

Fluidic Variable Inertia Flywheel

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Energy storage is important for many applications from hybrid vehicles to off-peak electric power to rotating machinery. A flywheel offers the combination of high energy density and high power density not attainable with other energy storage medium. In many situations, it is desirable to store energy at a constant angular velocity. This work proposes a novel self-governing fluidic variable inertia flywheel that can maintain a constant angular velocity across a range of energy storage. The fluidic flywheel uses a piston to separate the liquid filled chamber from a chamber vented to atmosphere. A force balance is created on the piston due to the radial pressure gradient of the liquid reacted by a constant force spring. Energy added to the system is stored in equally two forms: increases the kinetic energy of the flywheel at a constant angular velocity and increasing the potential energy of the constant force spring. A design example demonstrates that the fluidic flywheel enables a constant angular velocity with an order of magnitude lower mass moment of inertia than a conventional flywheel. This promising technology enables a simple constant angular velocity energy storage system, yet requires future work in numerous areas.

I. Introduction

NUMEROUS applications utilize flywheels, typically either for purely storing energy or for the purpose of minimizing the angular velocity fluctuation of a shaft. Examples of energy storage applications include flywheel hybrid vehicles, uninterrupted power supplies, cyclic alternative energy sources such as wind turbines, and space power systems. Applications utilizing a flywheel for smoothing angular velocity fluctuations include an internal combustion engine, industrial machinery such as camshafts, and AC generators. In the design of a system with a conventional flywheel used to minimize changes in angular velocity, the flywheel is sized for an allowable coefficient of fluctuation, defined as the change in angular velocity during a cycle divide by the average angular velocity.¹ To achieve a low coefficient of fluctuation, a flywheel with a large moment of inertia is required.

From the equation for the kinetic energy storage of a flywheel, $E = \frac{1}{2} I \omega^2$, where I is the mass moment of inertia and ω is the angular velocity, it can be noted that a change in energy can be accommodated with a change in the angular velocity, as in a conventional flywheel, or through a change in the moment of inertia. Utilizing a variable inertia flywheel theoretically enables a zero coefficient of fluctuation with a smaller and lighter flywheel. Alternatively, a variable inertia flywheel can eliminate the need for a continuously variable transmission between the flywheel and the load. The inertia of a flywheel can be changed in multiple ways including moving mechanical masses, allowing the flywheel material to strain, or adding fluid to the flywheel.

One of the earliest variable inertia flywheels was the flyball governor by James Watt.² Other moving mass variable inertia flywheels include designs using sliding masses on tracks³ and band-type variable inertia flywheels (BVIF)^{4,5}. The BVIF uses a thin metal band wrapped between an inner and outer drum. By changing the angular difference between the inner and outer hubs, the wrappings of the band are transferred between the two drums. To create this angular difference between the two hubs, Ullman and Velkoff propose recirculating power from the flywheel using a planetary gear train.⁵

Instead of moving fixed masses through mechanical means, another method of creating a variable inertia flywheel is to allow the centripetal acceleration of the flywheel create strain in the material. Harrowell presented the concept of using a vulcanized rubber elastomeric flywheel for this very purpose. Due to the non-linear stress-strain relationship of specific elastomers, Harrowell calculated that approximately 80% of the stored energy in the system could be extracted at a nearly constant angular velocity.⁶ Due to the limited strength of elastomeric materials with the required stress-strain behavior, this concept significantly limits the energy density of the flywheel. A

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commonality between Harrowell's work and that presented in this paper is that the angular velocity of the flywheel is self-governing by design.

While multiple works have discussed using a fluid to vary the inertia of a flywheel in passing, little academic research is available. Typical arguments against a fluidic flywheel include stress at the flywheel wall due to fluid pressure, fluid swirl, and hydraulic pumping power, which is discussed in more detail below.⁷ Despite little research literature available, multiple patents have been awarded for flywheels that use a fluid to change the inertia.⁸⁻¹⁰

As will be demonstrated below, and was previously pointed out by Ullman, that the energy required to change the moment of inertia of a flywheel is equal to one half of the energy change of the flywheel.⁵ The energy losses associated with hydraulic pumping at this power level is the primary reason that previous works have suggested that a fluidic variable inertia flywheel is impractical.⁵ The design discussed in this paper avoids these energy losses by using the pressure differential created between the external tank and the radial pressure gradient within the flywheel, which is created by centripetal acceleration, to drive fluid in and out of the flywheel.

