Stochastic Modeling of the Relationship Between the Surface Electromyogram and Muscle Torque

by

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Abstract

The surface EMG waveform is a complex spatial-temporal interference pattern of the electrical activity of the various underlying muscle tissues. Viewed as a random signal, the surface EMG has been shown to resemble a band-limited Gaussian random process. For isometric, isotonic contractions, the amplitude of the surface EMG has been observed to increase with the contraction level. Estimates of surface EMG amplitude are used as the command input to myoelectric prostheses and have also been investigated as indicators of muscle force. Typical surface EMG amplitude estimators have poor signal-to-noise ratio (SNR) performance. Previous investigators have demonstrated two separate techniques for improving performance: 1) temporal whitening of a single EMG channel (single channel optimization) and 2) the use of multiple EMG channels (spatial combination). This thesis presents theoretical techniques and experimental investigations which combined these two improvement techniques.

A stochastic, multiple channel, functional model of the surface EMG was constructed, from which optimal estimators of the EMG amplitude were derived. Experiment 1 investigated optimal amplitude estimators at one joint angle. Surface EMG waveforms from elbow flexors/extensors were recorded for non-fatiguing, constant-force, isometric contractions at 10, 25, 50 and 75% of maximum voluntary contraction (MVC). An SNR for EMG amplitude estimates was computed (deviations about the mean value of the estimate were considered as noise). A moving average root mean square estimator (245ms window) provided an average ± standard deviation SNR of 10.7 ± 3.3 for the individual channels. Temporal whitening improved the SNR to 17.6 ± 6.0. Combining temporal whitening and spatial combination with four channels improved the SNR to 24.6 ± 10.4. On one subject, an SNR of 35.0 ± 13.4 was achieved by optimally combining eight channels. Sensitivities of the temporal whitening and spatial combination algorithms were investigated experimentally and with simulation studies.

Experiment 2 investigated the influence of joint angle on the calibration and performance of amplitude estimators. Surface EMG waveforms from elbow flexors/extensors were recorded for non-fatiguing, constant-force, isometric contractions at 50% MVC from joint angles over a span of 90°. Results indicated that
SNR performance was similar at all angles and independent of the angle used to calibrate the optimized estimators.

An algebraic relation between joint torque and simultaneous flexor/ extensor contraction EMG amplitudes was assumed. A least squares method for identifying the parameters of this relation was investigated in a simulation study. Experiment 3 investigated the relationship between EMG amplitude and joint torque. Surface EMG waveforms from elbow flexors and extensors, and joint torque were simultaneously recorded for non-fatiguing, slowly force-varying (quasi-isotonic), isometric contractions spanning 0–50% MVC. Single/ multiple channel whitened/ unwhitened EMG amplitude estimates were used to predict joint torque via the parameterized relation. Each multiple channel predictor had a standard error (SE) approximately 70% of its respective single channel predictor. Predictors with whitened EMG amplitude estimates, however, performed more poorly than those without, because the performance of whitened estimators degraded when contraction levels were less than 10% MVC. A new surface EMG model, which included additive measurement noise, was consistent with these results. Based on the new model, an ad hoc solution to EMG amplitude estimation was proposed. The ad hoc multiple channel predictor had an SE approximately 90% of the unwhitened multiple channel predictor.

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Finally, I give thanks to my God for all He bestows upon me. I pray that I am able to sow some of what I learn.
To my grandparents and parents,
who have worked so hard all their lives,
that their progeny might not have to;

To my wife, Mary
who took each step of this project with me;

To my God,
who carried me for most of those steps.
# Table of Contents

Abstract .................................................................................................................. 3

Acknowledgements ............................................................................................... 6

Table of Contents ................................................................................................ 9

List of Figures ....................................................................................................... 14

List of Acronyms ................................................................................................. 21

1 Introduction ....................................................................................................... 23

2 Anatomy, Neuromuscular Physiology and Electrophysiology 
   of the Elbow ..................................................................................................... 37
   2.1 Introduction .................................................................................................. 38
   2.2 Skeletal Anatomy ....................................................................................... 38
   2.3 Muscle Structure ....................................................................................... 40
   2.4 Excitation-Contraction of the Muscle Fiber ............................................... 45
   2.5 Force Regulation in Muscle ...................................................................... 52

3 Electromyography ............................................................................................. 57
   3.1 Introduction ................................................................................................ 58
   3.2 The Surface Electromyogram .................................................................. 58
   3.3 Processing the Constant Force Surface Electromyogram ....................... 65
   3.4 The Surface Electromyogram and Joint Torque ....................................... 72
      3.4.1 Mechanical Indeterminacy of Muscles About a Joint ....................... 75
      3.4.2 EMG Recording and Processing Techniques ..................................... 78
      3.4.3 Individual Versus Universal Characterization ................................. 81

4 A Surface EMG to Joint Torque Model ............................................................ 83
   4.1 Introduction ................................................................................................ 84
   4.2 Modeling Overview ................................................................................... 84
   4.3 A Functional Mathematical Model of Surface EMG ................................. 87
      4.3.1 Model Formulation ........................................................................... 87
      4.3.2 Optimization of Several EMG Configurations ................................. 92
      4.3.3 The Mean Absolute Value Processor .............................................. 112
   4.4 A Functional Mathematical Model of Torque Generation 
      About the Elbow ....................................................................................... 114
4.5 Discussion .......................................................... 120
  4.5.1 Functional Modeling and the Physiology ...................... 120
  4.5.2 Surface EMG Model ......................................... 122
  4.5.3 Torque Generation About the Elbow ........................ 123
4.6 Conclusion .......................................................... 124

5 Experiment 1 — Part I: Single Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle .................................................. 127
  5.1 Introduction ...................................................... 128
  5.2 Experimental Apparatus ....................................... 129
  5.3 Experimental Methods ....................................... 134
  5.4 Methods of Analysis — Single Channel EMG Amplitude Estimation .............................................. 137
  5.5 Results and Discussion — Single Channel EMG Amplitude Estimation .............................................. 142
    5.5.1 Assessing Some Model Assumptions ...................... 142
    5.5.2 A Temporal Whitening Example ......................... 148
    5.5.3 Study of Variations in the Construction of Temporal Whitening Filters ...................................... 153
    5.5.4 Robustness — The Problem of Uncharacteristic Data .......... 170
    5.5.5 The Mean Absolute Value Processor .................... 174
    5.5.6 Predicted Versus Achieved SNR Performance ............. 175
  5.6 Summary and Conclusion — Single Channel EMG Amplitude Estimation .............................................. 177

6 Experiment 1 — Part II: Multiple Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle .................................................. 183
  6.1 Introduction ...................................................... 184
  6.2 Methods of Analysis — Multiple Channel EMG Amplitude Estimation .............................................. 184
  6.3 Results and Discussion — Multiple Channel EMG Amplitude Estimation .............................................. 188
    6.3.1 A Spatial Uncorrelation Example ...................... 188
    6.3.2 Study of Variations in the Construction of Spatial Uncorrelation Filters ...................................... 195
    6.3.3 The Mean Absolute Value Processor .................... 216
    6.3.4 Predicted Versus Achieved SNR Performance ............. 218
  6.4 Summary and Conclusion — Multiple Channel EMG Amplitude Estimation .............................................. 220
# 7 Experiment 2 — Influence of Joint Angle on the Construction and Performance of Optimized Single and Multiple Channel Constant Torque EMG Amplitude Estimators

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>228</td>
</tr>
<tr>
<td>7.2 Experimental Methods</td>
<td>228</td>
</tr>
<tr>
<td>7.3 Methods of Analysis</td>
<td>230</td>
</tr>
<tr>
<td>7.4 Results</td>
<td>231</td>
</tr>
<tr>
<td>7.4.1 Volume of Usable Data Collected</td>
<td>231</td>
</tr>
<tr>
<td>7.4.2 Single Channel Amplitude Estimation</td>
<td>231</td>
</tr>
<tr>
<td>7.4.3 Multiple Channel Amplitude Estimation</td>
<td>233</td>
</tr>
<tr>
<td>7.5 Discussion and Conclusion</td>
<td>238</td>
</tr>
</tbody>
</table>

# 8 Simulation Studies of EMG Amplitude Estimation

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1 Introduction</td>
<td>244</td>
</tr>
<tr>
<td>8.2 Functional Simulations</td>
<td>244</td>
</tr>
<tr>
<td>8.2.1 Design — Functional Simulations</td>
<td>244</td>
</tr>
<tr>
<td>8.2.2 Methods — Functional Simulations</td>
<td>247</td>
</tr>
<tr>
<td>8.2.3 Results — Functional Simulations</td>
<td>248</td>
</tr>
<tr>
<td>8.2.4 Discussion — Functional Simulations</td>
<td>261</td>
</tr>
<tr>
<td>8.3 Physiologic Simulations</td>
<td>264</td>
</tr>
<tr>
<td>8.3.1 Design — Physiologic Simulations</td>
<td>264</td>
</tr>
<tr>
<td>8.3.2 Methods — Physiologic Simulations</td>
<td>266</td>
</tr>
<tr>
<td>8.3.3 Results — Physiologic Simulations</td>
<td>268</td>
</tr>
<tr>
<td>8.3.4 Discussion — Physiologic Simulations</td>
<td>268</td>
</tr>
<tr>
<td>8.4 Conclusion</td>
<td>272</td>
</tr>
</tbody>
</table>

# 9 Estimation of Joint Torque from the EMG Amplitude:

## A Short Simulation Study

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1 Introduction</td>
<td>274</td>
</tr>
<tr>
<td>9.2 EMG to Torque Basis Functions</td>
<td>276</td>
</tr>
<tr>
<td>9.3 Investigation of Inputs to Identification Technique</td>
<td>277</td>
</tr>
<tr>
<td>9.3.1 Evaluation of Appropriate Inputs</td>
<td>277</td>
</tr>
<tr>
<td>9.3.2 Simulation Methods and Results</td>
<td>278</td>
</tr>
<tr>
<td>9.3.3 Discussion of Inputs to Identification Technique</td>
<td>285</td>
</tr>
<tr>
<td>9.4 Performance of EMG to Torque Identification Technique</td>
<td>287</td>
</tr>
<tr>
<td>9.4.1 Simulation Design</td>
<td>287</td>
</tr>
<tr>
<td>9.4.2 Simulation Methods</td>
<td>287</td>
</tr>
<tr>
<td>9.4.3 Simulation Results</td>
<td>292</td>
</tr>
<tr>
<td>9.4.4 Discussion of EMG to Torque Identification Performance</td>
<td>293</td>
</tr>
<tr>
<td>9.5 Conclusion</td>
<td>301</td>
</tr>
</tbody>
</table>
Appendix C: Processing the Constant-Force Contraction

Multichannel Electromyogram ........................................... 395
C.1 A Functional Model of Measured Surface EMG .................. 396
C.2 Optimization of Several EMG Configurations .................. 404
  C.2.1 Case I — Single Channel: Uncorrelated Samples ............ 405
  C.2.2 Case II — Single Channel: Correlated Samples ............... 411
  C.2.3 Case III — Multiple Channels: Uncorrelated Samples,
                 Uncorrelated Channels ...................................... 421
  C.2.4 Case IV — Multiple Channels: Uncorrelated Samples,
                 Correlated Channels .......................................... 429
  C.2.5 Case V — Multiple Channels: Correlated Samples,
                 Uncorrelated Channels ...................................... 439
  C.2.6 Case VI — Multiple Channels: Correlated Samples,
                 Correlated Channels .......................................... 445
C.3 Some Alternative EMG Processors .................................... 449
  C.3.1 The Mean Absolute Value Processor ............................ 449
  C.3.2 Gaussian Model with Additive Gaussian Noise ............... 456

Appendix D: Experimental Apparatus ..................................... 471
D.1 Introduction .......................................................... 472
D.2 Instrumented Chair .................................................... 472
  D.2.1 Mechanical Design .................................................. 472
  D.2.2 Strain Gauges and Associated Electronics ..................... 488
  D.2.3 Performance of the Instrumented Chair ....................... 488
D.3 EMG Recording Apparatus ............................................ 493
D.4 Informed Consent ..................................................... 493

