1. (10 points) Suppose you apply the bisection method to determine an approximate solution $x^*$ of $f(x) \equiv \cos(x) - e^x + 1 = 0$. Suppose the initial interval is $[a, b] = [0, \pi/2]$. How many iterations of the method are required in order to guarantee that the current approximate solution $c$ (the midpoint of the current interval) is within $10^{-10}$ of $x^*$? (Reason this out using the technique illustrated in class; don’t use a computer.)

2. (10 points) In each of the following cases, the given $f$ has a unique zero at $x^* = 0$. In each case, what can you say about the iteration sequence $\{x_n\}$ generated by Newton’s method starting with $x_0 \neq 0$? In particular, for $x_n \neq 0$, determine an expression for $x_{n+1}$ in terms of $x_n$. Then say whether the iterates converge to $x^* = 0$ and, if they do converge, whether they converge quadratically. If they don’t converge quadratically to $x^* = 0$, say why the convergence theorem for Newton’s method given in class can’t be applied, i.e., what assumption in the theorem fails to hold.

a. $f(x) = |x|^{1/2}$

b. $f(x) = x^2$