EFFICIENCY OPTIMIZATION FOR SYSTEMS AND COMPONENTS IN MICROWAVE POWER ENGINEERING

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The paper is concerned with one aspect of optimization of microwave thermal processing, namely, with optimization of energy coupling interpreted as a numerical characteristic of system efficiency. Since in computer-aided design coupling can be evaluated through the computed reflections, an optimization scheme is presented that is particularly suitable for minimizing the reflection coefficient in typical systems and elements of microwave power engineering. Based on response surface methodology and the sequential quadratic programming for constrained optimization, the procedure is linked with the full-wave 3-D FDTD electromagnetic simulator QuickWave-3D. Credibility and effectiveness of the method is illustrated by four examples: dimensional optimization is performed for a dry waveguide load, a waveguide T-junction with a partial-height post, a water cylinder in a cavity, and a slotted waveguide-backed radiating element.

INTRODUCTION

Advanced computer simulation of systems and processes of microwave (MW) heating finds now more applications in both R&D and industrial projects and allows the designers to get valuable characteristics of structures prior to constructing physical prototypes (Ehlers and Metaxas, 2001, Wäppling-Raaholt et al., 2002, Eves and Yakovlev, 2002). However, straightforward application of even highly sophisticated computational tools for analysis may not bring explicit recommendations as to how the construction of the system should be changed in order to improve its performance. Engineering practice usually requires solving specific optimization problems.

Computer-aided design (CAD) optimization options are available in a number of electromagnetic (EM) simulators, but they appear to be general-purpose and slowly converging procedures characterized by heavy demand on computer resources, so it is unlikely that they could be easily used in the majority of applied projects.

On the other hand, there are examples of optimization of MW heating structures when analysts choose to deal with some faster and simpler computational kernels, such as methods of moments (see, e.g., Sundberg et al., 1998), which, however, may be considered less robust and powerful compared to Finite Difference Time Domain (FDTD) and Finite Element Methods (FEM) and applicable only to certain classes of systems. This situation dictates the necessity of development of efficient CAD optimization techniques specifically for problems in MW power engineering. Corresponding computer procedures could become convenient and flexible instruments revolutionizing the design of applied microwave heating systems, most of which are still built on the basis of extensive and expensive experimentation.

Modern methods of advanced MW optimization consist of a number of approaches including a series of algorithms based on the space mapping technique (Bandler et al., 1997, 2001) and some other schemes (e.g., Alessandri et al., 1993, 1997, Zhang and Gupta, 2000). These techniques, developed for production-oriented optimization and demonstrating excellent performance for particular MW and millimeter-wave structures (filters, antennas, MMIC elements, etc.), are largely based on the detailed physics-EM models and specific optimization goals of the considered devices. The use of these refined methods in design of typical systems of microwave power

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engineering has never been attempted and would likely require them to be substantially modified and adjusted.

Another issue here is associated with the numerical methods backing optimization techniques. Recently it has become apparent that interfacing efficient EM simulations with optimization systems for direct application of powerful optimizers is a promising and growing design technology developed towards future integrated CAD systems (Bandler, 1997, Bandler et al., 2002). However, the work done so far in this area does not seem to be directly relevant to the specific needs of microwave heating. In literature one may find references to the procedures linking special optimizers with HP’s HFSS (Jain and Onno, 1997), HP-Eesof’s Touchstone (Bandler et al., 1997), Sonnet’s em (Bandler et al., 1997, Bandler et al., 2002, Ye and Mansour, 1997, Jain and Onno, 1997), Ansoft’s Maxwell SI Eminence (Jain and Onno, 1997), and CST’s MAFIA (Zhang and Weiland, 1997). In the meantime, the recent study (Yakovlev, 2001, 2004) has revealed the advanced capabilities of the two full-wave 3-D EM simulators, namely, of CST’s Microwave Studio and QWED’s QuickWave-3D in modeling reflections, electric field, dissipated power, and energy coupling in electrically large scenarios. The conformal FDTD software QuickWave-3D (2002) (QW3D) appears to be particularly convenient here due to a presence of a number of specific extensions and functions (such as field envelopes, the basic heating module, and others) beneficial for the field. Another advantage of interfacing efficient optimizers with a general solver comes from the fact that, unlike techniques based on the physical/EM models of particular devices, this approach should constitute a universal optimization procedure applicable to any structure which could be modeled with the use of this solver.

