Reconstruction of Complex Permittivity with Neural Networks

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Abstract

An innovative cavity-independent method of determination of dielectric properties of arbitrarily shaped materials is presented. Complex permittivity is reconstructed using a neural networking procedure matching the measured and FDTD modeled frequency characteristics of the reflection coefficient. High accuracy and efficiency of the method is demonstrated through numerical testing and determination of dielectric properties of fresh and saline water at 915 MHz.

Introduction

Nowadays, a heightened interest in complex permittivity \( \varepsilon = \varepsilon' - i\varepsilon'' \) results from the fact that contemporary electromagnetic (EM) modeling tools contribute to the design process by allowing the engineers to extensively characterize a constructed device prior to making a physical prototype. To prepare a trustworthy simulation, it is necessary to have good knowledge of the dielectric properties of all media involved.

Reliable and accurate determination of dielectric constant \( \varepsilon' \) and loss factor \( \varepsilon'' \) of practical materials is a challenging problem. The perturbation method (e.g., [1]) can give satisfactory results, but it has a principle limitation: the volume of the sample must be electrically small so as to produce a negligible effect on the field configuration in the cavity. To make the transmission/reflection techniques employing waveguides [2] or coaxial lines [3] accurate, the measured sample requires the laborious preparation of being very carefully machined to comply with strict dimensional tolerance requirements.

FDTD was used in [4] for simulation of EM processes in a cavity with a degenerate mode. The resonant frequencies computed with Finite Element Method (FEM) was compared in [5] with the measured ones and the best approximation for dielectric constant was sought by resorting to an evolution strategy. In [6], the experimental values of \( S \)-parameters around the fundamental resonant frequency were matched with the FEM data with subsequent error minimization. Since FEM and FDTD modeling are capable of handling arbitrary configurations of the materials, these methods reconstruct dielectric properties without requiring the sample under test to be of some specific geometry. This paper also contributes to the trend of development of the methods based on modeling.

Methodology

We consider an arbitrary cavity with the dielectric inclusion of arbitrary configuration and address a frequency response of the magnitude of the reflection coefficient \( |S_{11}| \) determined in the cavity’s input port (Fig. 1). Interpreting the measured curve as a goal function and the curve computed for arbitrary \( \varepsilon' \) and \( \varepsilon'' \) as an initial guess, we can arrange for subsequent simulations for some different input values of complex permittivity to closer approximate the measured curve. Let \( S_c(f) \) and \( S_e(f) \) be the characteristics computed for some particular known \( \varepsilon' \) and \( \varepsilon'' \) and measured when the complex permittivity of the sample is unknown (Fig. 2). If it happens that \( S_c = S_e \) in a certain frequency range \( (f_1, f_n) \), then we consider the values of \( \varepsilon' \) and \( \varepsilon'' \) (i.e., the permittivity for which \( S_c \) is computed) at \( f_0 \) the solution to our problem. In general, \( S_c \neq S_e \), so the simulation software could be run in an optimization loop until the both curves coincide at \( n \) points in the interval \( (f_1, f_n) \) such that \( f_0 \in (f_1, f_n) \). The goal is therefore to solve a 2-parameter optimization problem minimizing the increment \( \Delta_i = |S_{ei} - S_{ci}| \) as a function of \( \varepsilon' \) and \( \varepsilon'' \) simultaneously at \( n \)
points \( f_i \) \((i = 1, \ldots, n)\). Using conventional optimization methods for solving this type of problem would be unfeasible because of unrealistic resources, which may be required for simultaneous minimization of several functions of two variables. We suggest using artificial neural networks (NN) since they can work with numerical data, are suitable for dealing with optimization-type problems, and can handle nonlinearities (which we expect to find in \( S \)-parameter characteristics) much better than polynomial approximations [7].

