1. (5 points) In class, we outlined a predictor-corrector framework for implementing an implicit Adams–Moulton method of a given order, as follows: At the $n$th step, “predict” a next solution value $y_{n+1}^{AB}$ using an explicit Adams–Bashforth method of the same order, then “correct” to obtain $y_{n+1}^{AM}$ specified by the implicit Adams–Moulton method. As noted in class, this framework allows estimating local error using the difference between the “predicted” and “corrected” values $y_{n+1}^{AB}$ and $y_{n+1}^{AM}$, specifically

$$\text{local error} \approx \frac{C_{AM}}{C_{AM} - C_{AB}} (y_{n+1}^{AM} - y_{n+1}^{AB}),$$

where $C_{AB}$ and $C_{AM}$ are the constants in the leading local error terms for the Adams–Bashforth and Adams–Moulton methods, respectively. Suppose you use fourth-order Adams–Bashforth “predictor” and Adams–Moulton “corrector” methods. What is $C_{AM}/(C_{AM} - C_{AB})$ in this case? You can find expressions for the local errors on the PDF file “Adams Methods Formulas” posted on the “Handouts” page.²

2. (10 points) Recall from Homeworks 1 and 3 the IVP for the damped mechanical oscillator:

$$x'' + Dx' + x = 0, \quad x(0) = x'(0) = 1.$$  

As before, this can be recast as a first-order system $y' = f(t, y), y(0) = y_0$, by setting

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \equiv \begin{pmatrix} x \\ x' \end{pmatrix}, \quad f(t, y) = \begin{pmatrix} y_2 \\ -y_1 - Dy_2 \end{pmatrix}, \quad y_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$  

Use MATLAB’s ode113, which implements an Adams–Bashforth–Moulton predictor-corrector algorithm, to solve this IVP from $t = 0$ to $t = 20$ with $D = .1, 1, 10, 100$. Use odeset to set the RelTol and AbsTol error tolerances to $10^{-8}$ and also to set the

1If the ODE is linear, then the equation determining $y_{n+1}^{AM}$ is a linear system. If this system can be solved using a direct (i.e., non-iterative) linear-algebra method, then the “prediction” step isn’t necessary. In the general nonlinear case, as well as in the linear case when iterative linear-algebra methods are appropriate, the “predicted” value $y_{n+1}^{AB}$ is used as an initial approximate solution in an iterative procedure, which is applied to determine an acceptable approximation of $y_{n+1}^{AM}$.

2These expressions involve derivatives $y^{(k)}(\xi_n)$. The point $\xi_n$ differs in different formulas. However, the difference is usually very small in practice and, for error estimation, can be regarded as negligible.
Jacobian option to the system Jacobian $\partial f/\partial y$. Print out (or write up) and hand in a table showing the values of $D$ in the first column and the corresponding numbers of steps in the second column. Compare your results with those obtained in Homework 3.³

³In Homework 3, I asked you to include numbers of $f$-evaluations, based on ode45 using six $f$-evaluations per step. This isn’t strictly true, since adjustments in the step-size may require additional $f$-evaluations. Things are even murkier for ode113, since the corrector iterations as well as possible step-size adjustments may require an uncertain number of $f$-evaluations. The only sure way to count $f$-evaluations is to put a counter in the routine for evaluating $f$. In MATLAB, this can be done conveniently using global variables, which allow passing evaluation counts back to the main program.