References ............................................................... 501
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Surface EMG Waveform and Corresponding Joint Torque</td>
</tr>
<tr>
<td>1.2</td>
<td>EMG Amplitude Estimates</td>
</tr>
<tr>
<td>1.3</td>
<td>Prediction of Joint Torque From the Surface EMG</td>
</tr>
<tr>
<td>2.1</td>
<td>Bones of the Upper Limb</td>
</tr>
<tr>
<td>2.2</td>
<td>Muscles of the Arm</td>
</tr>
<tr>
<td>2.3</td>
<td>Neural Innervation of Skeletal Muscle</td>
</tr>
<tr>
<td>2.4</td>
<td>Structure of Mammalian Skeletal Muscle Fiber</td>
</tr>
<tr>
<td>2.5</td>
<td>Pattern of Actin and Myosin Filaments</td>
</tr>
<tr>
<td>2.6</td>
<td>Sliding Filament Mechanism of Sarcomere Contraction</td>
</tr>
<tr>
<td>2.7</td>
<td>Nerve Action Potential</td>
</tr>
<tr>
<td>2.8</td>
<td>Excitation-Contraction Coupling in the Myofibril</td>
</tr>
<tr>
<td>2.9</td>
<td>Diagram of Muscle Fiber Electro-Mechanical Response to Excitation</td>
</tr>
<tr>
<td>2.10</td>
<td>Muscle Force Versus Stimulation Frequency</td>
</tr>
<tr>
<td>2.11</td>
<td>Common Drive</td>
</tr>
<tr>
<td>2.12</td>
<td>Length-Tension Diagram for Skeletal Muscle</td>
</tr>
<tr>
<td>2.13</td>
<td>Biceps and Triceps Muscles Moment Arms</td>
</tr>
<tr>
<td>3.1</td>
<td>Surface Myoelectric Activity</td>
</tr>
<tr>
<td>3.2</td>
<td>Filtering Property of Tissue</td>
</tr>
<tr>
<td>3.3</td>
<td>Differential Electrode Filter Function</td>
</tr>
<tr>
<td>3.4</td>
<td>Motor Unit Action Potential</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic Representation of EMG Generation</td>
</tr>
<tr>
<td>3.6</td>
<td>Functional Mathematical Model of EMG</td>
</tr>
<tr>
<td>3.7</td>
<td>Non-Linear Isometric, Isotonic EMG to Torque Data</td>
</tr>
<tr>
<td>3.8</td>
<td>Theoretic Relation of EMG to Motor Unit Recruitment, Firing Rate</td>
</tr>
<tr>
<td>3.9</td>
<td>Evidence of Muscular Co-Contraction</td>
</tr>
<tr>
<td>4.1</td>
<td>Functional Model of Measured Surface EMG Waveform and Joint Torque</td>
</tr>
<tr>
<td>4.2</td>
<td>Functional Model of a Single Channel of EMG</td>
</tr>
<tr>
<td>4.3</td>
<td>Functional Model of Multiple Channels of EMG</td>
</tr>
<tr>
<td>4.4</td>
<td>EMG Model Case I — Single Channel: Uncorrelated Samples</td>
</tr>
<tr>
<td>4.5</td>
<td>Optimal EMG Processor — Single Channel: Uncorrelated Samples</td>
</tr>
<tr>
<td>4.6</td>
<td>Generic Optimal Filter — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>4.7</td>
<td>Alternate Generic Optimal Filter — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>4.8</td>
<td>Optimal EMG Processor — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>4.9</td>
<td>EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>4.10</td>
<td>Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>4.11</td>
<td>EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>4.12</td>
<td>Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>4.13</td>
<td>EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>4.14</td>
<td>Discrete Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>4.15</td>
<td>Optimal EMG Processor — Multiple Channels: Correlated Samples, Correlated Channels</td>
</tr>
<tr>
<td>4.16</td>
<td>Gaussian and Laplace Probability Density Functions</td>
</tr>
<tr>
<td>4.17</td>
<td>Simple Model of Torques About the Elbow</td>
</tr>
<tr>
<td>5.1</td>
<td>Instrumented Torque Chair</td>
</tr>
<tr>
<td>5.2</td>
<td>Electrode-Amplifier Locations</td>
</tr>
<tr>
<td>5.3</td>
<td>Chart of Subjects for Experiment 1</td>
</tr>
<tr>
<td>5.4</td>
<td>Measured Torque Versus Time at the Four Contraction Levels</td>
</tr>
<tr>
<td>5.5</td>
<td>Power Spectra of the Measured Torque</td>
</tr>
<tr>
<td>5.6</td>
<td>Autoregressive Modeling of the EMG Power Spectrum</td>
</tr>
<tr>
<td>5.7</td>
<td>Whitening Filter Design</td>
</tr>
<tr>
<td>5.8</td>
<td>Effect of Temporal Whitening on the EMG Signal</td>
</tr>
<tr>
<td>5.9</td>
<td>Effect of Temporal Whitening on the EMG Estimate</td>
</tr>
<tr>
<td>5.10</td>
<td>Single Channel Whitening — Effect of Number of Filter Coefficients and Length of Filter Calibration Record</td>
</tr>
<tr>
<td>5.11</td>
<td>Single Channel Whitening — Effect of Number of Filter Coefficients</td>
</tr>
<tr>
<td>5.12</td>
<td>Single Channel Whitening — Effect of Calibration Record Length</td>
</tr>
<tr>
<td>5.13</td>
<td>Single Channel Whitening at Each Contraction Level</td>
</tr>
<tr>
<td>5.14</td>
<td>Power Spectra at Different Sampling Rates</td>
</tr>
<tr>
<td>5.15</td>
<td>Effect of Aliasing on the Temporal Whitening Filter</td>
</tr>
<tr>
<td>5.16</td>
<td>Single Channel Whitening — Effect of Sampling Rate</td>
</tr>
<tr>
<td>5.17</td>
<td>Filter Coefficients Derived from One Record</td>
</tr>
</tbody>
</table>
Figure Title

5.18 Single Channel Whitening — One Whitening Filter Per Subject
5.19 Single Channel Whitening — Universal Whitening Filter
5.20 Whitening Uncharacteristic Data
5.21 Scatter Plots of Predicted Vs. Achieved SNR Performance
5.22 Tabulated Results of Single Channel Amplitude Estimation

6.1 Four Channels of Raw EMG
6.2 Four Channels of Temporally Whitened, Spatially Uncorrelated EMG
6.3 Power Spectrum of Four Channels of Raw EMG
6.4 Power Spectrum of Four Channels of Temporally Whitened, Spatially Uncorrelated EMG
6.5 Ensemble (Spatial) Correlation Coefficient Matrices
6.6 Multiple Channel Amplitude Estimate
6.7 Chart of SNR Performance of Four EMG Channels
6.8 Multiple Channel Uncorrelating — Effect of Number of Channels and Length of Spatial Filter Calibration Record
6.9 Multiple Channel Uncorrelating — Effect of Calibration Record Length
6.10 Multiple Channel Whitening at Each Contraction Level
6.11 Multiple Channel Whitening — Effect of Sampling Rate
6.12 Multiple Channel Whitening with Two Channels
6.13 Multiple Channel Whitening with Four Channels
6.14 Multiple Channel Whitening with Six Channels
6.15 Multiple Channel Whitening with Eight Channels
6.16 Multiple Channel Whitening — Single Record Calibration for Subjects DA and FA
6.17 Two Channel Processor with Adjacent and Non-Adjacent Channels
6.18 Four Channel Processor with Adjacent and Non-Adjacent Channels
6.19 Overlay of Multiple Channel Amplitude Estimate
6.20 MAV Processor Performance
6.21 Multiple Channel Predicted Vs. Achieved SNR Performance
6.22 Tabulated Results of Multiple Channel Amplitude Estimation

7.1 Chart of Subjects for Experiment 2
7.2 Tabulated Results of Single Channel Amplitude Estimation at Different Angles
Figure | Title
---|---
7.3 | SNR Performance Versus Joint Angle
7.4 | Tabulated Results of Four Channel Amplitude Estimation at Different Angles
7.5 | Tabulated Results of Four Channel Amplitude Estimation at One Angle
7.6 | Four Channel Processor Channel Correlations
8.1 | Functional Simulation Data
8.2 | Simulation EMG Amplitude Estimates
8.3 | Tabulated Single Channel Results for Functional Simulations and Subject DA
8.4 | Tabulated Multiple Channel Results for Functional Simulations and Subject DA
8.5 | MAV Versus RMS Results for Functional Simulations and Subject DA
8.6 | Simulation Predicted Vs. Achieved SNR Performance
8.7 | Model for Developing Monopolar AP
8.8 | Simulated Bipolar Motor Unit Action Potential
8.9 | Physiologic Simulation EMG Waveform and its Power Spectrum
8.10 | EMG Amplitude Versus Physiologic Model Parameters
9.1 | Method of Estimating Joint Torque From Surface EMG Waveforms
9.2 | Effect of Co-Contraction on Condition Number — Two Contractions
9.3 | Effect of Co-Contraction on Condition Number — Three Contractions
9.4 | Effect of EMG Amplitude Scale on Condition Number — Two Contractions
9.5 | Effect of EMG Amplitude Scale on Condition Number — Three Contractions
9.6 | Simulation of EMG to Torque Relationship
9.7 | Simulation EMG Waveforms
9.8 | Identification of a Quadratic Simulated Model
9.9 | Identification of a Cubic Simulated Model
9.10 | Standard Errors in Simulation Fits
9.11 | Simulation Standard Error Plot
9.12 | Combined Mutual Standard Errors in Simulation Fits
9.13 | Condition Numbers in Simulation Fits
<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>Single Channel EMG Amplitude Estimators</td>
</tr>
<tr>
<td>10.2</td>
<td>Power Spectrum of Amplitude Modulated EMG</td>
</tr>
<tr>
<td>10.3</td>
<td>Discrete-Time Functional Model of a Single Channel of EMG in Additive Gaussian Noise</td>
</tr>
<tr>
<td>10.4</td>
<td>Simulated EMG Power Spectrum With Additive White Gaussian Noise</td>
</tr>
<tr>
<td>10.5</td>
<td>EMG Power Spectra at Different Contraction Levels</td>
</tr>
<tr>
<td>10.6</td>
<td>Adaptive EMG Processor — Single Channel</td>
</tr>
<tr>
<td>10.7</td>
<td>Optimal Low Pass Cutoff Frequency at Unity EMG Amplitude</td>
</tr>
<tr>
<td>10.8</td>
<td>Adaptive Low Pass Filter Cutoff Frequency</td>
</tr>
<tr>
<td>10.9</td>
<td>Data From Isometric, Quasi-Isotonic Contraction</td>
</tr>
<tr>
<td>10.10</td>
<td>Chart of Subjects for Experiment 3</td>
</tr>
<tr>
<td>10.11</td>
<td>Correlation in Antagonist EMG Channel Pairs</td>
</tr>
<tr>
<td>10.12</td>
<td>Correlation Coefficients for Various Electrode-Amplifier Pair Locations</td>
</tr>
<tr>
<td>10.13</td>
<td>Extension EMG to Torque Fits</td>
</tr>
<tr>
<td>10.14</td>
<td>Flexion EMG to Torque Fits</td>
</tr>
<tr>
<td>10.15</td>
<td>EMG to Torque Fits With Multiple Channel Adaptive EMG Amplitude Estimators</td>
</tr>
<tr>
<td>10.16</td>
<td>Standard Errors in Standard Polynomial Basis Fits</td>
</tr>
<tr>
<td>10.17</td>
<td>Standard Errors in Orthogonal Polynomial Basis Fits</td>
</tr>
<tr>
<td>10.18</td>
<td>Plot of Standard Errors in Standard Polynomial Basis Fits</td>
</tr>
<tr>
<td>10.19</td>
<td>Condition Numbers in Single Channel Fits</td>
</tr>
<tr>
<td>10.20</td>
<td>Condition Numbers in Multiple Channel Fits</td>
</tr>
<tr>
<td>10.21</td>
<td>Prediction of Joint Torque</td>
</tr>
<tr>
<td>10.22</td>
<td>Standard Errors in Standard Polynomial Basis Predictions</td>
</tr>
<tr>
<td>10.23</td>
<td>Standard Errors in Orthogonal Polynomial Basis Predictions</td>
</tr>
<tr>
<td>10.24</td>
<td>Prediction of Joint Torque With Extrapolation Error</td>
</tr>
<tr>
<td>10.25</td>
<td>Standard Errors in Standard Polynomial Basis Predictions — Reduced Prediction Range</td>
</tr>
<tr>
<td>10.26</td>
<td>Multiple Adaptive Standard Polynomial Errors</td>
</tr>
<tr>
<td>11.1</td>
<td>Wiener Filter Adaptive EMG Amplitude Estimator</td>
</tr>
<tr>
<td>B.1</td>
<td>The Gaussian Probability Distribution</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>C.1</td>
<td>Measured Surface EMG Waveform and Corresponding Joint Torque</td>
</tr>
<tr>
<td>C.2</td>
<td>Multiplicative Model of Measured Surface EMG Waveform</td>
</tr>
<tr>
<td>C.3A</td>
<td>Discrete-Time Functional Model of a Single Channel of EMG</td>
</tr>
<tr>
<td>C.3B</td>
<td>Continuous-Time Functional Model of a Single Channel of EMG</td>
</tr>
<tr>
<td>C.4A</td>
<td>Discrete-Time Functional Model of Multiple Channels of EMG</td>
</tr>
<tr>
<td>C.4B</td>
<td>Continuous-Time Functional Model of Multiple Channels of EMG</td>
</tr>
<tr>
<td>C.5</td>
<td>EMG Model Case I — Single Channel: Uncorrelated Samples</td>
</tr>
<tr>
<td>C.6</td>
<td>Optimal EMG Processor — Single Channel: Uncorrelated Samples</td>
</tr>
<tr>
<td>C.7</td>
<td>Generic Optimal Filter — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>C.8</td>
<td>Alternate Generic Optimal Filter — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>C.9</td>
<td>Optimal EMG Processor — Single Channel: Correlated Samples</td>
</tr>
<tr>
<td>C.10A</td>
<td>Discrete-Time EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.10B</td>
<td>Continuous-Time EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.11A</td>
<td>Discrete-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.11B</td>
<td>Continuous-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.12A</td>
<td>Discrete-Time EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>C.12B</td>
<td>Continuous-Time EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>C.13A</td>
<td>Discrete-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>C.13B</td>
<td>Continuous-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels</td>
</tr>
<tr>
<td>C.14A</td>
<td>Discrete-Time EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.14B</td>
<td>Continuous-Time EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.15A</td>
<td>Discrete-Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
<tr>
<td>C.15B</td>
<td>Continuous-Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels</td>
</tr>
</tbody>
</table>
Figure  Title

C.16A Discrete-Time Optimal EMG Processor —
Multiple Channels: Correlated Samples, Correlated Channels
C.16B Continuous-Time Optimal EMG Processor —
Multiple Channels: Correlated Samples, Correlated Channels
C.17 The Laplace Probability Distribution
C.18 Discrete-Time Functional Model of a Single Channel of EMG in Additive Gaussian Noise
C.19 Optimal EMG Processor —
White Additive Gaussian Noise Model
C.20 Optimal EMG Processor —
Additive Gaussian Non-White Noise Model
C.21 Discrete-Time Functional Model of EMG in Additive Noise
C.22 Discrete-Time Multiple Channel Optimal EMG Processor —
Additive Noise, First Case
C.23 Discrete-Time Multiple Channel Optimal EMG Processor —
Additive Noise, Second Case

D.1 Straight-Back Metal Chair
D.2 Back Rest Plate — Front View
D.3 Subassembly One
D.4 Crossbar
D.5 Crossbar Spacer
D.6 Pivot Plate
D.7 Pivot Plate Mounting Spacer
D.8 Subassembly Two
D.9 Angle Bracket
D.10 Beam
D.11 Subassembly Three
D.12 Beam Spacer
D.13 Rear Beam Shield
D.14 Complete Mechanical Assembly of the Instrumented Chair
D.15 Strain Gauge Excitation Circuit
D.16 Strain Gauge Amplification Circuit
D.17 Instrumented Chair Performance
D.18 Liberty Mutual MYO111 Electrode-Amplifier Wiring Assignments
D.19 Inverting Amplifier Circuit
D.20 Informed Consent Document
D.21 Subject Interview Form
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADP</td>
<td>adenosine diphosphate</td>
</tr>
<tr>
<td>AP</td>
<td>action potential</td>
</tr>
<tr>
<td>AR</td>
<td>autoregressive</td>
</tr>
<tr>
<td>ARMA</td>
<td>autoregressive moving average</td>
</tr>
<tr>
<td>ATP</td>
<td>adenosine triphosphate</td>
</tr>
<tr>
<td>A±SD</td>
<td>average ± standard deviation</td>
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<tr>
<td>CE</td>
<td>correlation-ergodic</td>
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<tr>
<td>CPSD</td>
<td>cross power spectral density</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<tr>
<td>EMG</td>
<td>electromyogram</td>
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<tr>
<td>IPI</td>
<td>inter-pulse interval</td>
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<tr>
<td>JWSS</td>
<td>jointly wide-sense stationary</td>
</tr>
<tr>
<td>LTI</td>
<td>linear time-invariant</td>
</tr>
<tr>
<td>MAMAV</td>
<td>moving average mean absolute value</td>
</tr>
<tr>
<td>MARMS</td>
<td>moving average root mean square</td>
</tr>
<tr>
<td>MAV</td>
<td>mean absolute value</td>
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<tr>
<td>ML</td>
<td>maximum likelihood</td>
</tr>
<tr>
<td>MLE</td>
<td>maximum likelihood estimation</td>
</tr>
<tr>
<td>ms</td>
<td>millisecond(s)</td>
</tr>
<tr>
<td>MSE</td>
<td>mean square error</td>
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<tr>
<td>MSEMGr</td>
<td>measured surface electromyogram</td>
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<tr>
<td>MU</td>
<td>motor unit</td>
</tr>
<tr>
<td>MUAP</td>
<td>motor unit action potential</td>
</tr>
<tr>
<td>MUAPT</td>
<td>motor unit action potential train</td>
</tr>
<tr>
<td>MVC</td>
<td>maximum voluntary contraction</td>
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<tr>
<td>PDF</td>
<td>probability density function</td>
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<tr>
<td>PSD</td>
<td>power spectral density</td>
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<tr>
<td>RC</td>
<td>resistor-capacitor</td>
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<tr>
<td>RMS</td>
<td>root mean square</td>
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<tr>
<td>SE</td>
<td>standard error</td>
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<tr>
<td>SI</td>
<td>statistically independent</td>
</tr>
<tr>
<td>SNR</td>
<td>signal to noise ratio</td>
</tr>
<tr>
<td>T tubules</td>
<td>transverse tubular system</td>
</tr>
<tr>
<td>WSS</td>
<td>wide-sense stationary</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction
Accurate and reliable prediction of muscle forces from observation of the surface electromyogram (EMG) would provide a safe, non-invasive tool for the control of cybernetic prostheses, as well as for the study of human movement and biomechanics. In the field of prosthetics, it is often the case that muscle associated with actuation of a limb remain after amputation. For example, above elbow amputees most often have remnant biceps and triceps muscle — muscle which formerly contributed to flexion/extension of the elbow joint. Myoelectrically controlled prosthetic elbows utilize the electrical activity of such remnant muscle to control the prosthesis. The Boston Arm is an example of a myoelectrically controlled elbow prosthesis. (See Williams, 1990, for a review of upper-extremity prosthesis and their control modalities.) Such arms are limited in their ability to deduce intended elbow torque from the EMG signal. Myoelectric signal processing presently represents a “weak link” in providing a functional replacement for the amputation. Accurate and reliable interpretation of the surface EMG could vastly improve prosthetic function.