II. Approach

A self-governing fluidic variable inertia flywheel is proposed to achieve a constant angular velocity across a band of changing energy storage. The fluidic flywheel, seen in Fig. 1, is a cylindrical vessel with a moving piston that divides the internal volume into two chambers. One chamber, shown on the left of the figure, is vented to atmosphere, while the other chamber is filled with a liquid. The liquid is allowed to move freely between a central port in the flywheel and the tank at atmospheric pressure. Within the flywheel, a constant force spring, shown as a tension spring, applies a force to the piston in the direction of the liquid chamber.

When the flywheel is rotating, the liquid within the flywheel is subjected to centripetal acceleration. This acceleration creates a radial pressure distribution within the fluid. To aid in developing the equations to describe this pressure distribution, a free-body-diagram of an infinitesimal liquid element is provided in Fig. 2

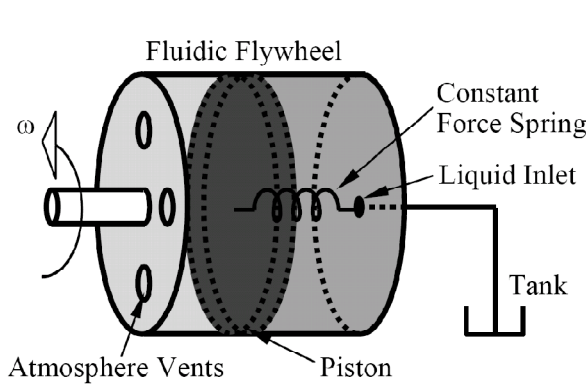


Figure 1. Self-regulating fluidic variable inertia flywheel.

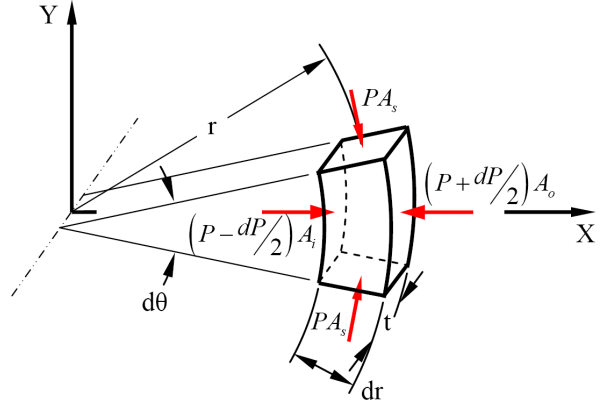


Figure 2. Free body diagram of an infinitesimal element of the liquid subject to centripetal acceleration in the flywheel. This acceleration develops a pressure gradient that varies with the radius.

Starting from Newton's second law, and assuming steady-state operation and a constant density of the liquid, the force balance along the positive X-axis is described by:

$$\sum F_x = ma \quad (1)$$

where the forces are due to the pressure acting on each side of the infinitesimal liquid volume, m is the mass of the fluid volume, and a is the centripetal acceleration of the fluid volume. Note that during transient operation, tangential, coriolis, and sliding acceleration are also present. Substituting the forces on the liquid volume yields:

$$\left(P - \frac{dP}{2} \right) A_i - \left(P + \frac{dP}{2} \right) A_o + 2PA_s \frac{d\theta}{2} = ma \quad (2)$$

where P is the liquid pressure, dP is the change in pressure across the element, $d\theta$ is the arc angle of the volume element, A_i is the area of the volume element on the inner radius, A_o is the area of the liquid element on the outer

radius, and A_s is the area of the top and bottom sides of the element. Note that the sine of a small angle, $\frac{d\theta}{2}$, has been substituted by the angle. Substituting the area of the surfaces into Eq. (2) yields:

$$\left(P - \frac{dP}{2}\right)\left(r - \frac{dr}{2}\right)d\theta \cdot l - \left(P + \frac{dP}{2}\right)\left(r + \frac{dr}{2}\right)d\theta \cdot l + 2Pdr \cdot l \frac{d\theta}{2} = \rho V(-\omega^2 r) \quad (3)$$

where r is the radius to the center of the element, dr is the radial length of the element, l is the thickness of the element, ρ is the mass density of the liquid, V is the volume of the element, and ω is the angular velocity of the flywheel. The volume of the element can be expressed as:

$$V = rd\theta dr \cdot l. \quad (4)$$

Expanding and simplifying Eq. (3) yields:

$$dP = \rho\omega^2 r dr. \quad (5)$$

Taking the integral of both sides:

$$\int_{P_I}^P dP = \int_0^r \rho\omega^2 r dr \quad (6)$$

where P_I is the liquid pressure at the inlet where $r=0$, yields an equation for the pressure of the liquid as a function of the radius:

$$P(r) = \frac{\rho\omega^2 r^2}{2} + P_I \quad (7)$$

As described above, the liquid inlet on the flywheel is directly connected to a tank that is vented to atmosphere. Note that the tank is assumed to be at the same elevation as the flywheel inlet, allowing the influence of gravity to be neglected. Because the inlet liquid pressure is held constant, Eq. (7) shows that a slight increase in the angular velocity will increase the fluid pressure away from the center of the flywheel. The increase in fluid pressure creates an imbalance of force acting on the piston, causing it to move to the left and force more liquid into the flywheel. The addition of liquid to the flywheel increases the mass moment of inertia, causing a decrease in the angular velocity due to conservation of momentum and thus the force on the piston returns to an equilibrium condition. During this process, the energy added to the system is stored in deflecting the constant force spring and increasing the kinetic energy of the flywheel. This self-regulating process results in a constant angular velocity, which is set by the force of the spring.

To aid in developing the equations to describe the force balance on the piston, Fig. 3 is presented. The forces are comprised of the parabolic pressure distribution of the liquid, described by Eq. (7), acting across the surface area of the piston, the force of the constant force spring, and the force created by atmospheric pressure acting on the left side of the piston.

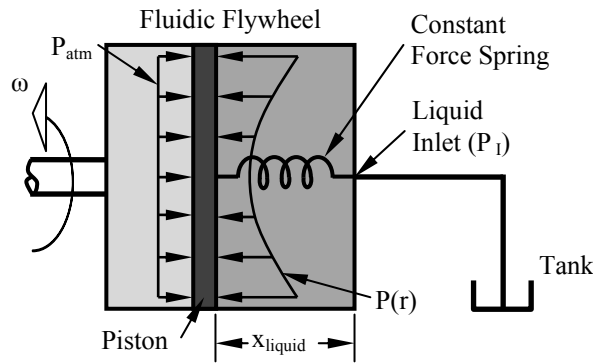


Figure 3. The forces acting on the piston in the flywheel. Note the parabolic liquid pressure distribution due to the centripetal acceleration. The constant force spring is drawn as a tension spring.

The forces on the piston must be equal for the piston to be in equilibrium, as described by:

$$\int P(r) dA = \int_0^{r_o} \left(\frac{\rho\omega^2 r^2}{2} + P_I \right) (2\pi r dr) = F_{spring} + P_{atm} \pi r_o^2 \quad (8)$$

where r_o is the outer radius of the flywheel piston, F_{spring} is the force of the constant force spring, and P_{atm} is atmospheric pressure. Integrating and recognizing that P_I is equal to atmospheric pressure due to the tank being vented to atmosphere, yields the angular velocity of the flywheel during equilibrium for a given spring force, liquid density, and flywheel radius:

$$\omega = \sqrt{\frac{4F_{spring}}{\pi\rho r_o^4}}. \quad (9)$$

It is important to note that the equilibrium of piston, described by Eq. (9), is independent of the position of the piston, or in other words, the inertia of the flywheel. If a slight perturbation is created in the angular velocity, a force imbalance on the piston will occur, which results in movement of the piston and a corresponding change in the moment of inertia. Due to conservation of angular momentum, a change in the moment of inertia will inversely change the angular velocity, re-establishing equilibrium.

