

Method of Control and Optimization of Microwave Heating in Waveguide Systems

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Abstract—A deterministic process of microwave heating in waveguide systems is considered. Use of the concept of optimal material design is suggested to secure a desired distribution of heat release within processed material via supplementary dielectric focusing structures. The method is illustrated by application to a rectangular waveguide with a thick E-plane dielectric layer and by finding parameters of the focusing structure maintaining uniform electric field within this layer.

Index terms—Control, lossless material, microwave heating, optimization, uniform field, waveguide.

I. INTRODUCTION

The use of microwave heating has proven to be effective for the processing of dielectric materials [1]-[3]. In many cases, a uniform temperature distribution within the product is required, and the literature on microwave processing suggest various means aimed to even up the pattern of heat release. Mechanical devices like stirrers and turntables provide an empirical solution to the problem in its non-deterministic setup. The multiple modes are thoroughly mixed up to achieve a random field distribution and therefore increase the probability of uniform heat release. Among other methods working on "statistical" principles, there should be mentioned active packaging [4], monitoring of the microwave power level [5], the variable frequency [6], and the multiple input of energy [2]. Despite evident attractive features, the practical implementation of these tools revealed the lack of uniformity in a number of important cases. These methods do not provide an effective control adapted to a specific material, and are based on a purely intuitive expectation that the diversity of participating modes will secure the uniformity of heat release in space-time.

There also were attempts to improve the temperature patterns generated by a single mode. In [7]-[8], there was considered a possibility of creation of the uniform field by formation of the TEM mode in a rectangular waveguide partially filled with dielectric slabs; a similar technique was used in [9] for a resonant cavity. Such an approach does not, however, provide realistic recommendations towards the elimination of higher modes that may well emerge once the operating chamber is filled with a processed material. That is particularly why the use of the TEM mode has a practical implementation only in applications related to irradiation [10].

Among other means used to correct fields generated by deterministic modes, there are the use of supplementary metal ridges [11] and grids [12], the mechanical change of the chamber's size [13], and the use of evanescent modes [14]. All these efforts are insufficient because the geometry of material inclusions as well as their deployment throughout the operating chamber are motivated by engineering experience

alone, both theoretical and experimental. In spite of its obvious merits, this experience is limited, and it therefore fails to completely exhaust the possibilities of improvement intrinsic in the system itself.

In order to release these additional resources, a rigorous mathematical consideration is necessary. Sophisticated methods of computer modeling [15]-[16] based on the numerical discretization cannot give direct recommendations as to how the system should be changed to guarantee improvement of the temperature distribution within the product. The necessity to apply "some kind of optimization" to arrange more effective processing was recognized in [17].

The present paper suggests an effective way to put the temperature distribution under direct control. The required formalization emerges from the idea of optimal material design (OMD) [18]. In the context of microwave thermoprocessing, this means an optimal placing of supplementary dielectric materials within a part of the operating chamber in order to appropriately focus the electromagnetic field onto the heated product to maintain the desired (e.g., uniform) heat release within it.

In contrast to attempts to enhance the electric field uniformity due to dielectric slabs partially filling the operating chambers [7]-[9], the location, structure, and permittivity of a focusing material are determined by the OMD method. They are found not empirically but rigorously as the means guaranteeing the required heat release.

II. METHOD

The microwave heating is known as a very complicated process depending on many physical parameters. The processed material may have non-uniform composition, non-smooth configuration, and temperature-dependent permittivity. The heating chambers differ by geometry, type of field excitation, the placement (orientation) of the product, etc. The problem of control of microwave heating becomes hard to formalize in such a general setting. Therefore, we narrow the problem and concentrate on fully deterministic, non-random processes. Specifically, we assume that:

- (i) the processed samples may have arbitrarily complex composition, but the distribution of their material properties is well determined;
- (i) the shape of the samples is known;
- (ii) a finite number of propagating modes subjected to deterministic control is allowed.