Therefore, what would be helpful in designing of variety of applied MW heating systems is an efficient optimization procedure working together with QW3D as a supporting computational kernel and focused on the issues of particular significance for the microwave power engineering. Clearly that development of a relevant algorithm and corresponding computer procedure appears to be of utmost importance for the field.

In the meantime, although optimization of MW thermal processing was addressed earlier in literature (Ryynanen and Ohlsson, 1996, Sundberg et al., 1998, Saa et al., 1998, Banga et al., 1999, Lee et al., 2002, Lurie and Yakovlev, 2002), it seems this concept still requires clarification and specifications. While possible approaches to controlling the uneven heating may include special findings in the applicator design, manipulation of the heat cycle, ingredient formulation, design of the package, and other means, in this paper we deal with parameters computable in the EM simulations.

For majority of MW heating applications, we identify the following two important characteristics mostly deserving optimization:

(a) reflections from the cavity, and
(b) pattern of heating.

The second issue has been a key subject of concern since the first steps of the MW heating technology (Thuery, 1992, Meredith, 1998). Uniformity of heating was directly addressed by Banga et al. (1999) in the framework of a model-based optimization. The complexity of the problem of CAD optimization of the heating pattern still seems to be prohibitively high for the methods and resources of modern computational electromagnetics. From a mathematical viewpoint, principal issues emerge already at the initial stage since the problem in fact allows multiple formulations; different uniformity measures have been recently reviewed by Bradshaw et al. (2003). Although there may be opportunities for progress in the future (e.g., availability of the mentioned advanced computational tools and quickly maturing productivity of hardware might be valuable preconditions for specialized research in this area), for now we leave the heating pattern out of optimization effort.

In the meantime, parameter (a) is also very important. The use of time-domain simulation in design of practical systems has recently made it evident that the efficiency of an applicator can be controlled when one can compute a frequency characteristic of the magnitude of the reflection coefficient |S| in the range adjacent to the operating frequency f. Indeed, in accordance with the approximate yet quite accurate formula for a one-port device, the coupling C (typically defined as a percentage of the source energy absorbed by the load) at f, can be found as:

\[ C \sim (1 - |S|^2 f^2)100\% \]  

and thus give the designer a good estimate of the energy absorbed in the cavity (i.e., not reflected back to the source). When the design of the system is supported by a simulator capable of computing the mentioned characteristic, and when the value of |S| at f turned out to be fairly high, the construction of the applicator (e.g., its geometry, position/orientation of the feed, etc.) in many cases can be properly changed towards dramatic decrease of reflections. The feasibility of such an approach was shown by Cresko and Yakovlev (2003): their study has illustrated that the coupling can be interpreted a numerical characteristic of system efficiency.

Responding to the circumstances outlined above, this paper addresses the problem of efficiency optimization via appropriate optimization of frequency characteristic of the reflection coefficient. Below we present a procedure working jointly with QW3D and particularly efficient for systems and components in MW power engineering.
In the terms of Jain and Onno (1997) we deal with dimensional (non-structural) optimization of the form of the $|S_{11}|$-versus-$f$ characteristic which is a function of geometrical parameters of the system. We outline an original deterministic optimization algorithm, consider quality of its performance, and describe examples of application of the corresponding computer procedure to a few practically meaningful elements and systems of MW power engineering.

CONCEPTUAL CONSIDERATION

Further analysis suggests that efficiency optimization would be improved if it is accomplished by minimization of $|S_{11}|$ not only at $f_0$, but also in its neighborhood. Since magnetrons, the traditional sources of energy in MW heating applications, usually generate not single-frequency signals but narrow-band frequency spectra covering some ranges adjacent to the operating frequency (illustrative examples of corresponding measurements can be seen in Chan and Reader (2000)), it would be clearly feasible to expect a system to possess low reflections at frequencies surrounding $f_0$.

This means that the problem of efficiency optimization can be formulated as follows:

**Statement A:**

Find a configuration of the structure such that the reflection coefficient $|S_{11}|$ is of less than the assigned level $S_0$ in the frequency range $(f_1, f_2)$ around the operating frequency $f_0$ ($f_1 < f_0 < f_2$).