**Neural Network Solution**

We have constructed a NN which consists of two identical multilayer perceptrons (MLP) dealing with \( \varepsilon' \) and \( \varepsilon'' \) (Fig. 3). The input of the network is represented by the \( n \)-vector \( X = [|S_{11}|(f_1), \ldots, |S_{11}|(f_n)]^T \) containing the values of the reflection coefficient at \( n \) points of the frequency interval around \( f_0 \). On the output of the network, we have the 1-vector determining either \( \varepsilon' \) (in Net \( A \)), or \( \varepsilon'' \) (in Net \( B \)). The feedforward computation of complex permittivity is carried out in accordance with the formulas

\[
\begin{align*}
\varepsilon' &= \sum_{j=0}^{N_A} w_{ij}^A \sigma \left[ \sum_{i=0}^{n} w_{ij}^A |S_{11}|(f_i) \right], \\
\varepsilon'' &= \sum_{j=0}^{N_B} w_{ij}^B \sigma \left[ \sum_{i=0}^{n} w_{ij}^B |S_{11}|(f_i) \right],
\end{align*}
\]

where \( N_A \) and \( N_B \) are the numbers of hidden neurons in Net \( A \) and Net \( B \), \( \sigma(\cdot) = \tanh(\cdot) \) is the hyperbolic tangent activation function used for the hidden neurons, and \( w_{pq}^{3,4,A,B} \) represents the network weights coming from the \( q \)th neuron in the 1st or 2nd layer to the \( p \)th neuron in the 2nd or 3rd layer of Net \( A \) or Net \( B \).

For network training and testing, we use information generated in the modeling phase of the method. A cluster of input vectors computed at different points on the complex permittivity plane is used in the training data. These are pairs of \((X_k, \varepsilon'_k), (X_k, \varepsilon''_k)\), \( k = 1, \ldots, P \), where \( \varepsilon'_k \) and \( \varepsilon''_k \) are the desired outputs of the network for inputs \( X_k \), and \( P \) is the number of training vectors. The aim is to adjust the network weights \( w \) to reduce the errors

\[
\begin{align*}
e^{e'} &= \frac{1}{2} \sum_{k=1}^{P} |\varepsilon'(X_k, w) - \varepsilon'_k|^2, \\
e^{e''} &= \frac{1}{2} \sum_{k=1}^{P} |\varepsilon''(X_k, w) - \varepsilon''_k|^2,
\end{align*}
\]

where \( \varepsilon'(X_k, w) \) and \( \varepsilon''(X_k, w) \) are the NN output for input \( X_k \). Minimization of (2) is carried out with the subsequent use of the backpropagation technique and the Levenberg-Marquardt method. When the network is well trained, a vector \( X_m \) consisting of \( n \) values of \( |S_{11}| \) measured at \( f_1, \ldots, f_n \) is supplied on the network input, and the NN determines
corresponding complex permittivity of the sample. This solution has been implemented in MATLAB 6 environment. EM modeling of frequency responses of $|S_{11}|$ is performed using the full-wave FDTD modeling software QuickWave-3D (QW3D) (www.qwed.com.pl).

**Numerical & Experimental Testing**

Network training was conducted using QW3D-generated $S$-parameter responses for a shorted WR284 waveguide section with a rectangular block (base side length 20 mm, height 30 mm) of dielectric material in the corner. Three frequency points were selected to represent each curve. The values of dielectric constant and loss factor used for generating the training and testing data sets were taken from the ranges $5 \leq \varepsilon' \leq 9$ (step 1.0) and $0.2 \leq \varepsilon'' \leq 1.0$ (step 0.2). The QW3D model representing this scenario was built with a non-uniform mesh having cubic 7.5 and 3 mm cells in air and in the dielectric respectively. When determining complex permittivity with $N_A = N_B = 10$ and a testing set of 51 samples, the network-generated results compared with corresponding exact data gave the sum-squared error of order of $10^{-3}$.

