

## A Neural Network Technique for Reconstruction of 2-D Complex Permittivity Profiles of Materials in Waveguide Systems

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### ABSTRACT

The problem of reconstruction of complex permittivity profiles in dielectric samples is considered. The spatial distributions of dielectric constant and the loss factor are approximated by functions with unknown coefficients which are reconstructed using a neural network technique. The inputs of the network are scattering parameters from a closed waveguide system containing the sample whereas the outputs are the associated coefficients of the function. FDTD modeling is applied on the step of the neural network training. Numerical examples obtained for rectangular and cylindrical samples show excellent reconstruction of permittivity profiles represented by Gaussian functions.

**KEYWORDS:** complex permittivity profile, neural network, FDTD modeling, waveguide.

### INTRODUCTION

Microwave imaging as reconstruction of permittivity and conductivity profiles of dielectric objects using microwave measurements has recently received an increased attention due to its potential in many applications [1]. Our interest in this technology is conditioned by substantial difficulties in building comprehensive numerical models for microwave sintering of particulate and powder materials. A principle problem there is the absence of data on spatial variation of material parameters of the processed sample in the course of its microwave sintering.

Numerical solutions of inverse problems are commonly considered a crucial part of the applied technologies of non-destructive evaluation, so a technique capable of reconstructing spatial profiles of complex permittivity appears to be one of the most needed computational tools required for modeling of microwave sintering. Existing techniques of reconstruction of 1- and 2-D permittivity profiles (e.g., [2, 3]) are associated with complex experimental systems and characterized by low resolution and accuracy. While it has been shown that artificial neural network (ANN)-based techniques may be efficient in determination of complex permittivity of materials [4], the known ANN methods of reconstruction of spatial distribution of dielectric constant  $\epsilon'$  and the loss factor  $\epsilon''$  are applicable to the coaxial [5] or layer [6] structures dealing effectively with 1-D reconstruction.

In this paper, we describe the ANN technique which reconstructs 2-D complex permittivity profiles from data obtained by elementary measurements. The technique deals with  $S$ -parameters of a simple waveguide system containing the tested sample. The permittivity profile is approximated by a continuous function defined by a small number of coefficients. In this contribution, we consider the profiles  $\epsilon'(x, y)$  and  $\epsilon''(x, y)$  described by three different functions, and an ANN is used for reconstruction of the coefficients controlling the profile. The network is trained by numerical results generated by full-wave FDTD modeling of the system.

### METHODOLOGY

The key idea of our approach to reconstruction of 2-D permittivity profiles is an approximation of spatial distribution of  $\epsilon'$  and  $\epsilon''$  by some surfaces described by smooth continuous functions defined by a small number of independent coefficients, and reconstruction of a set of the coefficients providing most close correspondence between the actual distribution and the model function.

In order to allow the approach to be flexible in reconstructing a variety of the profiles, the model function should be capable of taking different configurations depending on its coefficients.

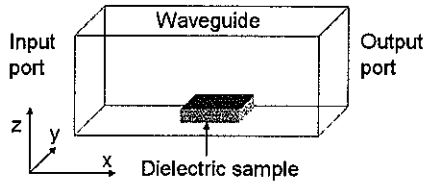


Fig. 1. Measurement system with dielectric sample inside.

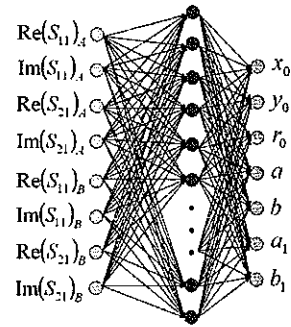


Fig. 2. Architecture of the RBF ANN for reconstruction of Gaussian coefficients.

In this work, we explore the potential of the reconstruction technique featuring linear, quadratic, and Gaussian approximations of  $\epsilon'(x, y)$  and  $\epsilon''(x, y)$ ; the latter are given by the formulas:

$$\epsilon' = a \left( 1 + b \exp \left( - \frac{(x - x_0)^2 + (y - y_0)^2}{r_0^2} \right) \right), \quad \epsilon'' = a_1 \left( 1 + b_1 \exp \left( - \frac{(x - x_0)^2 + (y - y_0)^2}{r_0^2} \right) \right) \quad (1)$$

In (1), parameters  $x_0, y_0$  are responsible for the position of the maximum/minimum of the function, parameter  $r_0$  is responsible for the width of the peak (i.e., it defines the radius of circle inside which the extreme point is located), parameter  $a$  defines the plateau's level of the function, and parameter  $b$  is responsible for the function value at the extreme point. By using the same  $x_0, y_0$  and  $r_0$  for both  $\epsilon'$  and  $\epsilon''$  we agree that the profiles of dielectric constant and the loss factors are not fully independent. Being a certain limitation on applicability of the technique, this, on the other hand, reduces the number of parameters responsible for spatial distribution of both characteristics to 7. At the same time, due to controlling mechanisms available in (1), the Gaussian surface may be suitably bendable in describing fairly different profiles when dealing with these independent coefficients.