The study of human movement and biomechanics has long been plagued with an inability to non-invasively determine the torques generated by muscular activation. Since muscles are the actuators of human movement, non-invasive assessment of their force output would be an indispensable tool. While EMG is certainly effective in providing an active/inactive (on/off) determination of muscle activation state, it is as yet ineffective in determining the torque produced by a muscle. Another problem is that external non-invasive kinematic observation of limbs can, at best, predict minimum muscular forces as the common force produced by two antagonist muscles cancels and is not observed externally. Thus, most studies of human movement rely little on muscle force information.
Figure 1.1 shows a surface EMG signal from the triceps muscle and the torque generated about the elbow. The surface EMG waveform is a zero mean random process whose amplitude distribution is often described as Gaussian. The amplitude of the random process seems to be related to the level of muscular contraction. For this reason, the problem of relating the EMG waveform to joint torque has typically been divided into two subproblems. First, the EMG amplitude is estimated from the EMG waveform. Then, second, joint torque is estimated from the EMG amplitude. This division is followed in this project.

Historically, Inman et al. (1952) are credited with the first EMG amplitude estimator. They likened the EMG signal to an amplitude modulated process and performed estimation with a full-wave rectifier (non-linear demodulator) followed by a simple resistor-capacitor low pass filter (smoother). Their standard estimator has been applied extensively in the study of the surface electromyogram and is considered the present day standard. Its performance, however, is poor.

Two methods have been demonstrated to improve the performance of EMG amplitude estimation. First, investigators have experimentally demonstrated (Harba and Lynn, 1981; Hogan and Mann, 1980a, 1980b; Kaiser and Peterson, 1974) or analytically predicted (D’Alessio, 1984; Filligoi and Mandarini, 1984) that temporal whitening (temporal optimization) of the EMG signal prior to demodulation improves the amplitude estimate. Improvements of 35–100% over the standard estimator were demonstrated. Second, the combined use of four EMG sensors (spatial combination) on a muscle provided a 40–180% improvement over the standard estimator (Harba and Lynn, 1981; Hogan and Mann, 1980a, 1980b).

The first two experimental studies of this project combined the above two
Figure 1.1: Surface EMG Waveform and Corresponding Joint Torque

A) Surface EMG waveform recorded from the triceps muscle with a bipolar electrode during an isometric non-fatiguing contraction. EMG is normalized to its maximum value in this trial.

B) Torque generated about the elbow during elbow extension for the same trial shown in (A) above. Torque is normalized to its maximum value in this trial.
improvement techniques (temporal-spatial optimization) and, for non-fatiguing, isometric, isotonic contractions about the elbow, studied the sensitivities, design considerations and performance of optimized multiple channel EMG amplitude estimators. A stochastic multiple channel functional model of the surface EMG waveform was constructed. Based upon the model, optimal estimators of the EMG amplitude were derived. Experimental trials evaluated the optimal estimators. Figure 1.2 shows a sample set of results. The two improvement techniques were shown to provide additive improvements. With an eight channel optimal estimator, an estimated 274% improvement over the standard estimator was realized (Clancy and Hogan, 1990).

The second portion of the EMG waveform to joint torque problem was prediction of joint torque from the EMG amplitude estimate. Many studies have been conducted for the case of non-fatiguing, isometric, isotonic contraction (see Chapter 3). Unfortunately, these studies have primarily debated whether or not the EMG to torque relationship is linear. Additionally, all of these studies used non-optimized, single channel EMG amplitude estimates. For the prediction of joint torque, the shape of the relationship is less important than the accuracy of the prediction.

The third experimental study of this project applied high fidelity EMG amplitude estimates to the problem of predicting joint torque. A functional model of the elbow, which parameterized the EMG amplitude to joint torque relationship, was developed. Since muscular co-contraction was always observed during moderate or greater muscular contraction, simultaneous flexor and extensor activity was considered. Methods for identifying the parameters of the model were investigated in a simulation study. Experimentally, surface EMG waveforms
Figure 1.2: EMG Amplitude Estimates

Top left plot is the measured torque for non-fatiguing, isometric contraction of elbow flexors at 25% maximum voluntary contraction. Top right plot is the moving average root mean square (MARMS) estimate (245ms window) of the amplitude of EMG channel 3. Bottom left plot is the temporally whitened MARMS estimate of the amplitude of EMG channel 3. Bottom right is the temporally whitened, spatially uncorrelated (8 channels) MARMS estimate of the EMG amplitude. Beginning portion of each amplitude estimate depicts the rise time of the estimator. All data are from the same trial. A signal to noise ratio (SNR) was computed for each plot with deviations about the mean value of the estimate considered as noise. [From Clancy and Hogan, 1990]
from elbow flexors and extensors, and joint torque were simultaneously recorded for non-fatiguing, quasi-isotonic, isometric contractions spanning 0–50% MVC. Single/multiple channel whitened/unwhitened EMG amplitude estimates were used to identify an EMG to torque relation, and then predict joint torque based on this relation. Each multiple channel predictor had an SE approximately 70% of its respective single channel predictor. Predictors with whitened EMG amplitude estimates, however, performed more poorly than those without, because whitening actually degraded EMG amplitude estimation performance when contraction levels were less than 10% MVC. A new surface EMG model, consistent with these results, was proposed. The new EMG model included an additive measurement noise. Based on the new model, an ad hoc adaptive solution to EMG amplitude estimation was proposed. The adaptive multiple channel predictor had an SE approximately 90% of the unwhitened multiple channel predictor. More formal solutions to the new EMG model might provide additional performance improvement. The best multiple channel predictor had approximately 67% the SE in predicting total joint torque of the unwhitened single channel predictor. Figure 1.3 shows a sample set of results.

The chapters which follow provide the details of these studies. First, some pertinent background material is reviewed. Next, EMG amplitude estimation is considered. Last, prediction of joint torque from the EMG waveform is presented.
Figure 1.3: Prediction of Joint Torque From the Surface EMG

Top plot is the predicted (dashed line) joint torque using a non-optimized single channel EMG amplitude estimator and a third order standard polynomial basis EMG to torque relationship. The standard error (SE) is 53.2 A/D units. Bottom plot is the predicted (dashed line) joint torque of the same trial using a four channel adaptive EMG amplitude estimator and a third order standard polynomial basis EMG to torque relationship. The SE is 23.8 A/D units. Solid lines are the measured torque. Subject began the trial at \(\approx50\%\) extension maximum voluntary contraction (MVC) and ended the trial at \(\approx50\%\) flexion MVC.
The Chapters Which Follow

Chapter 2: Anatomy, Neuromuscular Physiology and Electrophysiology of the Elbow

This chapter reviews the anatomy, physiology and electrophysiology of the upper limb. The skeletal anatomy and muscles which act about the elbow are presented. The process of muscular excitation-contraction and the manner by which a muscle regulates force is reviewed. Readers familiar with the anatomy, physiology and electrophysiology of the upper limb may skip this chapter without loss of continuity.

Chapter 3: Electromyography

Chapter 3 begins with a physiologic-based description of the surface electromyogram. Next, functional stochastic descriptions of the EMG waveform are described, followed by a review of EMG amplitude estimators. Then, previous studies relating the EMG amplitude to joint torque are presented. Finally, methods for improving the fidelity of the EMG to torque relationship are discussed. Readers familiar with the study of the relationship between the surface EMG and joint torque can skip all but the last section of this chapter without loss of continuity.

Chapter 4: A Surface EMG to Joint Torque Model

Two models are presented in this chapter. The first model is a functional stochastic model of the surface EMG waveform. Based on this model, optimum EMG amplitude estimators are derived for several types of EMG configurations. The
second model is a functional model of torque generation about the elbow. This model relates simultaneous flexor and extensor muscle group EMG amplitude to generated joint torque.

Chapter 5: Experiment 1 — Part I: Single Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle

Chapter 5 describes the first experimental trial and presents results for single channel amplitude estimates. The experiments consist of non-fatiguing, isometric, isotonic contractions at one joint angle. Sensitivities, design considerations and performance of the optimal single channel EMG amplitude estimation algorithms were investigated experimentally.

Chapter 6: Experiment 1 — Part II: Multiple Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle

This chapter presents results of the first experimental trial for multiple channel amplitude estimates. Sensitivities, design considerations and performance of the optimal multiple channel EMG amplitude estimation algorithms were investigated experimentally.

Chapter 7: Experiment 2 — Influence of Joint Angle on the Construction and Performance of Optimized Single and Multiple Channel Constant Torque EMG Amplitude Estimators

Chapter 7 describes the second experimental trial and results. The experiments consisted of non-fatiguing, isometric, isotonic contractions at five joint angles. Sensitivities, design considerations and performance of the single/ multiple chan-
nel EMG amplitude estimation algorithms, as a function of joint angle, were investigated experimentally.

Chapter 8: Simulation Studies of EMG Amplitude Estimation

Two simulation studies are reported. The first study generated surface EMG waveforms according to the stochastic model of Chapter 4. The second study generated the surface EMG waveform according to a physiologic-based description. Both models investigated EMG amplitude estimates derived from the simulated data. Results from the simulated data were compared to results from the experimental data.

Chapter 9: Estimation of Joint Torque from the EMG Amplitude: A Short Simulation Study

This short simulation study investigated two aspects of the EMG amplitude to torque identification problem. The first study attempted to determine the appropriate experimental data required to perform the identification. The second study evaluated the performance of the identification technique, based on a known EMG to torque model.

Chapter 10: Experiment 3 — Estimation of Joint Torque from the EMG Amplitude

Chapter 10 describes the third experimental trial and results. The experiments consisted of non-fatiguing, isometric, quasi-isotonic contractions at one joint angle. EMG amplitude to joint torque identification and prediction were investigated experimentally.
Chapter 11: Discussion and Prospectus for Future Research

A discussion of topics relevant to the complete project and directions for future research investigations are presented.

Chapter 12: Project Summary

This chapter is a short summary of the project.

Appendix A: Random Variables, Vectors and Processes

This appendix reviews random variables, vectors and processes and establishes the notation used throughout this document.

Appendix B: Gaussian Random Variables, Vectors and Processes

This appendix provides notation and certain results specific to Gaussian random variables, vectors and processes.

Appendix C: Processing the Constant-Force Contraction Multichannel Electromyogram

This appendix provides the complete mathematical details of all of the EMG waveform models and EMG amplitude estimators used throughout the main text.

Appendix D: Experimental Apparatus

This appendix serves as an operator's manual and maintenance document for the experimental apparatus.
References
Chapter 2

Anatomy, Neuromuscular Physiology and Electrophysiology of the Elbow
2.1 Introduction

This thesis will study the electrical and mechanical activity of the human elbow joint and those muscles which act to impart a torque about the joint. Thus, this chapter will review the anatomy, physiology and electrophysiology of the upper limb. Many of the details contained in this chapter will contribute to the development of the electrical and mechanical models used in this thesis. These details are presented together as one chapter to provide a background, or reference, from which ensuing chapters can draw. Readers familiar with the anatomy, physiology and electrophysiology of the upper arm may skip this chapter without loss of continuity. First, the skeletal anatomy will be presented. Next, details of the muscle structure are discussed. Then, the process of muscular excitation-contraction is detailed. Finally, the manner by which a muscle regulates force is examined.

2.2 Skeletal Anatomy

The three bones of most importance to the elbow joint are the bone of the upper arm — the humerus — and the bones of the lower arm — the radius and ulna. Figure 2.1 shows these bones as well as all other bones of the upper limb. The elbow joint is essentially a hinge joint with the upper arm (humerus) forming one link and the lower arm (radius and ulna) forming the second link. This hinge joint limits extension of the forearm with respect to the upper arm such that the angle between the forearm and the upper arm, measured in the plane of forearm extension, has a maximum of approximately 180 degrees.

Figure 2.2 shows the principal muscles of the upper arm which are responsible
Figure 2.1: Bones of the Upper Limb

Front view of all bones and many skeletal landmarks of the upper limb. [From Anderson, 1983, Figure 6-1]
for elbow flexion and extension. The muscles which produce elbow flexion are the biceps brachii, the brachialis, the brachio-radialis and the coracobrachialis (not shown in the figure). When the hand is supinated, the biceps brachii muscles are the prime flexors. Elbow extension is produced by the triceps brachii.