Here we consider $|S_{11}|$ as a multivariable objective function of frequency $f$ (i.e., an independent variable) and system parameters (e.g., geometrical characteristics) $y_1, \ldots, y_n$ (i.e., design variables) such that

$$|S_{11}| = S(f, y_1, \ldots, y_n) = S(X_1, X_2, \ldots, X_{n+1})$$  \hspace{1cm} (2)

Values of $S_0$, $f_1$, and $f_2$ in Statement A can be interpreted as the related constraints. In Figure 1 illustrating Statement A, the solid curve represents a conventional non-optimized design of an applicator with a high reflection ($|S_{11}| \sim 0.7$) at $f_1 = 2.45$ MHz. The goal of optimization process is to get the geometry which would be characterized by the $|S_{11}|$ frequency response passing the interval $(f_1, f_2)$ below the desired level $S_0$ – as the dotted curve in Figure 1. The behavior of $S(X)$ outside this frequency range does not matter; neither does an actual profile of this function below $S_0$.

Projecting the above formulation onto typical MW applicators and considering $S(X)$ as a function of two elements of vector $X$, namely, frequency and one of the geometrical parameters, we may get a surface like the one shown in Figure 2. This function may be very complex and characterized by essentially non-linear behavior and multiple resonances. When working with this sort of hypersurfaces in $(n+1)$-space, accurate solution of the problem would be unlikely possible without practically unaffordable time- and resource-consuming methods like genetic algorithm, techniques of stochastic modeling, or other methods of global optimization.

However, additional consideration suggests that for most applied systems getting a *global* minimum of $S(X)$ is not required: as long as the located extremum satisfies the applied conditions and constraints, the corresponding set of design variables $y_1, \ldots, y_n$ should be satisfactory as an optimal solution. Moreover, operation of a system would be more stable and efficient if $f_0$ is located outside narrow resonances of a frequency characteristic of $|S_{11}|$. This indicates that we may look for local extrema on the surface of some more smooth approximating function provided that such a surface exists in the domain bounded by the applied constraints.

This leads us to the concept of intended “rectification” of strongly fluctuating hypersurfaces. That is, we can assume that the original hypersurface can be roughly represented by some other simpler one. Clearly, the latter can be reconstructed by numerical simulations with much less computational effort. So we use this data to build a model (response surface) of $S(X)$ in $(n+1)$-space. Picking an appropriate approximation for this model, we may switch to working with smoother analytical function and easily perform its minimization.
Thus we arrive at the following key ideas placed in the background of our optimization procedure:

a) Actual hypersurface can be represented by a "rectified" one which may be reconstructed by a limited number of computational experiments.

b) The obtained model of the hypersurface can be approximated by some simple function, which, when it is reasonably adequate, could be processed by some efficient method of constrained optimization.

ALGORITHM AND IMPLEMENTATION

Hypersurface and Its Representation

Hereinafter, we use the term rectification to denote a series of EM simulations reconstructing the actual unknown hypersurface at a finite number of nodes of the grid in the \((n+1)\)-space formed by the variables \(x_1, \ldots, x_n\) varying in the corresponding ranges \((X_1^-, X_1^+), \ldots, (X_n^-, X_n^+).\) We suppose that the obtained \((n+1)\)-dimensional response surface will be represented with the use of an analytical approximation, specifically, a second-order polynomial function. Thus, it is feasible to introduce the original \(S(X)\) in the form of the quadratic polynomial as

\[
S(X) = \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} \sum_{j=i+1}^{k} a_{ij} x_i x_j + \varepsilon
\]

for \(k = n + 1\) input variables and \(p = (k+1)(k+2)/2\) coefficients. In (3), \(\hat{\beta} = [\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_1, \hat{\beta}_{11}, \ldots, \hat{\beta}_{12}, \hat{\beta}_{13}, \ldots, \hat{\beta}_{k+1}]^T\) and \(\varepsilon\) is the error term conditioned by the accuracy of the EM model. The accuracy and efficiency of the second-order approximation of response surfaces have been approved by many authors (e.g., Biernacki et al., 1989), and quadratic polynomials have proved to be versatile and practical when it comes to approximation of a variety of surface shapes over localized regions (Khuri and Cornell, 1996).