With these assumptions we still cover many practically significant situations. Then we further specify the problem and consider waveguide systems that appear to be the most suitable for setting the deterministic wave patterns.

The OMD concept says that the field focusing can be successfully implemented with the aid of an assemblage of two dielectric materials D_1 and D_2 differing in their permittivities and appropriately distributed within some part F of an opera-

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ting chamber embracing the heating zone P occupied by the processed sample (Fig.1). The layout of these materials serves as the key factor controlling the spatial distribution of the field, and, consequently, the heat release within P . Finding this layout is the main goal of the approach.

We are looking for the disposition of the focusing materials in F capable of minimizing the deflection of the acting heat release $q(x,t)$ within P from some desired distribution $q_0(x,t)$ of this parameter. Such a deflection becomes the cost (objective) functional; as an example, one may consider the mean square difference between the two distributions:

$$I = \int_0^{t_1} \int_P |q(x,t) - q_0(x,t)|^2 dx dt \quad (1)$$

where t_1 denotes the period of heating. This functional combines both spatial and temporal variations of the field and may be equally applied in the presence and/or absence of the temperature dependence of the product's permittivity. Along with (1), other types of cost functionals, e.g., linear functionals, may be introduced.

The goal of design is to minimize the functional by an appropriate layout of D_1 and D_2 in F . The field distribution within F and P is governed by the field equations and the relevant boundary conditions. The presence of such constraints is the key factor that distinguishes problems of optimal material design from traditional variational problems of minimization of functionals. In other words, the quantity q in (1) should be chosen not among the functions possessing just conventional smoothness properties, but it should rather be sought among solutions to a side boundary value problem depending upon control. The search of an optimal $q(x,y)$ therefore reduces to the search of optimal control. The complexity of this problem is due to the implicit influence produced by a control upon the heat release: this influence comes through the boundary value problem and is not reflected directly in the functional through its explicit dependence upon control. Such a concept is different from the approach addressed in many publications (e.g., [19-20]) where the functional appears to be explicitly control dependent.

The typical optimal layouts possess one common feature: certain parts of F (or the whole of it), become occupied by composites assembled from D_1 and D_2 [18]. The appearance of composites is crucial: they redistribute the electromagnetic field everywhere, and particularly in P , to the final effect of minimization of the cost functional (1).

In our problem, the Maxwell's equations are solved for domains F and P under suitable boundary conditions. Controls (permittivities of the focusing structure) are distributed throughout F , whereas the heat release to be controlled is distributed over P . Material properties of the substance within P are assumed given. Both domains have a common interface through which the control action is transmitted. The heat release generates the temperature pattern within the product according to the heat equation.

The domain F is occupied by two dielectrics with different permittivities ϵ_1 and ϵ_2 ; we look for their layout in F that finally makes the distribution of temperature within the domain P as close as possible to a desired pattern. This influence is implemented through the direct effect produced by a non-uniform dielectric material in F upon the field distribution, and consequently, the heat release within P .

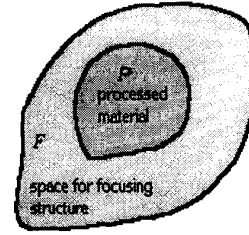


Fig.1. Domains P and F of the waveguide operating chamber

In this paper, we illustrate this approach by a simple example. Specifically, we assume the processed material to be lossless, and therefore, instead of heat release, we even up the distribution of the electric field itself.

III. APPLICATION TO RECTANGULAR WAVEGUIDE AND LOSSLESS MATERIAL

Consider a waveguide applicator of rectangular cross-section with sides a and b . The processed material is assumed isotropic and uniform with permittivity ϵ^* and permeability $\mu = 1$; it occupies a part P ($x \in [x_1, x_2]$, $0 < x_1 < x_2 < a$, $y \in [0, b]$) of the cross-section of the waveguide (Fig.2). Generally, the dielectric filling generates conditions for propagation of both dominant and higher modes. Any change in the fillings' parameters (permittivities or/and geometry) may affect the number of propagating modes, and this influence should be taken into consideration, particularly, when we optimize parameters of the focusing structure.

