and the backbone network in terms of scheduling, resource allocation, modulation schemes, and even MAC and network layer protocols, especially in disaster area scenarios.

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Authors' Biographies

Wenxuan Guo received his B.S. degree in computer science and technology and M.S. degrees in information security from Huazhong University of Science and Technology, Wuhan, China in 2004 and 2007, respectively. He is currently working toward the Ph.D. in Electrical and Computer Engineering at Worcester Polytechnic Institute, MA. His research interest is optimization in wireless networks.

Xinmeng Huang obtained his Ph.D. in Electrical Engineering from Virginia Tech, Blacksburg, VA in 2001. He is currently an Assistant Professor in the Department of Electrical and Computer Engineering at Worcester Polytechnic Institute, Worcester, MA. He previously worked at the wireless advanced technology laboratory, Bell Labs of Lucent Technologies from 2001 to 2003. He was a recipient of DARPA young faculty award in 2007. His research interests are circuits and systems design for embedded computing and wireless communications.

Youjian Liu received his Ph.D. and M.S. degree in Electrical Engineering from The Ohio State University in 2001 and 1998, respectively, M.S. degree in Electronics from Beijing University, China, in 1996, and B.E. degree in Electrical Engineering from Beijing University of Aeronautics and Astronautics, China, in 1993. Since August 2002, he has been an Assistant Professor of Department of Electrical and Computer Engineering, University of Colorado at Boulder, CO. From January 2001 to August 2002, he worked on space-time communications for 3G mobile communication systems as a Member of the Technical Staff in CDMA System Analysis and Algorithms Group, Wireless Advanced Technology Laboratory, Lucent Technologies, Bell Labs Innovations, Whippany, NJ. His current research interests include communications, coding theory, and information theory.