Least Squares in Surface Fitting

In the course of rectification, we make \(N\) computational experiments. Denoting the collection of \(N\) responses as \(Q\) and defining the residual as

\[
R = Q - S(X, \hat{\beta})
\]

we make the search region in the \(n\)-space of the design variables centered at the point \(X^0 = [X_1^0, \ldots, X_n^0]\) with the coordinates defined in accordance with \(X_i^0 = (X_i^- + X_i^+)/2, \ldots, X_n^0 = (X_n^- + X_n^+).\) As a part of surface fitting methodology (Khuri and Cornell, 1996), appropriate scaling is applied to make all the variables in the search region of the same order of magnitude. The terms of vector \(X\) are therefore modified to the form

\[
Z_i = (X_i - X_i^0)/S_i
\]

where \(i = 1, \ldots, k,\) and \(S_i\) is the chosen scale of \(X_i.\) In this case, (3) can be considered in the form in which \(y\)-elements

\[Figure 2: Example of conventional profile of \(S(X)\) as a strongly fluctuating function of frequency \(f = X_i\) and one of geometric parameters \(y_i = X_{ij}\) with other design variables set constant.\]
of vector $X$ are replaced by corresponding values of $Z$. The observed response can thus be represented in the matrix form:

$$Q = Z\beta + R$$  

(6)

where $Q = [Q_1 \ldots Q_N]^T$, $Z$ is an $N \times p$-matrix of the terms (5), and $R = [R_1 \ldots R_N]^T$ is a vector of residuals. For the set of data at $N$ points, we calculate the vector of coefficients $b$, which are the least squares estimates of $\beta$ and given by

$$b = (Z^T Z)^{-1} Z^T Q$$  

(7)

Substituting these estimates for the unknown vector $\beta$, we obtain the formula for the quadratic approximation of the hypersurface in the form:

$$\Phi = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j + R$$

(8)

Observation over $R$ should allow us to evaluate quality of model (8). To this end, we verify the assumption of normality of the error term, which requires determining that the residual distributions are approximately normal (Khuri and Cornell, 1996). Practically, this analysis is easily implemented with the use of normal quantile plots.

The actual form of model (8) is reconstructed with the use of the regression procedure allowing us to select probabilities for adding and deleting terms and thus possibly to simplify the model not losing a quality of representation of $S(X)$.

**Constrained Optimization**

After obtaining an adequate approximation for $S(X)$, we use this representation to locate the coordinates $X^* = [X_1^* \ldots X_k^*]_2$ of a critical point where the partial derivatives are equal to zero and which could, if it is a minimum, specify the optimized design variables.

To solve the optimization problem with the constraints specified in *Statement A*, we use the Sequential Quadratic Programming (SQP), the direct approach dealing with constraints and the objective function together without their transformation (Neittaanmäki, et al., 1996). The SQP method is not a technique for global optimization, so the achieved minimum is considered in general case as a local minimum. As mentioned above, when optimizing MW applicators, we may be satisfied with *local extrema* provided that the solution is in the region bounded by the applied constraints.

Under certain conditions, the solution to the constrained optimization problem may not exist, and any change of the constraints for the independent variable $f$ (i.e., the frequency range $(f_1, f_2)$) to expand those conditions may not be desirable. In this situation, we could check another initial guess (with the hope that it may lead us to a nearby local minimum), or narrow the initial range(s) of the design variable, or agree to change $S_0$. Any of these steps could be a part of a fruitful strategy leading to the required optimal solution.

**Computer Implementation**

The outlined algorithm has been implemented in MATLAB 6 R12 as a Windows NT/2000-based program. The diagram explaining organization of its major units is shown in Figure 3. MATLAB runs the executable file General_Optimizer.m that opens the input file Input.dat which is created in MATLAB and contains all necessary information to simulate the MW system (including the optimized parameters) and the constraints.
associated with the problem. Then General_Optimizer launches QW3D with the specified project and commands the simulator to export the results (S-parameters) to the file QuickWave_Output.dat. Finally, General_Optimizer reads this output file and saves its contents in a special array that then undergoes statistical test with SAS/STAT 8 (reconstruction of the polynomial model and error analysis with normal quantile plots) and is used for constrained optimization afterwards. As an operational unit of our optimization procedure, QW3D is completely controlled from the MATLAB programming environment. The procedure also uses the program implementation of the SQP method available as a MATLAB procedure fmincon.m.