2.3 Muscle Structure

A muscle is comprised of parallel muscle fibers (from a few hundred to as many as tens of thousands) ranging from ten to eighty microns in diameter. Each muscle fiber is a long, cylindrical, multinucleated cell which typically extends the full length of the muscle, originating and terminating in a tendon. Adjacent fibers have no direct mechanical, electrical, or chemical communication, being separated by an extracellular fluid. Neural innervation of the muscle fiber by a single alpha motoneuron occurs at approximately the midpoint of the muscle, as shown in Figure 2.3, forming the neuromuscular junction. All of the neuromuscular junctions of a muscle are dispersed within a contained region called the innervation zone. A single motoneuron, stemming from the spinal cord, branches to innervate several muscle fibers of a muscle. The motoneuron and its associated muscle fibers are called a motor unit (MU). MU’s in small muscles (extraocular for example) can be comprised of as few as 3–10 muscle fibers, while MU’s in large muscles (medial head of gastrocnemius for example) can be comprised of as many as 2000 muscle fibers. Within a particular muscle, smaller motoneurons are associated with a smaller number of muscle fibers, and larger motoneurons are associated with a larger number of muscle fibers. Thus, a gradation in MU size exists. It is important to note that all muscle fibers within an MU possess

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Figure 2.2: Muscles of the Arm

Lateral view of the principal muscles which act about the elbow. [From Anderson, 1983, Figure 6–37A]
Figure 2.3: Neural Innervation of Skeletal Muscle

[From Basmajian and DeLuca, 1985, Figure 1.7]
remarkably similar characteristics. Muscle fibers from a given MU are randomly distributed throughout a local area of the muscle and are interwoven with fibers from various (20–50) other MU’s.

Each muscle fiber is constituted of several hundred to several thousand parallel myofibrils, as shown in Figure 2.4. The myofibrils are surrounded by two tubular membrane structures, the transverse tubular system (T tubules) and the sarcoplasmic reticulum. The T tubules run transversely through the fiber, two T tubules per basic contractile unit. The T tubules invaginate the fiber from the cell membrane (sarcolemma), forming a membrane which is continuous with the sarcolemma. The sarcoplasmic reticulum is an irregular matrix shaped tubular system surrounding each myofibril and extending between the contacts of the myofibril with the T tubules. Adjacent to the T tubules, the sarcoplasmic reticula terminate in large chambers called terminal cisternae. The terminal cisternae store calcium ions in high concentration. Also present within the muscle fiber are a large number of mitochondria. The mitochondria lie between and parallel to the myofibrils. The mitochondria form adenosine triphosphate (ATP), the energy source for contraction.

Within each myofibril are approximately 1500 myosin filaments and 3000 actin filaments. The actin and myosin filaments lie side by side in a partially interdigitated pattern, as depicted in Figure 2.5. This pattern repeats itself over the length of the myofibril. The actin filaments attach to the Z membrane, an intracellular membrane that attaches to myofibrils throughout the entire cross-section of the muscle fiber. The portion of the myofibril between successive Z membranes, called a sarcomere, is the basic contractile unit of skeletal muscle. The normal resting length of a sarcomere is approximately 2.0 microns. The
Figure 2.4: Structure of Mammalian Skeletal Muscle Fiber

A single muscle fiber surrounded by its sarcolemma has been cut away to show individual myofibrils (1). The cut surface of the myofibrils shows the actin and myosin filaments. The sarcoplasmic reticulum (2) with its cisterns (3) surround each myofibril. The T system of tubules (4), which invaginates from the sarcolemma, contacts the myofibrils twice in every sarcomere. The T system and the adjacent cisterns of the sarcoplasmic reticulum constitute a triad. (6) Mitochondria. [Modified from Ganong, 1981, Figure 3-1]
myosin filaments have small projections, called cross-bridges, which protrude from the surfaces of the filament. In the presence of calcium, the cross-bridges of the myosin filaments interact with the actin filaments causing the actin and myosin filaments to slide together. This mechanism of contraction, shown in Figure 2.6 is known as the sliding filament mechanism. Energy for this contractile process is derived from the breakdown of ATP to adenosine diphosphate (ADP).

2.4 Excitation-Contraction of the Muscle Fiber

Both nerve and muscle fiber exhibit a resting membrane potential of between -75 and -95 mV (i.e. the inside of the cell is negatively charged with respect to the extracellular fluid). Upon excitation by the central nervous system, the lo-

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Figure 2.5: Pattern of Actin and Myosin Filaments

[Modified from Guyton, 1976, Figure 11-3]

Figure 2.6: Sliding Filament Mechanism of Sarcomere Contraction

The relaxed and contracted states of a myofibril showing sliding of the actin filaments into the channels between the myosin filaments. [Modified from Guyton, 1984, Figure 7-5]

...cal region of the motoneuron is depolarized, to a potential of approximately +35 mV, and current flows within the nerve cell. This current interacts with adjacent nerve tissue, causing depolarization in the adjacent tissue. As this process progresses, a nerve impulse conducts along the nerve with a conduction velocity of approximately 30–75 meters/second. In general, large MU’s have faster conduction velocities than small MU’s. A given region of the nerve remains depolarized for a fraction of a millisecond (ms) and then reverts to its polarized (resting potential) state. Once repolarized, the nerve fiber can be re-excited. Each time a motoneuron is excited an essentially identical pattern of depolarization/repolarization occurs. The rapid repolarization of nerve fiber allows the conduction of up to 250–2500 nerve impulses per second. The characteristic time course of the electric potential in a local area, shown in Figure 2.7, is known as the action...
Figure 2.7: Nerve Action Potential

Diagram of complete action potential of a large mammalian nerve fiber. [Modified from Ganong, 1981, Figure 2-7]
potential (AP). Upon reaching the neuromuscular junction, the nerve impulse is transmitted chemically to the cell membrane of the associated muscle fiber.

From the neuromuscular junction, depolarization spreads in both directions along the cell membrane, over the entire length of the muscle fiber. The muscle fiber AP has a conduction velocity of 2–6 meters/second and a duration of 1–5 ms. Once re-polarized, the muscle fiber can be re-excited. Each time the muscle fiber is activated, an essentially identical pattern of depolarization/re-polarization occurs. The AP travels to the deep interior of the muscle fiber via the T tubules. In fact, the T tubules contain extracellular fluid and are internal extensions of the cell membrane. As the AP travels through the T tubules of a sarcomere, current is caused to flow through the adjacent terminal cisternae of the sarcoplasmic reticulum. This current flow induces a rapid release of calcium ions from the terminal cisternae into the intracellular fluid, or sarcoplasm. The calcium ions facilitate contraction of the sarcomere. Contraction will continue while a high concentration of calcium ion remains in the sarcoplasm. A continuously active calcium pump, located in the walls of the sarcoplasmic reticulum, pumps the calcium ions from the sarcoplasm back into the sarcoplasmic reticulum. The pump reduces the calcium ion concentration in the sarcoplasm to a low enough level that contraction is no longer facilitated. The cycle of calcium ion release and re-uptake typically lasts approximately 30 ms. Figure 2.8 depicts excitation-contraction coupling in the myofibril.

When a muscle fiber is excited, the fiber contracts for a brief period. This response, diagrammed in Figure 2.9, is called a muscle twitch. The twitch begins approximately 2 ms after the onset of sarcolemma depolarization, prior to the completion of repolarization. Depending on the particular muscle fiber, the du-
Figure 2.8: Excitation-Contraction Coupling in the Myofibril

The action potential causes release of calcium ions from the sarcoplasmic reticulum and then re-uptake of the calcium ions by a calcium pump. [From Guyton, 1981, Figure 11-12]
Figure 2.9: Diagram of Muscle Fiber Electro-Mechanical Response to Excitation

The electrical response (mV potential change) and the mechanical response (T, tension in arbitrary units) are plotted on the same abscissa (time). [From Ganong, 1981, Figure 3–4]
Figure 2.10: Muscle Force Versus Stimulation Frequency

The figure shows the isometric tension developed in a single muscle fiber during continuously increasing and decreasing stimulation frequencies. The dots at the top of the figure are at 0.2 second intervals. [From Ganong, 1981, Figure 3–9]

...ration of the twitch may be as short as 7.5 ms or as long as 100 ms. Because the motoneuron and the muscle fiber have a short electrical refractory period, re-excitation of the motoneuron and muscle fiber can occur prior to completion of the muscle fiber twitch. The contractile mechanism within the muscle fiber, however, does not have a refractory period. Repeated excitation prior to mechanical relaxation produces a response that is added to the contraction already present. Hence, the mechanical response to excitation depends critically on the time history of excitation. If the muscle is repetitively excited at a rate greater than or equal to a critical frequency, a fused contraction results. During tetanus, successive contractions fuse together and can not be distinguished one from the other. Figure 2.10 shows the isometric tension developed in a single muscle fiber versus stimulation frequency.
2.5 Force Regulation in Muscle

The contraction of one or more MU's develops tension in the muscle. To sustain this tension, the individual MU's must be repeatedly activated and/or other MU's recruited. Since repetitive voluntary activation of an MU does not normally occur at a regular rate, MU excitation is typically characterized by an average firing rate. The average firing rate is the reciprocal of the average time interval between a few successive excitations of the MU. Average firing rates as low as 5 pulses/second and as high as 100 pulses/second have been observed.

The time interval between successive excitations of an MU is called the inter-pulse interval (IPI). Several authors (see Basmajian and DeLuca, 1985 and DeLuca, 1979 for a review) have studied the statistical dependence of successive IPI's of an MU. Little or no dependence has been demonstrated. Synchronization is the tendency for one MU to be regularly excited at or near the times that other MU's are excited. Basmajian and DeLuca (1985) and DeLuca (1979) report that few studies have examined synchronization. For non-fatigued muscle, evidence both for and against synchronization has been presented. Direct and indirect evidence suggests the existence of synchronization during muscle fatigue. Another important property of muscular excitation is common drive. Common drive is the tendency for the average firing rates of all active MU's to rise/ fall in unison as a function of both force and time. DeLuca et al. (1982a, 1982b) present evidence, some of which is shown in Figure 2.11, supporting the existence of common drive. Further, DeLuca et al. (1982a, 1982b) assert that common drive indicates that the nervous system does not control the firing rates of MU's individually. Rather, the nervous system acts upon a group, or pool, of

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3Adapted from Basmajian and DeLuca (1985), DeLuca (1979) and Freund (1983).
Firing rate records of concurrently active MU's (dashed lines) are shown superimposed on the force output (continuous line) recorded during triangular force-varying contractions of the deltoid and first dorsal intersosseous (F.d.i.). Force levels are given in percent of maximal voluntary contraction (m.v.c) at right. Note the presence of separate vertical scales for each of the displayed parameters. [From DeLuca et al., 1982a, Figure 2]
motoneurons in a uniform manner. Hence, force modulation can be facilitated by a common modulation in the firing rate of all active MU's. Common drive does not imply synchronization since it describes the relationship of average firing rates, not the timing of individual MU excitations.

During a constant-force isometric contraction, DeLuca (1979) summarizes the present knowledge of force regulation by first noting that once recruited, an MU tends to remain in use throughout a contraction. The average firing rate of an MU decreases in time. To maintain a constant force, the contractile strength of individual MU's may increase proportionally in time. Little change in recruitment has been reported.

Force regulation is considerably more complex for a force-varying isometric contraction. For many contractions, recruitment is based on MU size. MU's with fewer fibers are recruited first and MU's with more fibers are recruited last. This ordered recruitment has been called the "size principle". Smaller MU's generally display longer contraction times than larger MU's. Thus, as the force of a contraction rises, increasingly larger MU's, which have a shorter contraction time, are recruited.

The interrelation between recruitment and firing rate, which is a function of the contraction level, often follows the following characteristic paradigm: Recruitment is the dominant manner by which force is increased until approximately 30% of maximum voluntary contraction (MVC). At this point, the majority of the muscle fibers are active. Further increase in force is generated primarily through an increase in the firing rate. In the range of 60%-75% MVC, all of the MU's in a normal muscle have been recruited. Therefore, from 75%-100% MVC an increase in muscle force is generated solely through an increase
Figure 2.12: Length-Tension Diagram for Skeletal Muscle

The passive tension curve measures the tension exerted by the muscle at each length when it is not stimulated. The total tension curve represents the tension developed when the muscle contracts isometrically in response to a maximal stimulus. The active tension is the difference between the two. Drawn from data on the human triceps muscle. [Modified from Ganong, 1981, Figure 3–10]

in the firing rate.

Both the passive tension inherent in a muscle and the tension developed actively during stimulation vary with the length of the muscle. Figure 2.12 shows this relationship for human triceps muscle. When the muscle is flaccid (no increase in length) there is no passive or active tension. For small increases in muscle length, passive tension is not developed and active tension (total tension minus passive tension) increases approximately linearly. At large increases in length, passive tension rises sharply while active tension peaks.
Figure 2.13: Biceps and Triceps Muscles Moment Arms

The moment arms of biceps and triceps muscles are shown as a function of the elbow angle $\theta$. The elbow angle $\theta$ equals zero when the forearm is fully extended and equals 90 degrees when the forearm and upper arm are at right angles. [From Messier et al., 1971, Figure 3]

In the intact limb, static muscle length can be related to limb orientation. The orientation also determines the mechanical advantage, or moment arm, formed by the muscle and the bones to which it connects. Messier et al. (1971) investigated the moment arm of biceps and triceps muscles as a function of elbow angle. Their data, shown in Figure 2.13, indicate that passive/active tension must be considered simultaneously with the mechanical moment arm when assessing the tension exerted by an intact muscle.
Chapter 3

Electromyography
3.1 Introduction

There exist a wealth of investigations into the relationship between the surface EMG and torque generated about a joint. These prior studies form an important background for this thesis. This chapter reviews several studies. Readers familiar with the study of the relationship between the surface EMG and joint torque can skip all but the last section of this chapter without loss of continuity.

Initially, a description of the surface EMG, based upon the physiology, is provided. This description draws heavily on the details of the previous chapter. Next, functional descriptions of the surface EMG, which consider the surface EMG as an amplitude-modulated band-limited process, are described. Since many investigations (this thesis included) consider the surface EMG amplitude to be the dominant information contained in the surface EMG waveform, a review of techniques for estimating the surface EMG amplitude is presented. Then, previous studies relating the surface EMG amplitude to joint torque are reviewed. Finally, several areas in which the fidelity of the EMG to torque relationship can be improved are discussed.

3.2 The Surface Electromyogram

If an electrode is applied to the skin surface above a muscle, then electrical activity of the muscle can be observed. Figure 3.1 shows surface EMG from the triceps muscle recorded by a differential pair of electrodes during a constant force non-fatiguing, isometric contraction at approximately 25% of MVC. This surface activity is a complex spatial-temporal summation of the individual muscle fiber AP’s. These AP’s spread through the body tissue to be recorded at the body
Figure 3.1: Surface Myoelectric Activity

The surface EMG was recorded from the triceps muscle during a constant force, non-fatiguing, isometric contraction at approximately 25% of MVC.
Figure 3.2: Filtering Property of Tissue

The parameter h indicates the distance from an active muscle fiber to the detection electrode. This theoretic model assumes a muscle fiber conduction velocity of 4 m/s and a muscle fiber radius of 50 μm. [From Lindstrom and Magnusson, 1977, Figure 2]

As an AP spreads through the body tissue to the recording site, the signal is altered. The intervening tissue acts as a low pass filter. Lindstrom and Magnusson (1977) presented a theoretical model of the myoelectric power spectrum which predicts this low pass filter effect. Figure 3.2 shows the results of their theoretic analysis. The cutoff frequency of the tissue filter is inversely proportional to the distance to the signal source. Thus, those AP’s closest to the recording site make the greatest contribution to the observed surface potential. As a rough guide, Lynn et al. (1978) proposed that a differential electrode pair obtains most of its signal energy from muscle fibers whose distance from the electrode axis is
Figure 3.3: Differential Electrode Filter Function

The electrode pair separation distance is 2 cm and the muscle fiber conduction velocity is 4 m/s. [From Lindstrom and Magnusson, 1977, Figure 3]

equal to the electrode pair separation distance.

