

# UNIQUENESS OF COMPLEX PERMITTIVITY RECONSTRUCTION FOR AN ARBITRARILY-SHAPED BODY IN A PARALLEL-PLANE WAVEGUIDE

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**Abstract:** The paper presents a statement and a proof of uniqueness of solution to the inverse problem of determination of permittivity of a lossy dielectric inclusion in a parallel-plane waveguide from the reflection and transmission characteristics. The approach is based on the analysis of asymptotic representations of a solution to the direct problem of diffraction of a transverse electric wave and employs a generalization of the notion of partial far-field patterns applied for a guide.

## INTRODUCTION

With the recent remarkable progress of computational resources, computer-aided design has become a valuable component in developing systems of microwave power engineering. Knowledge of complex permittivity ( $\varepsilon = \varepsilon' - i\varepsilon''$ ) of materials involved in an application is critical for creating an adequate model and thus for successful system design. However, the dielectric constant  $\varepsilon'$  and the loss factor  $\varepsilon''$  are not always available. The lack of data regarding realistic materials motivates further development of practical technology of determining complex permittivity. Since  $\varepsilon'$  and  $\varepsilon''$  are not directly measured, but calculated given the data on some measurable characteristics, a related numerical simulator may be made involved in determination of material parameters through a numerical solution of a corresponding inverse problem. This approach has been taken in a number of techniques using the finite element method and the finite-difference time-domain (FDTD) method for modeling of the entire experimental fixtures. Further exploring this trend, Eves et al.[1] and Yakovlev et al.[2] have recently developed the novel neural-network-based FDTD-backed technique capable of efficiently determining the dielectric constant and the loss factor of diverse materials placed in a transmission-line-type cavity. The experimental part is reduced to measuring the reflection and transmission coefficients of the systems. The technique is demonstrated to be versatile, robust, frequency- and cavity-independent, and applicable to the samples and fixtures of arbitrary configuration. However, while it has been shown by Eves, et al.[3] that the reconstructed  $\varepsilon'$  and  $\varepsilon''$  can be easily validated and are proved to be accurate, uniqueness of this reconstruction remains to be an assumption. This contribution outlines the initial results of the original study aiming to show that determination of complex permittivity of a body in a waveguide is unique when  $\varepsilon'$  and  $\varepsilon''$  are reconstructed from the related reflection and transmission coefficients.

A goal of our study is to develop solution techniques elaborated by Shestopalov et al.[4, 5] for the direct and inverse boundary value problems (BVPs) for Maxwell's and Helmholtz

equations associated with the wave propagation in the waveguides with dielectric inclusions. The methods of reconstructing the shape of the scatterer or its permittivity are developed in Colton et al.[6] when obstacles are perfectly conducting or dielectric bodies in two- or three-dimensional space. The recent paper by Shestopalov et al.[7] suggests the technique for cylindrical scatterers in half-space. The uniqueness is proved when the data in the inverse problems of finding the shape of the scatterer or permittivity of the inclusion (i) consist of the scattered far-field patterns given for the plane wave irradiating the obstacle from all directions, and (ii) are available for all frequencies varying in a certain interval. However, when a dielectric body is situated in a waveguide, similar results concerning the unique solvability and efficient solution techniques for reconstructing permittivity are not available. This becomes a driving force of our effort in developing a new approach to the solution of both direct and inverse scattering problems in waveguides. The present paper proves the uniqueness for a parallel-plane waveguide; it is expected that the proof will be later extended to a more complex case of a general waveguide.

### DIFFRACTION PROBLEM

We consider a parallel-plane waveguide  $S = \{(y, z) : -\pi < y < \pi, |z| < \infty\}$  containing a nonmagnetic, isotropic, and inhomogeneous dielectric inclusion having the cross section  $D \subset Q = \{(y, z) : -\pi < y < \pi, -2\pi\delta < z < 2\pi\delta\}$ . The permittivity function  $\varepsilon = \varepsilon(y, z)$  is assumed to be continuously differentiable and such that  $\text{supp } m(y, z) \subset Q$ , where  $m(y, z) = 1 - \varepsilon(y, z)$ . The unit-magnitude TE electromagnetic wave is supposed to have only one nonzero component  $E_x^{inc}(y, z) = u^i(y, z) = \sin(y) \exp(i\Gamma_1 z)$ . When scattered by  $D$ , the longitudinal component of the total field  $u(y, z) = E_x(y, z) = E_x^{inc}(y, z) + E_x^{scat}(y, z) = u^i(y, z) + u^s(y, z)$  is the solution to the BVP stated in Shestopalov et al.[5]