Specifically, our goal is to locate controlling dielectric materials, also assumed lossless, in two lateral domains F_1 ($0 < x < x_1$, $0 < y < b$) and F_2 ($x_2 < x < a$, $0 < y < b$), in such a way as to obtain a uniform distribution of the electric field E_y inside the domain P .

The electric field in P is known to be superposition of propagating modes, and to make this field uniform we have to apply sophisticated composite layout aimed to even up these modes combined. This layout will be determined as we apply the technique of Section II in its entirety. This analysis will be substantially simplified if we allow propagation of a single (dominant) mode. This can be achieved, for example, by appropriate change of the waveguide's dimensions preserving, however, the material filling pattern.

To specify the starting points, we assume that the orientation of electric and magnetic vectors (\mathbf{E} and \mathbf{H}) in the dominant mode replicates the orientation of these vectors in the TE_{10} mode in an empty waveguide (see Fig.2). In these circumstances, the general method shows that the material layout that will not destroy the above orientation of the field vectors may only be a lamination with layers along y -axis. A uniform dielectric represent a special case of lamellar layout. We start with a question: what should be the permittivities ϵ_1 and ϵ_2 of uniform dielectrics occupying the domains F_1 and F_2 so that the relevant electric field E_y within the domain P becomes uniform?

The electric field of the dominant mode may be represented as:

$$E_y(x) = u(x)e^{-iz}, \quad (2)$$

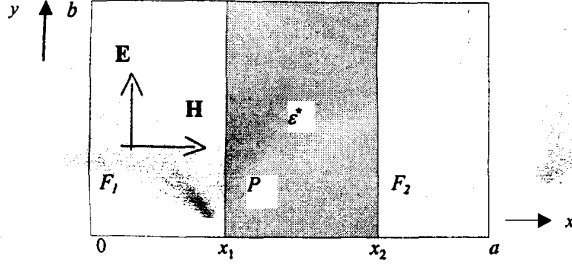


Fig. 2. Cross-section of a applicator waveguide applicator in case of no gap between P and $F_{1,2}$

where $u(x)$ is the complex amplitude, and γ is the propagation factor, or longitudinal wavenumber. Since the materials in P and F are assumed lossless, the propagation factor is equal to the phase constant β . The controlling dielectrics may occupy the lateral spaces either completely, or in part.

A. No Gap Between P and F

The boundary value problem combines the Helmholtz's equation for $u(x)$ and the relevant boundary conditions, i.e.:

$$u''(x) + hu(x) = 0, \quad h = \omega^2 \varepsilon(x) - \beta^2, \quad (3)$$

where h is a transverse wavenumber, ω is a circular frequency, and

$$\varepsilon(x) = \begin{cases} \varepsilon_1, & x \in (0, x_1), \\ \varepsilon^*, & x \in (x_1, x_2), \\ \varepsilon_2, & x \in (x_2, a). \end{cases} \quad (4)$$

For the TE_{10} mode, the following boundary conditions hold:

$$\begin{aligned} u(0) &= 0; \\ [u(x_1)]^+ &= 0; \quad [u'(x_1)]^- = 0; \\ [u(x_2)]^+ &= 0; \quad [u'(x_2)]^+ = 0; \\ u(a) &= 0. \end{aligned} \quad (5)$$

Within the processed material, we require the uniform field:

$$u(x) = 1; \quad x \in (x_1, x_2). \quad (6)$$

This means that the minimal requirement for the functional (1) might be expressed as:

$$\int_{x_1}^{x_2} [u(x) - 1]^2 dx \rightarrow \min. \quad (7)$$

The analysis of this problem shows that:

- (i) the uniform field E_y within P is maintained if the effective permittivities ε_{F1} , ε_{F2} of controlling layers are uniform in each of the F domains; thus, no composite structures arise there;
- (ii) the values of ε_{F1} , ε_{F2} exceed ε^* , and the permittivity increments $\Delta\varepsilon_{F1} = \varepsilon_{F1} - \varepsilon^*$, $\Delta\varepsilon_{F2} = \varepsilon_{F2} - \varepsilon^*$ depend on the operating frequency f and location of the processed material according to the formulas:

$$\Delta\varepsilon_{F1,2} = \left(\frac{h}{\omega}\right)^2, \quad (8)$$

where $\omega = 2\pi f$, and h is determined by

$$h = \begin{cases} \frac{\pi}{2x_1}, & \text{for domain } F_1, \\ \frac{\pi}{2(a-x_2)}, & \text{for domain } F_2. \end{cases} \quad (9)$$

For a symmetric case, (8) coincides with a similar relation in [7] (Eq. (5)) for TEM mode irradiated into the free space.

In a symmetric case, Fig. 3 illustrates the dependence between the relative permittivity increment $\Delta\varepsilon = \Delta\varepsilon_{F1}/\varepsilon_0 = \Delta\varepsilon_{F2}/\varepsilon_0$, where ε_0 is the permittivity of a vacuum, and the normalized widths of the controlling layers. One may see, for example, that if the processed material occupies 10, 30, and 50% of the cross-section of WR284, then $\Delta\varepsilon_F$ becomes equal to 0.9, 1.5, and 2.9 respectively. Since a dielectric filling leads to the increase of the cutoff wavelength, there appear higher modes that make the heat release non-uniform even if the dominant mode generates a uniform field. The higher modes will not appear, however, if the permittivities involved are not too high.

In Fig. 4, the maximal values ε_{\max}^* and ε^* are presented as functions of the material's thickness. For any $\varepsilon^* < \varepsilon_{\max}^*$ and corresponding $\varepsilon_{F1,2}$, the single mode range takes place, and, therefore, the field uniformity is maintained due to the use of the controlling dielectric. We may ensure the field uniformity also for materials with higher values of ε^* if we choose smaller waveguides. The computation of the cutoff wavelengths of the higher modes has been carried out using a 2D finite element algorithm and software described in [21].

B. An Air Gap Between P and F

The previous example can be extended to cover a practically significant case when there is no direct contact between the material $[x \in (x_1, x_2)]$ and the controlling layers $[x \in (0, x_{10}) \& x \in (x_{20}, a), x_{10} < x_1, x_{20} > x_2]$. Instead, they are separated by an empty space (or the space filled by the third material), and the relevant control action of the layers is implemented by a due choice of ε_{F1} and ε_{F2} .

The field representation (2) remains the same along with the Helmholtz's equation (3). The permittivity equals ε_0 in the gap between the focusing and processed materials, i.e., when $x \in (x_{10}, x_1) \& x \in (x_2, x_{20})$; the boundary conditions take into account the presence of the gap. The field uniformity in the interval (x_1, x_2) is expressed by (6), and the minimal requirement has the same form (7).

The analysis of this problem shows that:

- (i) the uniform field E_y within P is maintained if the effective permittivities ε_{F1} and ε_{F2} of controlling layouts in the x -direction are uniform in each of the F -subdomains occupied by controlling layers;
- (ii) the values of ε_{F1} and ε_{F2} exceed ε^* and the permittivity increment is determined by (8), but, instead of (9), h is defined as the root of the equations

$$\begin{aligned} \sqrt{h} \cot[\sqrt{h}x_{10}] &= -\sqrt{|h_0|} \tanh\left[\sqrt{|h_0|}(x_1 - x_{10})\right], & \text{for } F_1; \\ \sqrt{h} \cot[\sqrt{h}(a - x_{20})] &= -\sqrt{|h_0|} \tanh\left[\sqrt{|h_0|}(x_{20} - x_2)\right], & \text{for } F_2. \end{aligned}$$

where $h_0 = \omega^2(\varepsilon_0 - \varepsilon^*)$. The curves illustrating the dependence between the increment $\Delta\varepsilon$ and the normalized widths of the controlling layers are shown in Fig. 5. The curves