Normally, a pair of differential electrodes, placed parallel to the direction of AP propagation, is used to observe the electrical activity of the muscle. With such a configuration, the AP will reach one electrode prior to the other. Those frequency components of the AP which have a wavelength corresponding to an integer multiple of the electrode pair separation distance will present the same voltage to each electrode at all instants in time. Hence, no differential signal will be recorded. Similarly, each intermediate frequency component of the AP will be associated with a particular gain. Thus, the differential electrode configuration acts as a filter whose characteristics are a function of the electrode pair separation distance and the muscle fiber conduction velocity. Figure 3.3 depicts this filter function for an electrode pair separation distance of 2 cm and a muscle fiber conduction velocity of 4 m/s. Note that the first cancellation frequency occurs
Figure 3.4: Motor Unit Action Potential

The motor unit action potential, \( h(t) \) is a spatial-temporal summation of the electrical activities of the individual muscle fibers. [Modified from Basmajian and DeLuca, 1985, Figure 3.2]

at the muscle fiber conduction velocity divided by the electrode pair separation distance.

DeLuca (1979; Basmajian and DeLuca, 1985) has developed a model for the electromyogram which draws upon underlying physiology. (For similar models see Brody et al., 1974 and Parker et al., 1977.) Figure 3.4 is a schematic representation of the observed electrical activity of a single MU, denoted a motor unit action potential (MUAP). Each muscle fiber within the MU (shown on the left of the figure) makes a contribution to the surface potential (shown on the right of the figure) which is a function of, at least, the spatial arrangement of
the fiber, the time onset of the fiber AP, the shape of the fiber AP, the distance from the fiber to the electrode, the filtering effects of tissue, the geometric arrangement of the electrodes, and the electrode recording apparatus. The MUAP can be modeled as the response of the MU to a single neural excitation, where the neural excitation is modeled as an impulse to the MU. Repeated stimulation of an MU results in a series of the characteristic MU responses, denoted a motor unit action potential train (MUAPT). Typically, many MU’s are active during a normal contraction. Each MU in the vicinity of a surface electrode will make a contribution to the observed potential. Hence, a complete excitation process evolves as is depicted in Figure 3.5. Trains of impulses traverse the various nerve fibers and excite all muscle fibers within their respective MU’s. Each impulse train elicits a characteristic MUAPT response. The surface electrode observes the composite activity of those MU’s within its recording field. Electronics, required for amplification of the signal, add additional noise to the signal. The recorded EMG, therefore, is a complex interference pattern of the electrical activity of many MU’s. In order to provide some conceptual simplification to this physiologic model, DeLuca defined a generalized firing rate as the mean value of the firing rates of the MUAPT’s detected during a contraction. DeLuca and Forrest (1973) utilized empirical data to derive a time and force dependent generalized firing rate for the deltoid muscle during a constant force isometric contraction.

By considering the EMG as a sum of many independent probabilistic events (i.e. the summed contribution of many independent MU’s, each MU being excited by a sequence of independently timed nerve impulses) the law of large numbers can be used to describe the EMG as a signal with a Gaussian dis-
Figure 3.5: Schematic Representation of EMG Generation

[Modified from Basmajian and DeLuca, 1985, Figure 3.10]
tributed amplitude. Roesler (1974), studying the peak amplitudes of the EMG, found that the probability of deviation from a Gaussian distribution was less than 0.001, using a Chi-square test. Parker et al. (1977), studying the EMG as a stochastic process, found the first order PDF of the EMG to be well modeled by the Gaussian PDF. The variance of the PDF was a function of the contraction level. (Standard AC coupling of the EMG signal provided a zero mean.) Further, at a fixed contraction level, the EMG could be described as a random process. Kwatny et al. (1970) showed that the constant force electromyogram could be considered as a wide-sense stationary, correlation-ergodic, random process, if viewed over a short (1.23 seconds) time interval. Hogan and Mann (1980a, 1980b) utilized this probabilistic approach to present the functional mathematical model of EMG shown in Figure 3.6. Hogan and Mann (1980a) assume that the variance of the myoelectric signal is directly related to the level of muscle force. Shvedyky et al. (1977) discuss conditions by which a simplified realization of the more physiologic model and the probabilistic model can be related.

3.3 Processing the Constant Force Surface Electromyogram

The constant force surface electromyogram, shown previously in Figure 3.1, is a complex electrical interference pattern which is a function of both time and contraction level. As contraction level is increased, the amplitude of the EMG waveform is increased. For this reason, estimates of the EMG amplitude have been used as a single dimensional statistic of surface EMG waveform.

Since the electromyogram is typically AC coupled, early investigators treated the EMG waveform as a zero mean amplitude modulated signal. Inman et
Figure 3.6: Functional Mathematical Model of EMG

The EMG is represented as a band-limited, zero mean, Gaussian random process $N(t)$ which is amplitude modulated by a static function of muscle force. For the constant force case, $F(t) = F$. [From Hogan and Mann, 1980a, Figure 1]
al. (1952) suggested an amplitude estimator consisting of a full-wave rectifier (non-linear demodulator) followed by a simple resistor-capacitor low pass filter (smoother). They noted that a long filter time constant was desired to reduced noise in the estimate, while a short filter time constant reduces the time delay in tracking changes in the signal amplitude. This simple detector, with time constants from 0.1 to 1.0 second, has been applied extensively in the study of the surface electromyogram. Inman et al. (1952) called this standard estimator an integrator, a term which the authors noted is mathematically inaccurate.

In an effort to improve the performance of the standard estimator of Inman et al. (1952), Kreifeldt (1971) compared the performance of three smoothers – the standard resistor-capacitor (RC) low pass filter, a third-order Butterworth filter, and a third-order averaging filter. The performance of each amplitude estimator was evaluated from EMG data recorded during isometric, isotonic muscular contraction by computing a signal to noise ratio (SNR) from the output of each amplitude estimator. Because muscle contraction was constant, the mean value of an amplitude estimate was taken as the signal, and deviations about the mean were taken as noise. The settling times of the three filters were standardized. This performance criterion has been widely adopted in surface EMG amplitude estimation. Kreifeldt (1971) found that the SNR performance of the averaging filter was an approximate 44% improvement over the RC filter, while the Butterworth filter provided an approximate 11% improvement. Kreifeldt and Yao (1974) experimentally investigated the performance of six non-linear demodulators – a half-wave rectifier, a full-wall rectifier, a second-power demodulator, a fourth-power demodulator, a one half-power demodulator, and a one fourth-power demodulator. For proper comparison, the smoothed output of an $n^{\text{th}}$
power demodulator should be $n^{th}$ rooted so as to provide an estimate of the amplitude and not the $n^{th}$ power of the amplitude. With this comparison criterion, the second-power demodulator was found to be best for contraction levels of 10%, 25% and 50% of MVC. The fourth-power demodulator was found to be best at 5% of MVC. These power law demodulators improved the SNR performance of the full wave rectifier by approximately 5–20%, depending on the force level.

Hogan and Mann (1980a, 1980b) used their functional mathematical model of EMG (briefly discussed previously) to analytically predict that a second-power demodulator and an averaging filter would give the best maximum likelihood estimate of the standard deviation of the myoelectric signal. Experimentally, they confirmed that an averaging filter is superior in SNR performance to a low pass filter by approximately 26%.\(^1\) They found little SNR performance difference between the second-power demodulator and a full wave rectifier.

Evans et al. (1984) proposed an amplitude estimation scheme based on a multiplicative (signal multiplied by noise) functional mathematical model of EMG identical to that shown in Figure 3.6. The authors proposed a logarithmic transformation of the myoelectric signal. This transformation provides an additive (signal plus noise) representation of the EMG. The authors then applied the theory of Kalman filters to estimate the amplitude of the transformed signal. It is difficult to compare the results of Evans et al. (1984) to other studies because the authors designed and evaluated their estimator in the more complex setting of time varying muscle force. However, in a theoretic study, Filligoi and Mandarini (1984) predicted that for a constant force contraction, SNR performance

\(^1\)The authors suggested that the larger improvement found by Kreifeldt (1971) might be attributed to differences in SNR measurement methods.
with logarithmic detection is inferior to second-power detection by approximately 56%.

Several investigators have found that the inclusion of a whitening filter prior to demodulation and smoothing improves the performance of the amplitude estimate. A whitening filter is a filter whose output PSD is constant-valued when presented with the signal of interest as an input. Kaiser and Peterson (1974) found that the shape of the whitening filter should change as a function of the contraction level. They suggested that measurement noise, present in differing relative degrees depending on the absolute signal (contraction) level, may be a major factor in determining the shape of the whitening filter. Kaiser and Peterson (1974) designed an adaptive analog filter to achieve their desired whitening. Harba and Lynn (1981) used auto-regressive modeling of the EMG power spectrum to form a whitening filter in an off-line algorithm. Their sixth-order model found only small changes in the shape of the whitening filter as a function of the contraction level. Whitening approximately doubled the probability of correctly differentiating between one of four discrete contraction levels. Their off-line results were confirmed with an analog on-line implementation. Hogan and Mann (1980a, 1980b) found that whitening could be achieved by modifying the electrode geometry. By reducing the outer edge spacing of a pair of rectangular electrodes from 20mm to 10mm, a SNR performance improvement of approximately 35% was achieved. Presumably, the first cancellation frequency of the differential electrode filter function (see Figure 3.3) was moved to a higher frequency, thereby obscuring less of the EMG signal. The results of Hogan and Mann (1980a, 1980b) suggest that the effectiveness of electronic whitening must be considered in concert with the electrode geometry. D'Alessio (1984) and
Filligoi and Mandarini (1984) discussed whitening with respect to functional mathematical models of the EMG.

Amplitude estimation techniques based on the occurrence of peaks in the myoelectric signal have also been developed. Hof and Van den Berg (1981a) developed a technique in which each EMG peak made a contribution to the EMG amplitude estimate based upon the rising slope and height of each EMG peak. The contribution from each peak was summed with that of other peaks to form the EMG amplitude estimate. Wyss and Pollak (1984) used a similar technique in which each EMG peak made an identical contribution to the amplitude estimate. Estimator performance for the above investigations was accomplished with force varying trials, and can not be directly compared to the constant force case.

Recently, further improvements in EMG amplitude estimation have been achieved through the combination of multiple channels of the EMG waveform. As mentioned previously, a single differential electrode obtains most of its signal energy from a small portion of muscle adjacent to the electrode. Hogan and Mann (1980a, 1980b) suggested that dispersing multiple electrodes about a single muscle would provide a broader, more complete, measure of the underlying electrophysiologic activity. They derived an optimal amplitude estimator assuming that separate EMG channels were spatially correlated but temporally uncorrelated. Using four electrodes, they achieved an SNR performance improvement of approximately 91% compared to the single channel estimator of Inman et al. (1952). The combination of multiple channels and whitening via electrode geometry yielded an SNR performance improvement of approximately 176% compared to the single channel estimator of Inman et al. (1952). The SNR
performance of their algorithm was relatively insensitive to force levels over the range of 5–25% of MVC. Hogan and Mann (1980a, 1980b) implemented their algorithm off-line on a digital computer and on-line with analog circuitry. Murray and Rolph (1985) implemented this algorithm in real time on a digital microprocessor. Harba and Lynn (1981) used four electrode pairs to improve the quality of an EMG processor which tried to differentiate between four discrete contraction levels. They were able to improve the probability of correctly differentiating between contraction levels by 40–70% (compared to using one electrode).

Both Hogan and Mann (1980a, 1980b) and Harba and Lynn (1981) note that the electrode geometry is critical to improvement of the estimation of EMG amplitude. Tight intra-electrode spacing of a differential pair serves to whiten the EMG spectrum and limit the volume of muscle which contributes significantly to the measured EMG. The whitening property has been shown to improve processor performance. Limiting the muscle volume which contributes significantly to the EMG assures that each channel in a multiple channel estimator provides additional information to the estimation. There do appear to be limits, however, as to how tightly electrodes can be placed. Overly tight inter-electrode spacing of differential pairs might work to exclude contribution to the measured EMG from regions of the muscle distant from all electrode pairs. Multiple electrodes seem to gain their advantage by, overall, measuring the electrophysiologic activity of a broader region of the muscle than a single differential electrode. Additionally, both sets of above mentioned authors caution that sweat induced shorting of electrodes can occur if spacing is too tight.
### 3.4 The Surface Electromyogram and Joint Torque

There has been a great deal of research directed towards determining a relationship between the estimated surface EMG amplitude and torque exerted about the joint. Studies of isometric, isotonic contraction of the biceps brachii muscles have found both linear relationships (Inman et al., 1952; Knowlton et al., 1956; Messier et al., 1971) and non-linear relationships (Heckathorne and Childress, 1981; Lawrence and DeLuca, 1983; Solomonow et al., 1986; Vredenbregt and Rau, 1973; Woods and Bigland-Ritchie, 1983; Zuniga and Simons, 1969).

The non-linear relationship, shown in Figure 3.7, is such that at high torque levels an increase in EMG activity leads to a smaller torque increment than at lower torque levels. This relationship is the most generally accepted relationship between biceps brachii estimated EMG amplitude and exerted torque.

Several theoretic studies have explored the EMG to torque relationship (Bernshtein, 1967; Moore, 1967; Person and Libkind, 1970). By assuming that an increase in muscle force is achieved solely through recruitment of additional MU’s, a non-linear relationship of estimated EMG amplitude to MU recruitment was found. The same result was found if an increase in muscle force was achieved solely through an increase in the firing rate of active MU’s. These results are shown in Figure 3.8. In both cases, asynchronous activation of MU’s was assumed. Note that the resultant non-linear relationship was such that at high recruitment/firing rate levels an increase in EMG activity led to a larger recruitment/firing rate increment than at lower recruitment/firing rate levels. When synchronous activation of MU’s was allowed, the EMG to recruitment/firing rate relationship changed. For a fixed number of active MU’s, the recti-
Figure 3.7: Non-Linear Isometric, Isotonic EMG to Torque Data

Curves show the force (F) exerted under isometric, isotonic conditions in relation to the value of the EMG. The different curves are obtained at different muscle lengths. At each muscle length, contraction force spanned the range of 0–100% maximum voluntary contraction. [From Vredenbregt and Rau, 1973, Figure 1B]
Figure 3.8: Theoretic Relation of EMG to Motor Unit Recruitment, Firing Rate

Relation of rectified and smoothed EMG to the number of active MU's or the frequency of MU excitation. MU's are activated in an asynchronous manner. [From Person and Libkind, 1970, Figure 5]

...fied, smoothed EMG increased with the percent of synchronously acting MU’s. Completely synchronous MU activity led to a linear relationship between EMG and MU recruitment. These studies made no attempt to model all of the complexities of EMG generation. Most noticeably, these investigators did not model the relationship between MU activation and developed torque, the interplay between recruitment and firing rate, or the filtering properties of muscle tissue and electrodes. Each of these processes is complex and not yet fully understood. As such these models are best utilized in a theoretic discussion rather than as a predictive model of the relationship between EMG and torque.