EXAMPLES

In this section, we illustrate application of the described optimization scheme to four practical structures which one may encounter in microwave power engineering. Although the procedure does not apply any limitation on the number of the optimized parameters, the examples are limited to 3- and 4-parameter optimization. All computations referred below were performed on a PC with Pentium III 750 MHz processor and 128 MB RAM.

Waveguide Dry Load

First, we consider a typical waveguide dry load; its internal configuration is shown in Figure 4. The optimization goal is to find the geometry of the absorbing material, namely, the wedge length $d$ and the width of the edges facing the waveguide input $s$ guaranteeing lowest reflections from the load (and thus its highest absorption) at 2.45 GHz. In the market, there are samples of this type of load employing SiC as an absorbing material. Since experimental results available in literature on complex permittivity of SiC are characterized by high diversity, for this study, we use the data of three independent measurements presented in Table 1.

Optimization is performed for a section of WR284 ($a \times b = 72 \times 34$ mm) waveguide. Appropriate ranges for $d$ and $s$ are imposed as $20 \leq d \leq 80$ mm, $18 \leq s \leq 72$ mm. The frequency interval is bounded by $f_1 = 2.4$ GHz, $f_2 = 2.5$ GHz, and it is assumed that $\varepsilon'_s = 0.3$, i.e., we require the load to guarantee at least 90% coupling in this frequency range.

Graphs in the left column of Figure 5 illustrate typical contour plots of $S(X)$ for the fixed length of the wedge; corresponding approximating functions $Q(Z)$ are shown in the right column. The graphs suggest that in the considered structure we deal with relatively smooth hypersurfaces for all three types of SiC, and the contours of their approximations look particularly simple. The response surface models appear here in the following forms

\[
\begin{align*}
\text{HP SiC:} & \quad Q \approx 0.4285 - 0.0374s - 0.3207d - 0.0304f - 0.0253sd - 0.0344d^2 \\
\text{RB SiC:} & \quad Q \approx 0.6999 - 0.0160s - 0.1969d - 0.0531f - 0.0224sd + 0.0662sf + 0.1311ds + 0.0488s^2 - 0.3727d^2 + 0.0327f^2 \\
\text{S SiC:} & \quad Q \approx 0.4749 - 0.0301s - 0.0418d - 0.0170f + 0.0176sd - 0.0102sf - 0.0332s^2
\end{align*}
\]

(9a) \hspace{7cm} (9b) \hspace{7cm} (9c)

Figure 4: Waveguide dry load: configuration of lossy wedges and their placement in an $a \times b$ waveguide.

The quality of these approximations is evaluated by the study of normality of the error term. The quantile

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon'$</th>
<th>$\varepsilon''$</th>
<th>$\sigma$, S/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot Pressed (HP) SiC (Binner, 2002):</td>
<td>50</td>
<td>16.6</td>
<td>2.25</td>
</tr>
<tr>
<td>Reaction bonded (RB) SiC (Binner, 2002):</td>
<td>50</td>
<td>1.0</td>
<td>0.14</td>
</tr>
<tr>
<td>Staffordshire (S) SiC (Batt, et al. 1995):</td>
<td>92</td>
<td>88</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Table 1: Properties of different sorts of SiC measured by three independent experiments at 2.45 GHz and for room temperature.
Figure 5: The reflection coefficient in the dry load (Figure 4) with \( d = 70 \) mm for HP SC (a, b), RB SC (c, d), and S SC (e, f): rectified contour plots (a, c, e) and their approximations (7) (b, d, f).
plots for all three sorts of SiC presented in Figure 6 show a reasonable deviation from the straight line which corresponds to data perfectly matched with the Gaussian distribution, so we consider the approximations (9) satisfactory.

The optimized values of the design variables $s$ and $d$ are given in Table 2. It is seen that for all three materials, the best geometry is the one with the maximal possible length of the wedges. Physically, this is a quite natural result: with larger $d$, the angle at which the medium interface is inclined with respect to the direction of wave propagation becomes smaller. With extension of the range in which the optimum is sought beyond 80 mm, we may expect getting even larger optimal values of $d$.