The mass moment of inertia of the flywheel includes the inertia of the empty flywheel and the inertia of the liquid in the flywheel. Again, assuming the density of the liquid is constant with radius, the mass moment of inertia is described by:

$$I = I_{flywheel} + I_{liquid} = I_{flywheel} + \frac{mr_o^2}{2} \quad (10)$$

where $I_{flywheel}$ is the inertia of the empty flywheel, I_{liquid} is the inertia of the liquid, and m is the mass of the liquid in the flywheel. Substituting for the mass of the liquid in terms of the density and volume yields:

$$I = I_{flywheel} + \frac{\pi\rho x_{liquid}r_o^4}{2}. \quad (11)$$

where x_{liquid} is the axial length of the liquid volume.

There are two energy storage mediums in the presented fluidic flywheel: the kinetic energy of the flywheel and the potential energy of the constant force spring. As presented earlier, the total energy stored by the flywheel is:

$$E_{flywheel} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(I_{flywheel} + \frac{\pi\rho x_{liquid}r_o^4}{2}\right)\omega^2. \quad (12)$$

Expanding and substituting the equation for the angular velocity during equilibrium, Eq. (9), yields:

$$E_{flywheel} = \frac{1}{2}I_{flywheel}\omega^2 + F_{spring}x_{liquid}. \quad (13)$$

The total energy stored in the constant force spring is simply:

$$E_{spring} = F_{spring}x_{spring} \quad (14)$$

where the displacement of the spring, x_{spring} , is equal to the axial length of the liquid in the flywheel, x_{liquid} .

Energy is added or removed from the system by applying a torque to the shaft of the flywheel. By recognizing that the angular velocity of the flywheel remains constant due to the self-governing behavior, the first term of Eq. (13) remains constant as energy is added or removed from the system. Thus, energy added to the system is stored in increasing the potential energy of the spring, Eq. (14), and increasing the inertia of the flywheel, the second term of Eq. (13). It can be further noted that the change in energy stored in these two mediums is equal and described by the force of the spring multiplied by the axial travel of the piston, which is equal to the displacement of the spring. Described in other terms, for a given energy input to the system, half is stored in increasing the kinetic energy of the flywheel and the other half is stored in increasing the potential energy of the spring.

III. Design Example

To illustrate further the behavior of the constant velocity fluidic flywheel, an example is presented. Consider a portable AC electric generator utilizing an internal combustion engine as a prime mover. To produce 60 Hz AC frequency, the single-cylinder engine must operate continuously at 3600 rpm (377 rad/s). Due to the gas torque and shaking torque of the engine, the engine torque fluctuates significantly over the two full crankshaft revolutions of a four-stroke cycle. By integrating the torque function with respect to the average torque for two full crankshaft revolutions, the maximum change in energy can be found. Following the method described by Norton, the required moment of inertia of a fixed inertia flywheel for a given angular velocity and a desired coefficient of fluctuation is:¹

$$I = \frac{E}{k\omega_{avg}^2} \quad (15)$$

where E is the energy variation associated with the largest torque pulse, k is the coefficient of fluctuation, and ω_{avg} is the average angular velocity of the shaft.

For a 200 cc four-stroke single-cylinder engine operating at 3600 rpm, the energy fluctuation during the maximum torque pulse is approximately 350 J. This calculation combines both the gas torque and the shaking torque as described by Norton.¹ Using a coefficient of fluctuation of 0.02, the required inertia of the flywheel, from Eq. (15), is 0.123 kg·m². If the flywheel is made of a solid steel disc and the radius is set equal to the thickness, the diameter would be 0.2 m and the thickness would be 0.1 m. The flywheel would have a mass of 24.7 kg.