Some of the discrepancy in the isometric, isotonic EMG to torque relationship developed out of the comparison of dissimilar sets of data. Some investigators who determined a linear relationship studied force levels only up to approxi-
mately 50% of MVC. Non-linear relationships were found by other investigators who studied force levels over a broader range of MVC. Upon close inspection, the non-linear data sets appear essentially linear over the lower force range. The issue was further confounded by the compilation of data from several different muscle groups. Recent studies suggest that different muscle groups have different EMG to torque relationships. Both Lawrence and DeLuca (1983) and Woods and Bigland-Ritchie (1983) found a linear EMG to torque relationship for the small first dorsal interosseous muscle, but a non-linear relationship for the large biceps brachii muscles.

Although the above mentioned historical confusions no longer hinder the study of an EMG to torque relationship, fundamental problems remain. First, from a mechanics standpoint, the resolution of torque about a joint is an indeterminate problem. Antagonist muscles may co-contract about a joint, but only the composite torque — not the contribution from individual muscles — can be measured. Second, EMG recording and processing techniques may be inadequate or may misrepresent the underlying electrophysiologic events of the muscle. Third, the EMG to torque relationship may differ between subjects. High fidelity EMG to torque quantification may require a technique by which all parameters are uniquely fit to each subject. The above fundamental problems are discussed in the sections which follow.

3.4.1 Mechanical Indeterminacy of Muscles About a Joint

In order for a joint to both flex and extend, the joint must have agonist and antagonist muscle groups. At present, there is no non-invasive technique for measuring the torque contribution from a single muscle within a group of mus-
cles acting simultaneously about a joint. Only the composite torque can be measured. Thus, the resolution of torque contributions from the agonist(s) and antagonist(s) is indeterminate. In order to resolve this indeterminacy, most investigators have assumed, either explicitly or implicitly, that an agonist muscle (or group of muscles) can be contracted while the antagonist muscle is inhibited (Knowlton et al., 1956; Lawrence and DeLuca, 1983; Vredenbregt and Rau, 1973; Woods and Bigland-Ritchie, 1983; Zuniga and Simons, 1969). This muscular inhibition is called "reciprocal inhibition". For such a case, the measured composite torque is the torque due to the agonist.

It is suggested herein that reciprocal inhibition may exist at low levels of muscular contraction, but does not exist at high levels of muscular contraction. Hasan and Enoka (1985), studying isometric flexion of the elbow joint over a wide range of angles, found no discernible triceps EMG activity in three of four subjects when the biceps was exerting 20% of maximum flexion-effort EMG. In the fourth subject, triceps activity of 1.3% maximum triceps-effort EMG was detectable at one angle. The authors concluded that, at this level of activation, co-contraction was generally non-existent. No flexion trials were performed at high contraction levels. Soechting and Roberts (1975), utilizing sinusoidal contractions of the biceps, found no significant triceps force as long as the modulation amplitude of biceps tension was no more than 20% of the mean. Again, no trials were performed at high contraction levels. However, for a strenuous contraction, An et al. (1983) found the magnitude of the antagonist muscle EMG (extensor pollicis longus) to be as high as 100% of its maximum value, and Solomonow et al. (1986) found magnitudes as high as 50% (biceps brachii). The data of Solomonow et al. (1986) are shown in Figure 3.9. At the
Figure 3.9: Evidence of Muscular Co-contraction

Normalized EMG mean absolute value (MAV) shown as a function of normalized force for the agonist and antagonist for elbow flexion at three angles (top) and for extension at three angles (bottom). The solid and dashed lines through the data points are first- and second-order polynomial fits, respectively. (B–biceps, T–triceps, based on data from twelve subjects.) [From Solomonow et al., 1986, Figure 3]
low force levels, the six plots — representing first flexion then extension trials for three different elbow orientations — show little or no antagonist muscle EMG activity. However, for force levels above approximately 50% of MVC, the antagonist EMG activity is clearly different from zero. Unfortunately, it is difficult to differentiate co-contraction from antagonist muscle EMG crosstalk. Nonetheless, in the absence of direct evidence to the contrary, it must be assumed that muscular co-contraction does, in fact, exist.

Clearly, unaccounted co-contraction will influence the observed EMG to torque relationship. In particular, the observed torque will necessarily be less than that actually produced by the agonist muscle. A portion of the produced torque must combat the torque produced by the antagonist muscle. Determination of an EMG to torque relationship must account for the contributions from both the agonist and antagonist muscles.

3.4.2 EMG Recording and Processing Techniques

The predominant EMG recording technique presently is that of recording a surface bipolar potential difference from a single site. EMG processing typically consists of rectifying and smoothing the raw EMG signal. Several smoothers are commonly employed; a low pass filter, a moving averager, and integration with time reset. Siegler et al. (1985) have shown these three techniques to be essentially identical with respect to predicting the relationship between EMG and torque. Although these techniques are common, they all suffer from what Hogan and Mann (1980a) have called a “spatio-temporal sampling artifact”. That is, for a muscle such as the biceps brachii, a single bipolar electrode can effectively represent only a small fraction of the total muscle. A large portion of the muscle
contributes a very small amount of information to the observed EMG signal.

A systematic effect on the estimated EMG amplitude to torque relationship based upon such a spatial-temporal sampling artifact has been theorized (Woods and Bigland-Ritchie, 1983). As discussed previously, an increase in muscular force is the result of two complementary factors — recruitment of additional MU’s and an increase in the firing rate of active MU’s. Initially, recruitment is the dominant factor. At higher torque levels, an increase in the firing rate is responsible for torque increases. With respect to surface EMG, the above description is important because Clamann (1970) has shown, for the biceps brachii muscle, that deep MU’s tend to be recruited first and superficial MU’s tend to be recruited later. Since surface bipolar electrodes heavily weight the activity of local MU’s, muscles such as the biceps brachii will have surface EMG which is dominated by the electrical activity of superficial MU’s. Thus, at low torque levels, few MU’s close to the electrodes will be active, and low levels of observed EMG amplitude will result. At progressively higher torque levels, more MU’s close to the electrodes will become active, giving the appearance of a marked increase in EMG amplitude/ torque slope when, in fact, overall EMG amplitude/ torque slope has not increased. (See Basmajian and DeLuca, 1985 for a more complete discussion.) Therefore, the shape of the EMG amplitude to torque relationship may be influenced by the EMG recording technique.

In order to investigate the role of the EMG recording technique, two investigations briefly contrasted the use of bipolar recordings versus monopolar recordings. A monopolar electrode is less selective of local muscle fibers than a bipolar electrode. Moritani and DeVries (1978) found a linear relationship for the biceps brachii when using a monopolar electrode, but a non-linear relation-
ship when using bipolar electrodes. Woods and Bigland-Ritchie (1983) found non-linear relationships (similar in form) for the biceps brachii with both electrode configurations. These results are conflicting and further investigation is warranted.

Other electrode configurations and processing techniques whose intent it is to provide a more equal weighting of MU’s throughout the muscle to the EMG have been investigated. Wyss and Pollak (1984) developed a processing technique based on the occurrence of peaks in the EMG waveform. Since the EMG peaks of remote MU’s can be detected in the surface EMG waveform, the authors suggested that such a technique is more representative of the composite muscle. Hogan and Mann (1980a, 1980b) introduced a multielectrode array and processing technique which simultaneously observes various areas of the muscle, combining all of the independent EMG information into a single processed waveform. This technique inherently observes a greater portion of the muscle, thus weighting MU’s throughout the muscle more equally. Harba and Lynn (1981) presented a similar multielectrode technique. (Both techniques have been discussed previously.) Clearly, further investigation is required in order to determine the effects of electrode configuration and processing techniques on the EMG to torque relationship.

Finally, the inter-trial repeatability of EMG amplitude estimators has received little attention. Any useful application of an EMG to torque relationship must perform reliably over the course of many muscular contractions. Siegler et al. (1985) found that the greatest variant in the EMG to torque relationship was not different processing schemes but, rather, poor inter-trial repeatability in the estimated EMG amplitude. Siegler et al. (1985) suggest that the conflict-
ing results of prior investigations are primarily caused by the large intra-subject variability in the EMG signal recorded from repeated contractions. Development of methods to reduce this variability should be of prime concern.

3.4.3 Individual Versus Universal Characterization

Several EMG to joint torque models require the coalescing of modeling parameters from multiple sources, yet apply this information to a single subject. For example, cadavers have been used to measure the moment arm as a function of limb angle formed by a muscle and the bone to which it inserts (Messier et al., 1971; An et al., 1983). These data are applied to a subject to determine the actual torque developed during a contraction trial at a particular limb angle which differs from the reference angle. Such a technique is certainly inaccurate (as the moment arm derived from one cadaver need not accurately represent the moment arm of a particular subject) and ignores all other factors which could influence the intact limb. In particular, it has long been known that the force exerted by a muscle at a fixed rate of stimulation depends upon the length of the muscle (Gordon et al., 1966), and therefore the angle of orientation of the intact limb. Thus, accounting for the mechanical moment arm only is inadequate to assess the influence of limb orientation on joint torque. Hasan and Enoka (1985) investigated the influence of limb orientation for the biceps brachii by measuring the torque produced at different angles from a fixed level of EMG amplitude. For all subjects, the isometric torque varied with elbow angle as a singly-peaked function, yet the location of the peak varied by as much as 50 degrees between subjects. Individual subjects, however, appeared to exhibit specific tendencies in the coordinated use of their elbow. Although this is but one example, it is
clear that the EMG to joint torque relationship need not be characterized by a universal set of parameters defined over all subjects. Yet, for a given subject, a consistent relationship may exist.
Chapter 4

A Surface EMG to Joint Torque Model
4.1 Introduction

The preceding two chapters provided a background of the anatomy, physiology and electrophysiology, as well as a review of past investigations pertaining to the relationship between surface EMG and joint torque. Clearly, a reliable relationship has yet to be established. However, several important contributions in the literature have yet to be applied to the EMG to torque problem. Prime among these contributions are 1) temporal whitening of the surface EMG signal to improve the performance of EMG amplitude estimation, 2) the combination of multiple surface EMG signals to improve the performance of EMG amplitude estimation, and 3) modeling agonist/antagonist coactivation over a wide range of joint torques. Each of these contributions could significantly decrease the error in estimating joint torque from the EMG. All three contributions will be utilized in this thesis.

This chapter presents a model for relating surface EMG to joint torque. The model is applied to the human elbow, but is generally applicable to many joints. The chapters which follow attempt to parameterize, as well as test, this model.

4.2 Modeling Overview

A complete anatomic model of the relationship between the EMG waveform, as measured by a particular configuration of electrode-amplifiers on the skin surface, and joint torque requires observation of the measured surface EMG (MSEMG) waveform, decomposition of the MSEMG waveform into the contributions from all of the active constituent MU's, knowledge of the relationship between excitation and contraction for each MU, and knowledge of the effect of
each MU’s contractile state upon the composite joint torque. Such information is not presently available. For example, decomposition of the MSEMGl waveform into the contributions from constituent MU’s is presently possible, but only if the number of active MU’s is limited to approximately less than ten (Basmajian and DeLuca, 1985; McGill et al., 1985). Additionally, the processing time required for this decomposition is presently unrealizable in a real-time application. Mechanically, there presently exists no technique to measure in situ the contribution of individual MU’s to composite joint torque. Typically, the torque produced by all MU’s in all muscles acting about a joint is measured. Thus, a complete physiologic model will not be utilized in this thesis. Rather, a functional mathematical model will be developed.

As has been discussed previously, the summation of the electrical activity of many randomly excited MU’s leads to an MSEMGl waveform which is well described as a Gaussian random process (see Chapter 3). As the firing rate of active MU’s or the number of active MU’s increases/ decreases, so does the amplitude (standard deviation) of the Gaussian random process. Similarly as the firing rate of active MU’s or the number of active MU’s increases/ decreases, so does the isometric torque developed about the joint. If a single parameter, denoted the EMG amplitude, is assumed to characterize the firing rate and recruitment of the various MU’s in a muscle, then, in the model to be developed herein, EMG amplitude reflects the input of a system which has two outputs — the MSEMGl waveform and joint torque. The EMG amplitude, in effect, describes the intrinsic intensity of muscular contraction. Given a probabilistic description of the MSEMGl waveform, mathematically optimal estimators of its standard deviation (amplitude) can be developed. Hence, a functional model in
Figure 4.1: Functional Model of Measured Surface EMG Waveform and Joint Torque

The measured surface EMG waveform and joint torque are generated from the EMG amplitude. The model assumes that the EMG amplitude can be identified from the measured surface EMG waveform and that joint torque is an identifiable function of the EMG amplitude.

which EMG amplitude is the driving function for the MSEMG waveform and joint torque can be presented. Figure 4.1 shows the model which is used in this thesis. Three fundamental assumptions must be made in order that the model of Figure 4.1 be useful. First, it is assumed that the EMG amplitude can be identified from the MSEMG waveform. Second, for the case of isometric, constant-force, non-fatiguing muscle contraction, it is assumed that the EMG amplitude has a constant value. Third, it is assumed that joint torque is an identifiable function of the EMG amplitude.

With the above assumptions, the MSEMG waveform to joint torque problem
can be divided into two subproblems. The first subproblem is that of estimating the EMG amplitude from the MSEMNG waveform. Since the EMG amplitude cannot be measured directly (it is a functional parameter only), the first problem is best addressed using relative measures of performance. The second subproblem is then estimation of joint torque from the estimated EMG amplitude.

The above approach is further justified by recognizing that the first problem, estimation of the EMG amplitude from the MSEMNG waveform, is immediately applicable to proportional myoelectric prosthetic control. In proportional myoelectric prosthetic control, an estimate of the EMG amplitude is utilized as the input command signal to the prosthesis controller. Typical EMG amplitude estimators have poor signal-to-noise performance and poor repeatability. Improved estimators might be implemented immediately in existing prosthesis designs.

A model for the MSEMNG waveform and a model of generated joint torque, respectively, will be provided in the two sections which follow. Both models are functional in nature. The first model seeks to estimate the EMG amplitude from multiple channels of the MSEMNG waveform. The multiple channel work of Hogan and Mann (1980a, 1980b) is extended by adding individual channel temporal prewhitening to the optimal processor. The second model relates the EMG amplitude to generated joint torque.

