PROBABILITY OF A SHUTOUT IN RACQUETBALL*

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Abstract. The probability of a player winning a shutout in racquetball is calculated as a function of the probability that he wins any particular rally.

To win a game of racquetball it is necessary to win 21 points, and to win a point it is necessary to have the serve. If the loser has zero points at the end of the game, he is said to be shut out. We seek the probability $P$ that player $A$ shuts out player $B$, assuming that $A$ has probability $p$ of winning any particular rally.

Let $P(n)$ be the probability that $A$ wins $n$ points while $B$ wins zero points with $A$ having the serve initially. Then $P(n)$ satisfies the recursion relation

$$P(n) = P(n-1)P(1).$$

Repeated application of (1), or induction on $n$, yields

$$P(n) = [P(1)]^n.$$

To find $P(1)$, we note that $A$ can win one point while $B$ wins zero points in either of two ways. Either $A$ can win the first rally and thus win one point, or he can lose the first rally and thus lose the serve but then win the second rally to regain the serve. After that he is in the same condition he started from, and again he has the probability $P(1)$ of winning one point before $B$ wins any. Thus $P(1)$ satisfies the equation

$$P(1) = p + (1-p)pP(1).$$

Solving (3) yields

$$P(1) = \frac{p}{1 - (1-p)p}.$$

Now (2) and (4) give

$$P(n) = \left(\frac{p}{1 - p + p^2}\right)^n.$$

When $A$ has the serve initially, then the probability that he shuts out $B$ is just $P(21)$, where $P(n)$ is given by (5). However if the initial server is determined by the toss of a fair coin or racquet, then $A$ has probability $1/2$ of having the serve initially. If $B$ has the serve, $A$ must win it and then win 21 points. Thus the probability $P$ of a shutout is

$$P = \frac{1}{2}P(21) + \frac{1}{2}pP(21).$$

Upon combining (5) and (6) we get the final result

$$P = \frac{1 + p}{2} \left(\frac{p}{1 - p + p^2}\right)^{21}.$$
The values of $P$ for a few values of $p$ are:

\[
\begin{array}{ccccccc}
 p & 1. & .9 & .85 & .84 & .842 & .5 \\
 P & 1. & .753 & .534 & .490 & .500 & .0001504 \\
\end{array}
\]

Thus the probability of a shutout is .5 when $p = .842$, which means that on the average $A$ wins 5.33 rallies for each one that $B$ wins. For evenly matched players $p = .5$ and the probability that $A$ shuts out $B$ is only .0001504 or one in 6649 games.

I want to thank Ralph Levine for having proposed this problem, and for his comments on the results.