The fixed inertia flywheel could be replaced with a fluidic self-governing variable inertia flywheel to achieve theoretically zero angular velocity fluctuation. As previously discussed, the change in energy of the flywheel system is:

$$\Delta E = 2F_{spring} \Delta x_{spring} \quad (16)$$

By solving Eq. (9) for the spring force and substituting into Eq. (16), the maximum energy variation can be solved for in terms of the geometry, angular velocity, and fluid density. By again setting the radius equal to the length, in this case Δx_{spring} , and using water as the liquid, the diameter of the liquid volume would be 0.138 m and the axial length would be 0.069 m. Additional size would be added for the cylindrical shell of the flywheel and the piston. For reference, the mass of the liquid in the flywheel when full would be 1.0 kg. The cylindrical shell of the flywheel would not need to be very thick as the pressure exerted by the water on the outer radius, from Eq. (7), is only 812 kPa (118 psi). The required spring force for this situation is 2.5 kN.

IV. Discussion and Conclusion

The above example demonstrates that a self-governing fluidic variable inertia flywheel can replace a fixed inertia flywheel. The fluidic VIF is capable of absorbing and releasing the required energy with no change in angular velocity with an order of magnitude lower mass than a steel flywheel with a 2% coefficient of fluctuation. It does need to be noted that the example did not include the design of the fluidic flywheel shell; however, the pressure exerted by the liquid on the outer perimeter is quite low, allowing the use of a relatively thin walled shell.

When the energy in the flywheel is either above or below the constant angular velocity operation zone, determined by being either completely empty or full of liquid, the flywheel acts as a fixed inertia flywheel. As demonstrated in the results of the example, the moment of inertia of the fluidic flywheel when empty or full is significantly less than the fixed inertia flywheel. The result of the lower inertia is that less energy is required to bring the system up to the operating angular velocity, and similarly less energy is required to stop the system, which is important for emergency stop situations.

When operating in the constant angular velocity zone, the energy is stored in increasing the kinetic energy of the flywheel and increasing the potential energy of the constant force spring. The energy stored in both of these forms is equal, meaning that the work done by the spring to change the inertia of the system is equal to half of the input energy. The work done by the spring is analogous to the work performed by a spinning figure skater pulling in their limbs to decrease their moment of inertia to increase the rate of spin.

To create a force on the piston opposing the pressure of the liquid, a constant force spring was suggested. In some situations, conventional constant force spring may not offer sufficient force to achieve the desired angular velocity. Other option to create an approximate constant force is to use gas pressure. A pre-charged gas reservoir could be connected to the left side of the piston to create an opposing force. If the volume of the reservoir were significantly larger than the change in volume of the liquid in the flywheel, the change in gas pressure would be small. Recognize that any change in force would result in a variation in the angular velocity during operation. One benefit of using an external gas reservoir is that changing the gas pressure would allow external variation of the angular velocity.

One challenge that will have to be overcome in the fluidic flywheel is preventing fluid swirl. As a torque is applied to the flywheel, there will be a tendency for rotation of the fluid relative to the shell of the flywheel. The fluid shearing due to fluid swirl would create heat and thus an energy loss. This fluid motion can be minimized by using baffles inside the flywheel. To allow movement of the piston, the baffles could be designed to interlock with the end of the flywheel or have a telescoping design.

The fluidic variable inertia flywheel has numerous applications both as a means to create a constant angular velocity and for purely energy storage. One important application area is energy storage for the electric grid, including off-peak storage and emerging alternative power generation such as wind turbines and solar power. The

ability to store a significant amount of energy at a constant angular velocity is important for these systems and presents an alternative to other options such as compressed air energy storage and batteries. Another important application for the fluidic variable inertia flywheel is hydraulic hybrid vehicles. By using the flywheel to store energy as both a hydraulic accumulator and in rotating kinetic form, the energy density of hydraulic energy storage is greatly increased. Furthermore, by controlling how much energy is stored in gas compression versus rotating kinetic form, the hydraulic system pressure becomes independent of the quantity of energy stored.

Numerous areas of future work exist relating to the fluidic variable inertia flywheel. From a modeling perspective, a dynamic model of the system operation is required to observe the system response to an applied torque. This dynamic model needs to account for inertial forces and flow resistance in the liquid passages that will delay the system response. Further design work is required to quantify energy losses in the system such as aerodynamic drag on the flywheel rotor, bearing drag, and fluid dynamic losses. The system models then need to be applied to the design for specific applications and then validated experimentally.

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