4.3 A Functional Mathematical Model of Surface EMG

4.3.1 Model Formulation

The MSEMNG waveform is a complex spatial-temporal interference pattern of the electrical activity of the various underlying muscle tissues. As discussed previ-
ously, if the MSEMГ waveform is viewed as a random signal, a given sample of
the MSEMГ waveform closely resembles a Gaussian distributed random variable.
(See Appendix A for a review of random variables, vectors and processes. See
Appendix B for a review of Gaussian random variables, vectors and processes.)
If the MSEMГ signal is observed over a window of time, the observed signal has
been shown to resemble a band-limited Gaussian random process. Further, there
is evidence (see Chapter 3) that the random process is both wide-sense stationary
(WSS) and correlation-ergodic (CE) for isometric, isotonic contractions. Thus,
the discretely sampled MSEMГ waveform will be modeled as being formed from
the multiplication of a unit intensity, zero mean, WSS, band-limited, CE, jointly
Gaussian process and a control signal. The control signal represents the EMG
amplitude, \( s_i \). Such a model describes the electrical phenomenon, but in no way
accounts for details of the underlying physiology.

The band-limited Gaussian process will be modeled as being formed from a
zero mean, WSS, white, CE, jointly Gaussian process of unit intensity passed
through an LTI shaping filter which is stable, causal, and whose inverse exists
and is stable and causal. Only the magnitude response of the shaping filter is
essential for spectral characterization of the EMG. The stability and causality
of the shaping filter and its inverse assure that both filters are realizable. Since
the input to the shaping filter is WSS and the filter is LTI, the filter output is
also WSS (see Appendix A). Additionally, the LTI assumption guarantees that
a jointly Gaussian input to the filter will yield a jointly Gaussian output. It
will be assumed that the shaping filter characterizes the filtering effects of AP
propagation through the body tissues, as well as the recording properties of the
electrodes. The present MSEMГ waveform analysis will be limited to the case of
constant force, non-fatiguing contractions. Thus, the EMG amplitude simplifies to the constant $s$, and the shaping filter accounts for all of the time dependence in the MSEMГ waveform. This single channel model is shown in Figure 4.2. It is assumed that the level of constant force contraction is related to the EMG amplitude $s$ (the standard deviation of the MSEMГ signal). Thus, the single channel MSEMГ waveform processing problem is formulated as performing a standard deviation estimate on a zero mean, WSS, CE, jointly Gaussian process.

When multiple electrodes are placed over a muscle, several, possibly correlated, MSEMГ signals are recorded. Figure 4.3 extends the single channel MSEMГ waveform model to a multiple channel model. $L$ independent, zero mean, JWSS, white, CE, jointly Gaussian processes of unit intensity are passed through an $L$-input, $L$-output, LTI shaping filter, $H_{space}$, which is stable, causal, and whose inverse exists and is stable and causal. This multi-dimensional shaping filter is restricted to account only for the spatial dependence between channels, including differences in signal strength. Such a restriction requires that outputs of the multi-dimensional filter can only be based on knowledge of the present inputs. Any use of past inputs would imply a contribution to the temporal correlation in the MSEMГ signal. Hence, the multi-dimensional filter has no dynamics and can be represented as a linear transformation. The $L$ outputs from the multi-dimensional shaping filter are passed through a bank of LTI shaping filters to form $L$ dependent, zero mean, JWSS, non-white, CE, jointly Gaussian processes. The bank of shaping filters account for all of the time dependence in the MSEMГ signal. These shaping filters are stable, causal, and have an inverse which is stable and causal. The multiple channel MSEMГ waveform processing problem is, therefore, formulated as estimating the common standard deviation
Zero Mean, WSS, 
CE, Jointly Gaussian, 
White Process 
of Unit 
Intensity 

Filtering 
Effects of 
Muscle Tissue, 
Bone, Skin and 
Electrodes 

Measured 
Surface 
EMG

Figure 4.2: Functional Model of a Single Channel of EMG

A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity is passed through the stable, causal, inversely stable, inversely causal, linear, time-invariant shaping filter \( H_{\text{time}}(e^{j\omega}) \) and multiplied by the EMG amplitude \( s \) to form the measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure 4.3: Functional Model of Multiple Channels of EMG

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{space}$ which accounts only for the spatial dependence between channels. These filter outputs are each passed through a shaping filter $H_{time,j}(e^{j\omega})$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
(EMG amplitude) of $L$ zero mean, JWSS, CE, jointly Gaussian processes.

### 4.3.2 Optimization of Several EMG Configurations

In practice, the complete single and multiple channel models may neither be necessary nor desired. For example, Cavanaugh et al. (1983a, 1983b) considered successive samples of MSEM activity to be essentially uncorrelated (correlation coefficient less than ten percent of its maximum value) if the signal was sampled at a rate of 100 Hz. or less (bipolar electrodes placed longitudinally along the muscle at a spacing of approximately 1cm). Xiong and Shvedyk (1987) considered successive samples of MSEM activity to be statistically independent if the signal was sampled at a rate of 250 Hz. or less (bipolar recordings from Beckman silver electrodes placed longitudinally along the muscle at a spacing of approximately 3cm). Thus, if the MSEM signal is sampled slowly, the temporal correlation of samples within a channel can be ignored. Harba and Lynn (1981), utilizing multiple electrodes, found little correlation between channels if the spacing between the bipolar contacts of a particular electrode was approximately 9mm and the spacing between adjacent electrodes was approximately 20mm. For many muscles, such an electrode configuration would preclude the need to consider the spatial correlation between channels. These examples point out that particular applications will determine the appropriate MSEM waveform model. Accordingly, MSEM waveform processing will be discussed separately for the following six cases:

I) Single Channel: Uncorrelated Samples

II) Single Channel: Correlated Samples

III) Multiple Channels: Uncorrelated Samples, Uncorrelated Channels
IV) Multiple Channels: Uncorrelated Samples, Correlated Channels
V) Multiple Channels: Correlated Samples, Uncorrelated Channels
VI) Multiple Channels: Correlated Samples, Correlated Channels

For each case, an optimal estimate of the EMG amplitude will be derived. In all of the optimization cases, “optimal” will be taken to mean optimal estimation of the EMG amplitude \( s \) in the maximum likelihood sense. Only causal estimators will be considered since they are appropriate for eventual implementation in real-time systems. The analysis will assume that the estimate is made by observing the MSEM signal over a finite time duration. Each formulation will consider only the sampled-data case where the continuous-time MSEM waveform is sampled by a digital computer and further processed in discrete-time. Finally, note that the mathematics involved in determining the optimal estimates and their performance measures are too laborious to include in the main text of this thesis and are presented in Appendix C.

Case I — Single Channel: Uncorrelated Samples

To achieve uncorrelated samples in the single channel model, the shaping filter becomes an all-pass filter, yielding the model shown in Figure 4.4. In the discrete-time case, if a single channel of continuous MSEM activity \( m(t) \) is sampled periodically over a finite time duration \( T \), the \( N \) samples of MSEM activity can be denoted as the \( N \) random variables \( m_1, m_2, m_3, \ldots, m_N \). By assumption, these random variables are uncorrelated and each random variable has the Gaussian PDF with mean value zero and standard deviation \( s \). To perform MLE, it will be assumed that \( s \) has the known value \( s \). This event gives
Zero Mean, WSS, CE,
Jointly Gaussian,
White Process
of Unit
Intensity

Measured
Surface
EMG

Figure 4.4: EMG Model Case I — Single Channel: Uncorrelated Samples

A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity is multiplied by the EMG amplitude $s$ to form the measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
the conditional density

\[ p_{m_i | s}(M_i | \hat{s}) = G(M_i; 0, \hat{s}^2) = \frac{1}{\hat{s} \sqrt{2\pi}} e^{-\frac{(M_i - \hat{s})^2}{2\hat{s}^2}} \quad -\infty \leq M_i \leq \infty \]

for all \( i \). Since the random variables are both jointly Gaussian and uncorrelated, they are independent. Hence, the conditional joint PDF for the \( N \) random variables is just the product of the individual conditional PDF's;

\[ p_{m_1, m_2, m_3, \ldots, m_N | s}(M_1, M_2, M_3, \ldots, M_N | \hat{s}) = \]

\[ = \prod_{i=1}^{N} p_{m_i | s}(M_i | \hat{s}) \]

\[ = \prod_{i=1}^{N} \frac{1}{\hat{s} \sqrt{2\pi}} e^{-\frac{M_i^2}{2\hat{s}^2}} \quad -\infty \leq M_i \leq \infty \]

\[ = \left( \frac{1}{(2\pi \hat{s}^2)^{N/2}} e^{-\frac{1}{2\hat{s}^2} \sum_{i=1}^{N} M_i^2} \right) \quad -\infty \leq M_i \leq \infty \]

The maximum likelihood estimate of the standard deviation is the value of \( \hat{s} \) which maximizes the above density. Appendix C shows that the desired estimator is

\[ \hat{s} = \left[ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right]^{1/2} \]

which is often referred to as the root mean square (RMS) estimator. The estimate bias is found as

\[ \mu_{error} = E_{m_i | s}[s - \hat{s}] = s \left[ 1 - \sqrt{\frac{2}{N} \frac{\Gamma\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)}} \right] \]

In practice this bias is quite small. For example, with \( N = 30 \), the bias is already less than one percent of the true value of the parameter. The estimate \( MSE \) is

\[ MSE_{\hat{s}} = E_{m_i | s}[(s - \hat{s})^2] = 2s^2 \left[ 1 - \sqrt{\frac{2}{N} \frac{\Gamma\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)}} \right] \]

The estimate error variance is

\[ \sigma_{\hat{s}}^2 = s^2 \left[ 1 - \frac{2}{N} \frac{\Gamma^2\left(\frac{N+1}{2}\right)}{\Gamma^2\left(\frac{N}{2}\right)} \right] \]
Finally, it is of interest to define a signal to noise ratio (SNR) for this problem. At constant force the true EMG amplitude $s$ is modeled as constant. For the purposes of a SNR calculation, the true value of $s$ will be taken to be the expected value of the estimate $\hat{s}$. A SNR can then be defined as

$$SNR = \left[ \frac{E^2[\hat{s}]}{E[(\hat{s} - \mu_s)^2]} \right]^{1/2}$$

For the present problem,

$$SNR_2 = \left[ \frac{2 \Gamma^2(\frac{N+1}{2})}{N \Gamma^2(\frac{N}{2}) - 2 \Gamma^2(\frac{N+1}{2})} \right]^{1/2} = \left[ \frac{N}{2} \left\{ \frac{\Gamma(N)}{\Gamma(N+1/2)} \right\}^2 - 1 \right]^{-1/2}$$

Hogan and Mann (1980a)\textsuperscript{1} give an approximation to this SNR when $N$ is large as

$$SNR_2 \approx \sqrt{2N} \quad \text{for } N \text{ large}$$

In practice, muscle force, and thus the EMG amplitude $s$, is not constant and, viewed as a stochastic process, the discrete MSEM signal is not WSS. However, the peak bandwidth for voluntary oscillation of the arm about the elbow and, by assumption, the EMG amplitude is from zero to approximately six Hz. (Neilson, 1972; Zahalak and Heyman, 1979), while the MSEM waveform contains little information below approximately twenty Hz. (Hogan and Mann, 1980b). Thus, the MSEM signal can be considered quasi-stationary. That is, if viewed via short time windows, the EMG amplitude is constant within a particular window, but can vary from one window to the next. Therefore, if a channel of continuous

\textsuperscript{1}Hogan and Mann (1980a) actually provide the SNR approximation for the related estimate

$$\hat{s}^* = \left[ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right]^{1/2a}$$

where $a$ is a constant. By assigning the value one to the constant $a$, the relevant SNR approximation has been calculated.
Figure 4.5: Optimal EMG Processor — Single Channel: Uncorrelated Samples

Samples of measured surface myoelectric activity are denoted $m_i$, $\hat{s}_i$ are the optimal causal estimates of the EMG amplitude, and $N$ is the window length.

MSEMG activity is sampled periodically for an indefinite period of time, the discrete-time non-linear filter depicted in Figure 4.5 can be utilized to provide sequential optimal, causal estimates of the EMG amplitude.

Case II — Single Channel: Correlated Samples

When successive samples of MSEMG activity are correlated, the general single channel model of Figure 4.2 models the generation of the MSEMG signal. The shaping filter has a non-constant value. Again in discrete-time, denote $N$ periodically sampled values of the continuous MSEMG activity as the random vector $m$. Given that each element of $m$ has the standard deviation $\hat{s}$,

$$p_{\text{ml}}(M|\hat{s}) = \frac{e^{-\frac{M^T K_{mm}^{-1} M}{2}}}{(2\pi)^{N/2} |K_{mm}|^{1/2}} \quad -\infty \leq M \leq \infty$$

Unfortunately, there is no simple form for the inverse of the covariance matrix $K_{mm}$. 

97
Figure 4.6: Generic Optimal Filter — Single Channel: Correlated Samples

The filter $G_{optimal}(e^{j\omega})$ produces sequential causal optimal estimates of $s$ from an indefinite length sequence of periodically sampled values of the continuous measured surface myoelectric signal.

A solution to this discrete time estimation problem can be based on the filtering concept discussed in Case I. In particular, Case I showed that a filter could be utilized to produce successive estimates of the EMG amplitude from successive samples of the MSEM signal. For the present case, successive estimates of the EMG amplitude are also desired. Thus, begin the solution by assigning a filter $G_{optimal}(e^{j\omega})$ which, given an indefinite length sequence of periodically sampled values of the continuous MSEM signal, produces sequential causal optimal estimates of $s$. Figure 4.6 depicts this filter. Since an optimal estimate exists for any, and thus every, sequence of $N$ sequential MSEM samples, the filter $G_{optimal}(e^{j\omega})$ must exist.

Next, consider the output $v_i$ of a stable, causal filter $G_{white}(e^{j\omega})$, whose inverse exists and is stable and causal, formed from the input $m_i$. Since the filter $G_{white}(e^{j\omega})$ is invertible, no information is lost due to the filter. Thus, optimal estimation of $s$ from the sequence $v_i$ is equivalent to optimal estimation of $s$
Figure 4.7: Alternate Generic Optimal Filter — Single Channel: Correlated Samples

Cascade of the two filters $G_{white}(e^{j\omega})$ and $G_{white}^{-1}(e^{j\omega})$ forms an all-pass network which, combined with the filter $G_{optimal}(e^{j\omega})$, produce the optimal estimates $\hat{s}_i$. From the samples $m_i$. This equivalence must be so, since optimal estimation of $s$ from the sequence $v_i$ could always be accomplished by cascade of the filters $G_{white}^{-1}(e^{j\omega})$ and $G_{optimal}(e^{j\omega})$. Figure 4.7 illustrates this argument.