Further, we combine the idea of functional approximation of the permittivity profile with the model imitating an experimental system chosen as a simple waveguide structure proven to be convenient in ANN-based reconstruction of complex permittivity of uniform samples [3]. The measurement system consists of a rectangular waveguide with two (input and output) ports containing a rectangular dielectric block lying on the narrow waveguide's wall (Fig. 1). The measured parameters are complex reflection ( $|S_{11}|$ ) and transmission ( $|S_{21}|$ ) coefficients.

An ANN used for reconstruction of the Gaussian coefficients is shown in Fig. 2. The ANN inputs are the values of 8  $S$ -parameters obtained for two positions (A and B) of the sample. The ANN outputs are the independent coefficients defining Gaussian spatial distribution of complex permittivity. This conveniently leaves the control over the resolution of permittivity reconstruction with the coefficients of approximating function. Two sets of  $S$ -parameters associated with the ANN input (Fig. 2) correspond to the original and 90°-rotated positions of the sample.

The ANN used in our algorithm is an RBF network with cubic basis functions [7]. The network is trained by numerical data obtained with the use of the full-wave 3-D conformal FDTD simulator *QuickWave-3D* [8]. The training points are generated as random combinations of function coefficients uniformly distributed in the specified intervals. When the ANN is found to be sufficiently trained by modeling data, it is able to reconstruct the function coefficients (and thus permittivity profile in the sample) using the 8  $S$ -parameters obtained from the related measurements. The reconstruction algorithm is implemented as a *MATLAB* code.

## RESULTS

In this section, we present the numerical results obtained for two shapes of the tested sample – a rectangular block and a cylinder. Computations were performed for the section of WR975 (248 × 124 mm) of the length 612 mm operating at 915 MHz and containing the rectangular ( $L_x \times L_y \times L_z =$

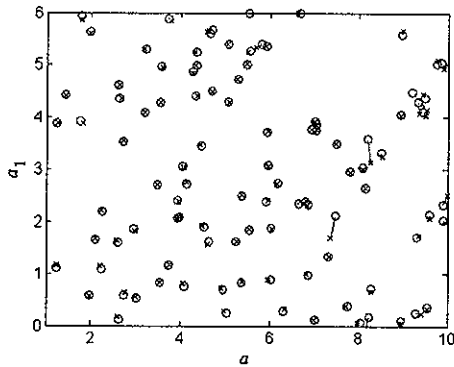


Fig. 3. Network responses (x) to testing points (o) in reconstruction of  $a$  and  $a_1$  for a rectangular sample,  $x_0 = 10$  mm,  $y_0 = -10$  mm and  $r_0 = 30$  mm; 240 training and 100 testing points.

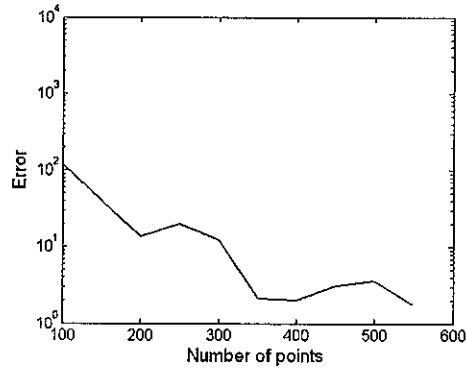


Fig. 4. Convergence of the average error upon 100 testing points in the reconstruction of  $x_0, y_0, b$  and  $b_1$  in rectangular samples for  $a = 5, a_1 = 2$  and  $r_0 = 30$  mm.

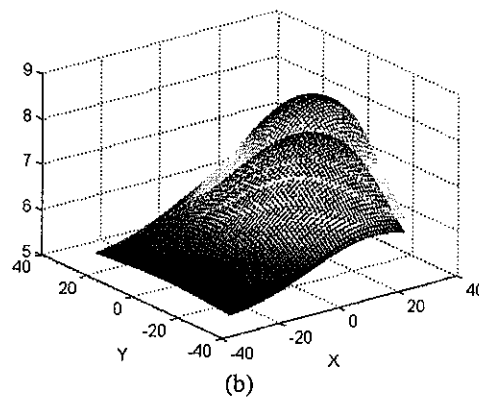
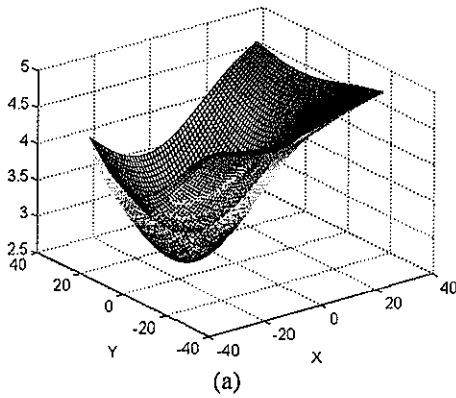


Fig. 5. Actual (transparent surface) and reconstructed (solid surface) profiles of  $\epsilon'(x, y)$  in a rectangular sample: functions (1) with  $x_0 = 10$  mm,  $y_0 = -10$  mm,  $r_0 = 30$  mm,  $a = 5, a_1 = 2, b = -0.5, b_1 = 0.5$  (a) and with  $x_0 = 0$  mm,  $y_0 = 20$  mm,  $r_0 = 30$  mm,  $a = 5, a_1 = 2, b = 0.5, b_1 = 0.5$  (b).