$$[\Delta + \kappa^2 \varepsilon(y, z)]u(y, z) = 0 \text{ in } S, \quad u(\pm\pi, z) = 0, \quad (1)$$

$$u(y, z) = u^i(y, z) + u^s(y, z), \quad u^s(y, z) = \sum_{n=1}^{\infty} a_n^{\pm} \exp(i\Gamma_n z) \sin(ny), \quad z \begin{matrix} > \\ < \end{matrix} \pm 2\pi\delta, \quad (2)$$

where  $\Gamma_n = (\kappa^2 - n^2)^{1/2}$  satisfy the conditions  $\text{Im } \Gamma_n \geq 0$ ,  $\Gamma_n = i|\Gamma_n|$ ,  $|\Gamma_n| = \text{Im } \Gamma_n = (n^2 - \kappa^2)^{1/2}$ ,  $n > \kappa$ , and it is assumed that the series in (2) converges absolutely and uniformly and allows for double term-wise differentiation.

### UNIQUENESS OF THE PERMITTIVITY RECONSTRUCTION

Assume that the cross-section  $D$  occupied by the dielectric inclusion is fixed and there are given sets of the partial far-field patterns

$$U_{\infty, \pm}(y, \kappa) = U_{\infty, \pm}(y, \kappa; \varepsilon; u) = \left\{ u_{\infty, \pm}^{(n)}(y, \kappa) \right\}_{n=1}^{\infty} = \left\{ \sin(ny) u_{\infty, \pm}^{(n)}(\kappa) \right\}_{n=1}^{\infty} \quad (3)$$

for all  $y \in [-\pi, \pi]$  and (i) for one frequency or (ii) for all values in an interval  $\kappa \in K = (\kappa_1, \kappa_2) \subset (1, 2)$ . Variable  $y$  in (3) plays the role of the observation angle of the far field pattern. However, the angle of incidence of the plane wave (which enters the far field pattern representation in case of scattering by an obstacle in space) has no counterpart in this statement. Therefore, it is not possible to prove the uniqueness of finding permittivity of the inclusion directly using the methods by Colton et al.[6]. So

we let  $\varepsilon_1(y, z)$  and  $\varepsilon_2(y, z)$  be two different permittivity functions (continuous in  $S$ ) such that corresponding (nontrivial) partial far-field pattern vectors (3) coincide,

$$U_{\infty, \pm}^1 - U_{\infty, \pm}^2 \equiv 0, \quad y \in [-\pi, \pi], \quad U_{\infty, \pm}^j = \left\{ u_{\infty, \pm}^{(n), j}(y, \kappa) \right\}_{n=1}^{\infty} = U_{\infty, \pm}(y, \kappa; \varepsilon_j; u_j), \quad j = 1, 2. \quad (4)$$

Using the reduction to a volume integral equation (VIE) with the kernel involving Green's function of the empty guide (used in Shestopalov et al.[5]) it can be shown that the scattered fields generated by  $U_{\infty, \pm}^j(y, \kappa)$  are

$$u_j^{s, \pm}(y, z) = -\sin(y) \exp(-i\Gamma_1 z) u_{\infty, \pm}^{(1), j}(\kappa) - \sum_{n=2}^{\infty} \frac{i}{\pi} \frac{\exp(\mp |\Gamma_n| z)}{|\Gamma_n|} u_{\infty, \pm}^{(n), j}(y, \kappa), \quad j = 1, 2. \quad (5)$$

Based on the unique solvability of the VIE we prove the following statements: (i) conditions (4) imply that the functions specifying the scattered fields coincide in the whole domain  $S$ , and (ii) the total field  $u = u^i + u^s$  is uniquely determined from (5) by  $U_{\infty, \pm}(y, \kappa)$  in the domains  $z > 2\pi\delta$  (+) and  $z < -2\pi\delta$  (-), respectively, and thus in  $S$ . This means that  $u(y, z)$  satisfies equation (1) with two different permittivity functions  $\varepsilon_1(y, z)$  and  $\varepsilon_2(y, z)$ , which yields  $\varepsilon_2(q) \equiv \varepsilon_1(q)$  because  $\kappa^2[\varepsilon_2(q) - \varepsilon_1(q)]u(q) = 0$  and  $u(y, z) \neq 0$ .

## CONCLUSION

We have proved the uniqueness of the solution to the inverse problem of finding permittivity of a lossy arbitrarily-shaped body inside a parallel-plane waveguide on the basis of knowledge of the partial far-field patterns of the transmitted and reflected fields. The technique enables one to consider the case when the domain containing the scatterer is a waveguide complementing thus the inverse scattering theory. The obtained asymptotic representation of the scattered field justifies the way of choosing the data for reconstructing complex permittivity in the form of the reflection and transmission coefficients.

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