The last step is the fine analysis of the waveguide loads with the optimal configurations. Graphs in Figure 7 show that in the specified frequency range (2.4, 2.5 GHz), $|S_{11}|$ is notably less than the assigned level $S_0 = 0.3$. In this figure, the characteristics of corresponding non-optimized WR284 dry loads with $d = 53$ mm and $s = 51$ mm are also plotted. It is seen that for each material the quality of the optimized load is much higher: at 2.45 GHz, the values of $|S_{11}|$ correspond to the couplings near 98-99%. With the wedges made from RB SiC and characterized by comparatively low losses, the load performs like a resonance structure whereas with highly absorbing HP and S SiC, the inclusions completely eliminate a resonant behavior.

To guarantee high accuracy of simulation, the QW3D model was created with the non-uniform mesh (subjected to the values of $\varepsilon'$ and $\varepsilon''$ as well as the dimensions of the absorbing material) which used 30,000-60,000 cells.

Table 2. Optimal configurations of the wedges of different sorts of SiC in the WR284 dry load and corresponding coupling.

<table>
<thead>
<tr>
<th>Material</th>
<th>$s$, mm</th>
<th>$d$, mm</th>
<th>Coupling, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP SC</td>
<td>18</td>
<td>80</td>
<td>99</td>
</tr>
<tr>
<td>RB SC</td>
<td>72</td>
<td>80</td>
<td>99</td>
</tr>
<tr>
<td>S SC</td>
<td>30</td>
<td>80</td>
<td>98</td>
</tr>
</tbody>
</table>
Figure 7: Frequency characteristics of the reflection coefficient in non-optimized (thin curves) and optimized (see Table 2, thick curves) dry loads with the HP SiC (a), RB SiC (b), and S SiC (c) wedges.
(cell size from 3 mm in the medium to 9 mm in air) at the stage of optimization and 200,000 to 260,000 cells (cell size from 1 mm in the medium to 5 mm in air) when performing fine analysis. For the considered 3-parameter optimization problem, reconstruction of the hypersurfaces took about 1.5 hours (HP SC and S SC) and 2.5 hours (RB SC). Compared to that, computer time required for constrained optimization of the approximating functions (the SQP method) was negligible (several minutes).

**Water Load in a Cavity**

This example is concerned with a metal cavity containing a plastic cylindrical microwave-transparent cup filled with water (Figure 8). A somewhat similar construction has been considered by Eves et al. (2004) in the experimental part of the modeling-based method of determination of complex permittivity of materials. Computational trials show that random positioning of the cup with water leads to noticeably different frequency characteristics of the reflection coefficient, but the value of $|S_{11}|$ at $f_0$ is always high. The optimization goal here is to find the load’s dimensions and location which would provide as low reflections in the frequency range near $f_0$ as possible.

We consider the cup with internal radius $r$ and internal height $h$ filled up with fresh tap water located on the shorting wall of the 200 mm section of WR975 ($a \times b = 248 \times 124$ mm) waveguide. It is supposed that the cup is placed along the $x$-axis at the distance $s$ from the waveguide center as shown in Figure 8. The operating frequency is $f_0 = 915$ MHz. The cup’s wall and bottom thickness is constant (10 mm); dielectric constant of plastic is $\varepsilon' = 2.55$, and its loss factor $\varepsilon'' = 0$. Complex permittivity of water at 915 MHz at the room temperature $20^\circ C$ is determined from the model described by Eves and Yakovlev (2002) ($\varepsilon' = 79.9$, $\varepsilon'' = 4.10$). The goal is to find $r$, $h$, and $s$ for which the reflection coefficient is less than 0.5 in the range $910 \leq f_0 \leq 920$ MHz. The design specifications for these variables are: $24 \leq r \leq 124$ mm, $10 \leq h \leq 50$ mm, and $-62 \leq s \leq 62$ mm.

The quadratic model of the response surface is obtained in the following form:

$$Q \approx 0.8028 + 0.0131f - 0.0789h - 0.0789f - 0.093r + 0.0277hr + 0.0244r^2 + 0.0343f^2 + 0.0343f^2$$  \hspace{1cm} (10)$$

Quality analysis for model (10) is illustrated by Figure 9, a. The curve demonstrates certain divergence of $R$ from normal distribution and thus warns us that (10) may not be a good approximation. Such a divergence can be naturally caused by the fact that the actual function $S(X)$ and its approximation $Q(Z)$ are highly non-linear surfaces; indeed, all the four coefficients with the quadratic terms are non-negligible. In this situation, we have two options:

1) We may ignore this information and proceed to constrained optimization for (10) with the hope that the anticipated solution will be physically valuable. Doing so, we find the optimal design of the structure as $h = 33.6$.
mm, r = 44.9 mm, and s = 22.1 mm.