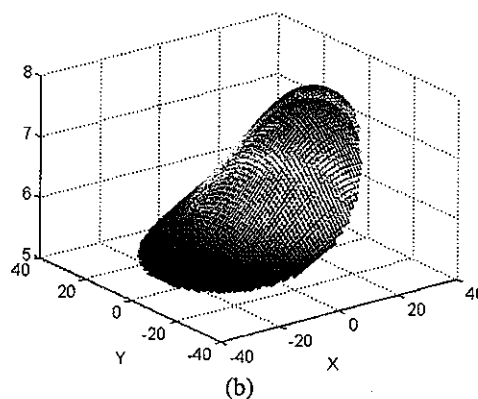
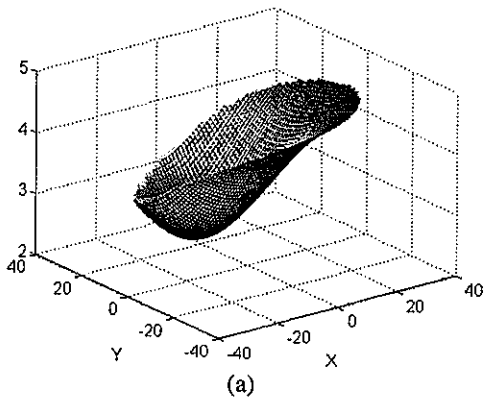


Fig. 6. Actual (transparent surface) and reconstructed (solid surface) profiles of  $\epsilon'(x, y)$  in a cylindrical sample: functions (1) with  $x_0 = 10$  mm,  $y_0 = -10$  mm,  $r_0 = 30$  mm,  $a = 5, a_1 = 2, b = -0.5, b_1 = 0.5$  (a) and with  $x_0 = 0$  mm,  $y_0 = 20$  mm,  $r_0 = 30$  mm,  $a = 5, a_1 = 2, b = 0.5, b_1 = 0.5$  (b).

60 × 60 × 20 mm) or cylindrical (60 × 20 mm) sample (Fig. 1). The values of dielectric constant and the loss factor of the profiles were supposed to be within the intervals  $1 \leq \epsilon' \leq 10$ ,  $0 \leq \epsilon'' \leq 6$ . FDTD cell sizes in the sample were  $2 \times 2 \times 2$  mm and in the rest of the waveguide  $9 \times 9 \times 9$  mm.

The efficiency of the algorithm is illustrated in Fig. 3 showing the ANN performance in the reconstruction of coefficients  $a$  and  $a_1$  responsible for the function values in the maximum and for the plateau's level; other parameters ( $x_0$ ,  $y_0$  and  $r_0$ ) are fixed in these experiments. The curve in Fig. 4 shows a typical average error versus a number of training points. It has been shown that with a sufficiently high number of training points (in the present setting, nearly 550), the accuracy of this technique becomes about the same for both, rectangular and cylindrical, samples.

Functionality of the technique is illustrated here by the results of computational experiments showing the surfaces of actual  $\epsilon'$ -profiles and their Gaussian reconstruction in the rectangular and cylindrical samples (Figs. 5 and 6). It is seen that the reconstructed surfaces are very similar to the ones represented by Gaussian functions.

## CONCLUSION

In this contribution, we have presented an applied technique for reconstruction of 2-D profiles of complex permittivity of the samples in a waveguide system. The technique is based on the approximation of the profile by a continuous smooth function defined by small number of coefficients and reconstruction of these coefficients by a neural network inversion; here, we have demonstrated the functionality of the algorithm featuring the Gaussian approximations (1) and illustrated its satisfactory performance in finding 4 out of 7 independent parameters which can be responsible for building a variety of profile's configurations. Reconstruction of spatial distribution of  $\epsilon'$  and  $\epsilon''$  has been demonstrated for rectangular and cylindrical samples.

Our study suggest that the concept of reconstruction of function parameters is highly promising as it has the potential to improve the accuracy and resolution by applying more sophisticated approximating functions with sufficiently small number of unknown coefficients. The proposed technique can be thus directly extended to the general case of profile reconstruction in 3-D.

## ACKNOWLEDGEMENT

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