We also worked with an experimental fixture measuring reflections from a cavity holding a liquid dielectric sample. The setup consisted of the metal box (a section of a WR975 waveguide), a Teflon block with the cylindrical cutout suitable for holding liquids, and a Teflon plate designed to locate the block at a specific position on the bottom of the cavity (Fig. 4). The reflection coefficient was measured using an Agilent 8753 Automated Network Analyzer. The dimensions of the Teflon block (70 x 70 x 50 mm) and radius and height of the cylindrical container (25 mm and 40 mm respectively) were selected, the position of the Teflon block in the cavity was determined by modeling to ensure the minimal sensitivity of $|S_{11}|$ characteristics to accidental deviations of the sample’s location: it was centered on the cavity’s axis at 40 mm from the waveguide’s shorting wall.

This experimental setup has been used to determine the dielectric properties of fresh and saline water. Some divergence of the measured and simulated data at $f_1$ and $f_2$ seen from Fig. 5 (average disagreement is 2.4%) can be attributed to the strong dependence of water’s permittivity on frequency – a phenomenon which can not be explicitly taken into
Table 1. Complex Permittivity of Fresh and Saline Water at 915 MHz

<table>
<thead>
<tr>
<th>Conditions</th>
<th>(\varepsilon')</th>
<th>(\varepsilon'')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_w = 0.033\ %\text{, } T = 18.6^\circ\text{C})</td>
<td>80.6</td>
<td>80.5</td>
</tr>
<tr>
<td>(S_w = 0.1\ %\text{, } T = 18.5^\circ\text{C})</td>
<td>80.8</td>
<td>80.9</td>
</tr>
</tbody>
</table>

account in our simulations. However, the adequacy of the model can be surely acknowledged. The training and testing sets were prepared for the ranges \(60 \leq \varepsilon' \leq 90\) and \(1 \leq \varepsilon'' \leq 20\). Out of totally 332 samples in the database, 108 were used for training, and 224 for testing. The optimal structure of the network was found as having \(N = 15\) hidden neurons for dielectric constant (Net \(A\)) and \(N = 19\) for loss factor (Net \(B\)). The normalized sum of squared differences between the desired and actual network responses at the training stage in both cases was less than \(10^{-4}\) while the testing error assessed as a maximum of all differences between the desired and actual responses was found to be 3.3\% for \(\varepsilon'\) and 4.8\% for \(\varepsilon''\). Given the measured values of \(|S_{11}|\) at \(f_1 = 910, f_2 = 915,\) and \(f_3 = 920\ \text{MHz},\) our trained NN has generated a dielectric constant and the loss factor for water samples; the respective numbers are shown in Table 1 along with the data computed from the accurate polynomial model [8]. The results are in close agreement: for fresh water, we have 0.12\% and 1.2\% divergence for \(\varepsilon'\) and \(\varepsilon''\) respectively.

**Discussion and Conclusion**

Our NN-controlled method of determination of dielectric properties possesses a clear practical advantage. While a relatively large computational effort may be required for creation of a database of training and testing samples, network training requires nearly negligible time. So whenever we work in one frequency range with materials that can take some pre-defined form (e.g., any liquid in a holder, like the Teflon block in our fixture), the database is created only once; one can do that prior to actual experimental testing. Thereafter each new material can be processed practically in real time provided that its dielectric constant and loss factor are within the ranges specified in the database.

We have proposed a novel technology employing an optimization technique based on the use of artificial neural networks (NN) controlling full-wave FDTD simulation. The method relies on the measurement of reflections. No requirements are applied on the size and geometry of the sample and the fixture. Further study of the chosen experimental setup has revealed a notable sensitivity of the frequency response to the sample’s height and led us to the conceptual conclusion that although the methods of complex permittivity determination involving general numerical algorithms (like our NN technique and the ones in [4-6]) in principle work with cavities and samples of arbitrary configurations, in practice the methods’ options may be to some extent limited by underlying physical phenomena. Truly efficient application of these techniques may require using specially designed experimental fixtures that both conserve the universal features of the method and allow sample preparation to less strict dimensional tolerances.

**References**