(2) Alternatively, we can take some measures to improve the approximation. In general, if it is not possible to reduce the number of design variables $y_1, \ldots, y_n$, changing their ranges can help. Making them smaller, we may cut the part of the surface that is responsible for its essentially non-linear behavior and thus improve the quality of our quadratic approximation. However, in this example, we find it practically suitable to cut one of the design variables. We agree to deal with a given cup (i.e., with a fixed radius $r$) and look for the optimal amount of water in it (height $h$) and the cup’s best position along the $x$-axis ($s$). In other words, we reduce the problem to 3-parameter (frequency plus two geometrical characteristics) optimization.

Application of our algorithm to the problem with $r\text{ const}=50$ mm leads us to the following approximation:

$$Q \approx 0.8852 + 0.0293f + 0.1786h + 0.0343fh + 0.900h^2 - 0.2858h^2$$  (11)

The quadratic polynomial (11) turns out to be $s$-independent. It generates the normal quantile plot which is shown in Figure 9, b and looking somewhat better than the curve for the 4-parameter optimization. The optimized geometric parameters obtained from minimization of (11) are: $h = 33$ mm, $s = 0$.

The frequency characteristics of $|S_{11}|$ shown in Figure 10 for 4- and 3-parameter optimization imply that they both comply with all the constraints and make the reflection coefficient at 915 MHz reasonably small (0.27) making C $\sim 93\%$. We therefore conclude that the residual characterized by the curve in Figure 9, a can still be associated with an acceptable polynomial approximation.

Computer time spent for the 4- and 3-parameter optimizations of this structure was 6 and 1.5 hours respectively.

Waveguide T-Junction

In many microwave power applications, the energy to be delivered from a generator to an applicator has to be split into two arms of a perpendicularly oriented waveguide as shown in Figure 11. In this case, it is critical to minimize reflections from this waveguide junction to the generator. A post (screw-bolt) inserted through a waveguide wall is a popular and convenient tool for matching the waveguide lines. In its presence, the level of reflection strongly depends on the dimensions and position of the post: it creates an additional reflected field which may add up with the field reflected by the T-junction in an opposite phase and thus sharply reduce $|S_{11}|$. The theoretical and experimental studies of a waveguide T-junction with a partial-height post by Wu and Wang (2001) suggest that this construction may be characterized by a acceptably low reflection. So finding the optimal configuration of the matching post seems to be a valuable practical problem.

In order to validate the $QW3D$ model used in optimization process, we first computed the magnitudes and the phases of $S$-parameters of the H-plane WR75 (19.05 x 9.53 mm) T-junction with a post studied by Wu and Wang (2001) and found that our model generates the results very well consistent with their data. Then we solved the optimization problem for the WR975 T-junction typical for industrial microwave heating systems.

In this illustration, we optimize the following design variables: the diameter of the post $2r$, its height $h$, and its position along the $y$-axis $s$. The design specifications for these parameters are: $2 \leq r \leq 10$ mm, $5 \leq h \leq 60$ mm,
and \(-50 \leq s \leq 50\) mm. Since the partial-height post is supposed to provide a wide-band matching, we seek the frequency characteristic of \(|S_{11}|\) being less than \(S_0 = 0.2\) at the frequency range from \(f_1 = 900\) to \(f_2 = 930\) MHz.

The approximating polynomial \(Q\) obtained in this case contains many coefficients which are negligibly small, and we thus get a simpler function for constrained optimization. The optimal geometry is: \(r = 2.0\) mm, \(h = 58.8\) mm, and \(s = -38.7\) mm. Optimization took about 2 h. The \(|S_{11}|\) characteristic for the optimized structure is shown in Figure 12: at 915 MHz, the reflection coefficient becomes 0.16 instead of 0.85 corresponding to the arbitrary configuration with \(r = 6.0\) mm, \(h = 31\) mm, and \(s = 0\).