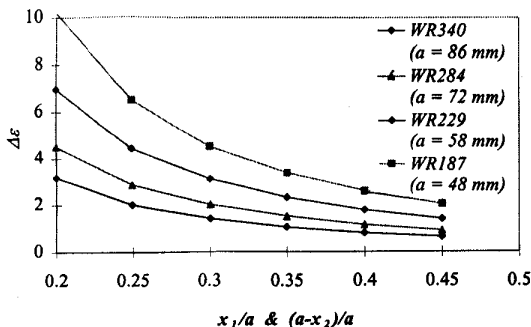


Fig. 3. Permittivity increment versus normalized thickness of controlling layers for various standard waveguides at 2450 MHz

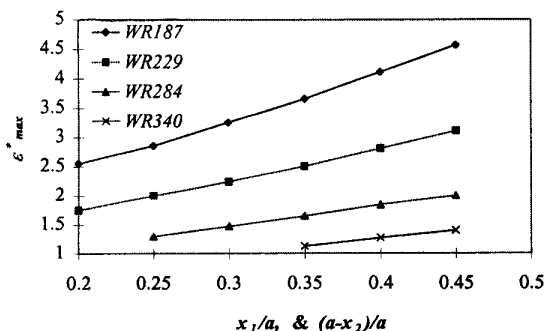


Fig. 4. Maximal value of processed material permittivity preserving a single-mode regime for various standard waveguides at 2450 MHz

come to the points $x_{10}/a = x_1/a$, or $(a-x_{20})/a = (a-x_2)/a$ (lower points on each curve) corresponding to the case of no gap between the controlling layers and processed material. It is obvious that the air gap produces a notable increase of the permittivity increment.

IV. CONCLUSION

The effective control over electromagnetic fields is one of the most significant (and yet unsolved) problems in microwave power engineering. The suggested concept offers a systematic means to maintain a desired (e.g., uniform) field pattern within a material placed in a waveguide chamber and characterized by well-determined shape and dielectric properties. In both examples, the controlling materials appear to be isotropic with the permittivities higher than that of a processed material. This feature is a consequence of a geometric simplicity of domains F and P along with the field patterns associated with the relevant modes. To secure the uniform field within P , we need dielectrics with suitably defined permittivities ϵ_1 , ϵ_2 . If we do not dispose of such dielectrics originally, they should be created artificially as composites assembled from the available constituents. Composites will also emerge in situations involving the higher modes.

In the present paper, the method of control of the electric field distribution was discussed with no account of the influence produced upon it by the presence of dielectric losses. This influence can, however, be taken into account without affecting the basic control procedure. When the processed material features losses, the uniform distribution of E_y within

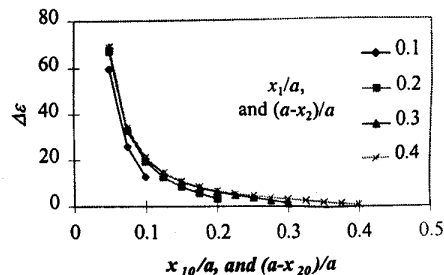


Fig. 5. Permittivity increment versus normalized thickness of controlling layers for WR340 ($a = 86$ mm), $\epsilon' = 6$, and various gap sizes at 2450 MHz

it can no longer be maintained by lossless focusing materials. We must either allow for losses in the focusing structures, or agree on minimization of the integral deflection of the acting field from some desired (e.g., uniform) field. In the latter case, the minimum is not zero, and it is achieved via composite laminate construction assembled from the lossless dielectric components originally available.

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