### Slotted Waveguide

Our optimization scheme was also applied to a waveguide-backed slotted radiating element. The motivation for this problem is generated by an interest that has recently emerged in industry regarding the excitation systems alternative to a traditional open-end waveguide connected with the cavity (Meredith, 1998, Metaxas, 2000, Chan and Reader, 2002, St-Denis, et al., 2001, Al-Rizzo, et al., 2002, Cresko and Yakovlev, 2003).

The scenario in which this element is regarded as radiating in free space is simulated as an open structure: in the QW5D model, the waveguide with 5 equidistant slots is considered surrounded by a box with the faces imitating
the near-to-far field transformation and the absorbing boundary conditions. The configuration of the slots in the broad wall of WR430 (Figure 13) is optimized for the fixed slot’s sizes \((w = 8 \text{ mm}, l = 65 \text{ mm})\) and variable distance from the last slot to the waveguide shorting wall \(d\), separation of the slots \(s\), and their inclination \(\theta\). Imposing appropriate ranges for the geometrical parameters \((30 \leq d \leq 120 \text{ mm}, 30 \leq s \leq 70 \text{ mm}, 0 \leq \theta \leq 60^\circ)\) and assuming \(f_1 = 2.4 \text{ GHz}, f_2 = 2.5 \text{ GHz},\) and \(S_0 = 0.3\), we get \(Q\) as an incomplete quadratic polynomial. The related normal quantile plot appears to be a reasonable approximation to the straight line.

The optimal solution turned out to be \(\theta = 27^\circ, s = 56 \text{ mm},\) and \(d = 118 \text{ mm}\). Figure 14 shows that, as it was required, in the specified frequency range, \(|S_{11}|\) is less than \(S_0 = 0.3\). At \(f_0 = 2.45 \text{ GHz}, |S_{11}| \sim 0.2,\) i.e., about 96% of electromagnetic energy leaves the waveguide for the outer space. In the considered radiating element, an adequate approximation of the slot pattern requires from 90,000 to 180,000 cells in the coarser mesh, so the reconstruction of a rectified hypersurface needs more CPU time than the models analyzed above. Furthermore, here we deal with a 4-parameter optimization. As a result, it took about 11 hours of CPU time.

This consideration allows us to think of the slotted waveguide as of a potentially efficient radiator and to suggest that with appropriate optimization it could be practically used without the matched load routinely applied in the conventional traveling-wave slotted-waveguide antennas. The obtained optimal design makes it usable in high power applications, but, of course, only when radiating in free space. If the goal is to optimize a slotted waveguide as an excitation system of a MW oven, this would require another model involving also the oven and the processed material. In this case, the model should be built as a closed (not open, like in this example) scenario,
and the $|S|_{11}$ characteristic will be strongly stipulated by parameters of the contiguous cavity and the enclosed lossy load. Clearly, the presented optimization scheme could be applied to such a scenario as well, but in this case it would deal with a more complicated model and thus demand more computational resources.

Conclusion

This paper has outlined and illustrated the efficient yet simple algorithm suitable for optimizing $S$-parameters in systems and components of MW power engineering. Although in the described examples we have dealt with 3 and 4 design variables (frequency plus 2 or 3 geometric parameters), the method obviously works with an arbitrary dimension of vector $X$. In many applications, our procedure may be associated with response surfaces approximated by reduced quadratic models, so the presented approach could be fairly quick and efficient in generating sets of optimized design variables for complex problems requiring multivariable optimization. The described deterministic technique converges quicker than the approaches in applicator design working with global optimization (simulated annealing used by Sundberg, _et al._, 1998, genetic algorithm applied by Domínguez-Tortajada, _et al._, 2003, or stochastic methods).

While all the presented examples have been concerned with minimization of the reflection coefficient, the optimization algorithm can be easily adjusted to process other $S$-parameters (transmission coefficient, etc.). Its implementation does not need any special computer resources and requires very reasonable CPU time for solution of many practical problems. Although in this paper the procedure has been described as working with _QuickWave-3D_, it naturally can be connected with other back ing FDTD simulators. It has been shown that the developed computational scheme is able to dramatically improve characteristics of practically valuable devices. This work, therefore, contributes to further progress in efficient CAD instruments for design and optimization of MW systems and components.

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References