As shown in Appendix C, filtering of the sequence $m_i$ to the sequence $v_i$ can be exploited if $G_{white}(e^{j\omega})$ is selected as $H_{time}^{-1}(e^{j\omega})$. Since $H_{time}(e^{j\omega})$ was constrained to be linear, time-invariant, stable, causal, invertibly stable and invertibly causal, its inverse must exist and be stable, causal and linear time-invariant. The output of this filter must be a white Gaussian process of intensity $s$. Thus, the optimal estimate is completed as derived in the previous analysis case. Intuitively, this “whitening filter” in a sense orthogonalizes the data samples, allowing the detection algorithm to operate on each output sample independently. The resultant discrete-time optimal causal filter for a single MSEM channel with correlated samples is shown in Figure 4.8. The filter $G_{white}(e^{j\omega})$ is known as a whitening filter.
Figure 4.8: Optimal EMG Processor — Single Channel: Correlated Samples

The estimate is formed as the square root of the smoothed, whitened measured surface myoelectric signal.

Performance for the discrete-time maximum likelihood estimator can utilize the results of the previous analysis case. For the previous case of uncorrelated samples, $N$ denoted the sample size. More generally, the previous performance formulae are based on the number of degrees of freedom in the observed MSEM signal. For uncorrelated data, there are precisely $N$ degrees of freedom. In practice, however, perfect whitening can not be achieved. For correlated data, Hogan and Mann (1980b) suggest the use of an effective number of degrees of freedom $N_{\text{effective}}$ as

$$N_{\text{effective},II} = 2 B_s T$$

for Case II

where $B_s$ is the statistical bandwidth of the MSEM signal. (Recall that for discrete data $T = N \Delta \tau$.) For a continuous waveform, $B_s$ is given as

$$B_s = \frac{\int_{-\infty}^{\infty} S_{mm}(j\omega) d\omega}{\int_{-\infty}^{\infty} S_{mm}(j\omega) d\omega}$$

for a continuous waveform.
For a discrete waveform, $B_s$ will be estimated as

$$B_s = \frac{\int_{-\pi}^{\pi} S_{mm}(e^{j\omega}) \, d\omega}{\int_{-\pi}^{\pi} S_{mm}^2(e^{j\omega}) \, d\omega}$$

for a discrete waveform

Case III — Multiple Channels:

Uncorrelated Samples, Uncorrelated Channels

To achieve uncorrelated samples and uncorrelated channels, the multiple channel MSEM model reduces to that of Figure 4.9. The channel gains $g_j$, where $g_j > 0$, reflect the relative signal level of the different channels. (More generally, the $g_j$ represent the most simple form of a shaping filter — only the magnitude of the signal is altered.) In the discrete-time case, Figure 4.10 shows the optimal processor. Gains are applied to the MSEM signal to equalize the contribution from each channel to the estimate. After applying these gains, the data from all channels and over the complete observation window are squared, averaged, and then square rooted. Performance for this discrete-time maximum likelihood estimator can again be based on the performance formulae of Case I. For the present case, all $N \cdot L$ samples are uncorrelated. Since these samples are jointly Gaussian, they are also independent. Thus, the number of degrees of freedom in the data is $N \cdot L$.

Case IV — Multiple Channels:

Uncorrelated Samples, Correlated Channels

To achieve uncorrelated samples and correlated channels, the multiple channel MSEM waveform model becomes that shown in Figure 4.11. Again, this case provides no simple form for the inverse of the covariance matrix $K_{mm}$. A solution can be based on the filtering concept discussed in Case II. Consider filtering the
Figure 4.9: EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are multiplied first by the gain factors $g_j$ ($g_j > 0$) and second by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure 4.10: Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

The optimal estimate is formed as the square root of the average of $L$ channels of smoothed, variance adjusted measured surface EMG waveform.
Figure 4.11: EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{space}$, which accounts only for the spatial dependence between channels. The outputs of $H_{space}$ are each multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
$L$ MSEM signals via an $L$-input, $L$-output, linear, stable, causal filter whose inverse exists and is stable and causal. Denote a channel of output from this filter as $v_j$. This filtering operation can be exploited if the output channels $v_j$ have uncorrelated samples, uncorrelated channels and are of equal variance $s^2$. Namely, the optimum processor is specified by the results of Case III. Appendix C shows that such a filter exists, and is the filter $H_{sp}^{-1}$. Further, this filter can be derived from the eigenvalues and eigenvectors of the covariance matrix $K_{m_i,m_j}$, where the ensemble random vector $m_{*,i}$ is formed from the $L$ channels of MSEM activity corresponding to time $i$ as

$$m_{*,i} = \begin{bmatrix} m_{1,i} \\ m_{2,i} \\ m_{3,i} \\ \vdots \\ m_{L,i} \end{bmatrix}_{L \times 1}$$

The discrete-time optimal causal filter for multiple MSEM waveform channels with uncorrelated samples and correlated channels is given in Figure 4.12.

Performance for the discrete-time maximum likelihood estimator can again be based on the performance formulae of Case I. If perfect spatial uncorrelation is achieved, then the number of degrees of freedom in the data is $N \cdot L$. In practice, however, perfect spatial uncorrelation is not achieved. Thus, consider the use of an effective number of degrees of freedom. Since each time frame is independent for this case, the effective number of degrees of freedom in the data is equal to the sum of the effective number of degrees of freedom in each time frame;

$$N_{\text{effective, IV}} = \sum_{i=1}^{N} N_{\text{effective, i}}$$

for Case IV

where $N_{\text{effective, i}}$ is the number of degrees of freedom in time frame $i$. The effective number of degrees of freedom in time frame $i$ is equal to the effective
Figure 4.12: Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels

The filter $H_{space}^{-1}$ produces $L$ channels of spatially uncorrelated data from $L$ channels of spatially correlated data. The uncorrelated data are then smoothed, averaged and square rooted to form the optimal estimate.
number of degrees of freedom in the ensemble random vector $m_{*,i}$. Since $m_{*,i}$ is stationary, $N_{\text{effective},i}$ has the same value $N_{\text{effective,ensemble}}$ for all $i$. Thus,

$$N_{\text{effective},IV} = N \cdot N_{\text{effective,ensemble}} \quad \text{for Case IV}$$

From the discussion in Case II,

$$N_{\text{effective,ensemble}} = 2 B_{*,m_{*,i}} T$$

where $B_{*,m_{*,i}}$ is the statistical bandwidth of the discrete random vector $m_{*,i}$. Thus,

$$N_{\text{effective,IV}} = 2 N T B_{*,m_{*,i}} \quad \text{for Case IV}$$

A problem exists in applying the above formula. For the ensemble vector $m_{*,i}$, $T$ is ill-defined. $T$ is meant to be the time duration for observing a single signal. For this ensemble formulation, an intuitive empirical solution is to take $T$ as the product $L \triangle \tau_{\text{equiv}}$, where $\tau_{\text{equiv}}$ is the sampling period of a single MSEM waveform channel which will yield a covariance matrix similar to the covariance matrix of the ensemble random vector $m_{*,i}$. Akin to the case of a discrete-time signal where $T = N \triangle \tau$, the term $L$ represents the number of discrete entries and the term $\triangle \tau_{\text{equiv}}$ represents the equivalent time spacing.

Case V — Multiple Channels:
Correlated Samples, Uncorrelated Channels

When multiple uncorrelated channels of the MSEM waveform have correlated samples, the MSEM waveform model of Figure 4.13 results. Appendix C derives the optimal processor, given in Figure 4.14. Each channel of MSEM activity is filtered by the stable, causal, invertibly stable and causal, LTI filter $H_{t1}^{-1}(e^{j\omega})$. The output of each filter is a zero mean, JWSS, CE, jointly
$w_{1,i} \rightarrow H_{time,1}(e^{j\omega}) \rightarrow n_{1,i} \rightarrow m_{1,i}$

$w_{2,i} \rightarrow H_{time,2}(e^{j\omega}) \rightarrow n_{2,i} \rightarrow m_{2,i}$

$w_{3,i} \rightarrow H_{time,3}(e^{j\omega}) \rightarrow n_{3,i} \rightarrow m_{3,i}$

$\vdots$

$w_{L,i} \rightarrow H_{time,L}(e^{j\omega}) \rightarrow n_{L,i} \rightarrow m_{L,i}$

Independent Zero  Filtering  Measured
Mean, JWSS, CE,  Effects of  Surface
Jointly Gaussian,  Muscle Tissue,
White Processes of  Bone, Skin and  EMG
Unit Intensity  Electrodes

**Figure 4.13:** EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are each passed through a shaping filter $H_{time,j}(e^{j\omega})$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure 4.14: Discrete Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels

The estimate is formed as the square root of the average of $L$ smoothed, whitened channels of measured surface EMG waveform.
Gaussian, white process of intensity \( s^2 \). Optimal estimation of \( s \) based on \( L \) such outputs is given as the result of Case III, where all gains \( g_j \) are equal to unity. The result of Case III is cascaded with the \( L \) whitening filters \( H_{time,j}^{-1}(e^{j\omega}) \) to form the estimator. Performance for the discrete-time maximum likelihood estimator can again be based on the performance formulae of Case I. If perfect temporal whitening is achieved, then there are \( N \cdot L \) degrees of freedom in the data. In practice, however, perfect temporal whitening can not be achieved. Since the \( L \) channels are uncorrelated, and therefore independent, the effective number of degrees of freedom in the data \( N_{\text{effective},V} \) must be the sum of the effective number of degrees of freedom in each channel;

\[
N_{\text{effective},V} = \sum_{j=1}^{L} N_{\text{effective},j} \quad \text{for Case V}
\]

where \( N_{\text{effective},j} \) is the effective number of degrees of freedom in the data of channel \( j \). Case II gives the effective number of degrees of freedom in a particular channel. Thus,

\[
N_{\text{effective},V} = 2T \sum_{j=1}^{L} B_{s,j} \quad \text{for Case V}
\]

where \( B_{s,j} \) is the statistical bandwidth of the MSEMG signal of channel \( j \) as defined in Case II.

Case VI — Multiple Channels:

Correlated Samples, Correlated Channels

When multiple channels of the MSEMG waveform are correlated both in time and space, the general multiple channel model of Figure 4.3 is applicable. Appendix C derives the optimal processor, given in Figure 4.15. Each channel is first prewhitened by passing the data through the filters \( H_{time,j}^{-1}(e^{j\omega}) \). The \( L \) output
Figure 4.15: Optimal EMG Processor — Multiple Channels: Correlated Samples, Correlated Channels

The bank of filters $H^{-1}_{time,j}(e^{j\omega})$ uncorrelate each channel temporally. The filter $H^{-1}_{space}$ uncorrelates the data spatially. The resultant signals are smoothed, averaged and square rooted to form the optimal estimate.
channels $v_j$ are now uncorrelated in time, but still correlated in space. Estimation of $s$ from the channels $v_j$ is now exactly the problem solved in Case IV. Note that the filter $H^{-1}_{space}$ must perform a transformation based on the covariance structure of the intermediate random variable $v_j$ and not the original MSEM signal. Also, since the bank of whitening filters and $H^{-1}_{space}$ are linear, their position in the solution figure can be switched. Performance for the discrete-time maximum likelihood estimator can again be based on the performance formulae of Case I. Again, if perfect temporal whitening and spatial uncorrelation is achieved, then there are $N \cdot L$ degrees of freedom in the data. In practice, perfect temporal whitening or spatial correlation can not be achieved. Thus, for the present case, $N_{effective,VI}$ must account for both the number of effectively uncorrelated time frames and the number of effectively uncorrelated channels within a time frame.

Since each channel can have a different statistical bandwidth, each channel can have a different number of effectively uncorrelated time frames. Thus, use of the following ad hoc value of $N_{effective,VI}$ is suggested;

$$N_{effective,VI} = N_{channel,average} \cdot N_{effective,ensemble} \quad \text{for Case VI}$$

$N_{channel,average}$ is the average over $L$ channels of the number of effectively uncorrelated time frames. For a single channel, the number of effectively uncorrelated time frames is given in Case II. $N_{effective,ensemble}$ is the number of effectively uncorrelated channels, as given in Case IV.

### 4.3.3 The Mean Absolute Value Processor

An EMG amplitude estimator common in the literature is the mean absolute value (MAV) processor. The MAV processor can be formed from the above derived optimal estimators by replacing the RMS detector with an MAV detector.
For example, the MAV processor for a single MSEMG channel with uncorrelated samples (Case I) is
\[ \hat{s}_{\text{MAV}} = \frac{1}{N} \sum_{i=1}^{N} |M_i| \]

where \( M_i \) are the MSEMG waveform samples. The MAV processor is important to study for two principal reasons. First, since amplitude estimation via this technique is common, a comparison of the predicted and actual performances of the MAV processor and the optimal estimator should be informative. Second, the MAV processor is simpler to realize, either in analog hardware or on a digital computer, than the optimal estimator. A simpler estimator, with comparable performance, may be appropriate for certain applications.

The performance of the MAV processor, based upon the Gaussian models discussed above, can be derived analytically (see Appendix C). The SNR performance for Case I is
\[ SNR_{\hat{s}_{\text{MAV}}} = \sqrt{\left( \frac{2}{\pi - 2} \right) N} \]

For Cases II-VI, the SNR performance can be found by inserting the corresponding effective number of degrees of freedom in the data for \( N \). Thus, compared to the optimal estimator, the MAV processor is predicted to provide an approximate 6–7% decrement in SNR performance.

Alternatively, the MAV processor can be shown to be optimal if the underlying PDF for the myoelectric samples is distributed as a Laplace random variable, and successive myoelectric samples are independent (see Appendix C). The temporal whitening and spatial uncorrelation filters used in the Gaussian model estimator would be assumed to produce temporal and spatial independence, respectively, in the data of the Laplacian model. Note that independent
is a stronger condition than uncorrelated. A zero mean Laplace random variable \( x \), shown in Figure 4.16 with a zero mean Gaussian random variable of identical variance, has the PDF

\[
p_x(X) = \frac{a}{2} e^{-a |X|} \quad -\infty \leq X \leq \infty
\]

and the standard deviation

\[
\sigma_x = \frac{\sqrt{2}}{a}
\]

for some constant \( a > 0 \).

4.4 A Functional Mathematical Model of Torque Generation About the Elbow

As discussed previously, a fundamental assumption of this thesis is that joint torque is an identifiable function of the EMG amplitude. Mathematically, if the non-fatiguing, isometric, isotonic torque due to elbow flexor muscles is denoted \( T_F \), then the above assumption can be written as

\[
T_F = \mathcal{F}_F(s_F, \theta)
\]

where

\[
\mathcal{F}_F(\cdot) = \text{functional operator for flexors,} \\
s_F = \text{flexor EMG amplitude,} \\
\theta = \text{joint angle (angle between the forearm and the upper arm).}
\]

The effect of joint angle is parameterized explicitly in this formulation. Similarly, the torque \( T_E \) due to elbow extensor muscles can be written as

\[
T_E = \mathcal{F}_E(s_E, \theta)
\]
Figure 4.16: Gaussian and Laplace Probability Density Functions

Zero mean, equal variance Gaussian and Laplace PDF's are graphed in the same event space.
where
\[
\mathcal{F}_E(\cdot) = \text{functional operator for extensors,}
\]

\[
s_E = \text{extensor EMG amplitude.}
\]

A third torque \( T_{\text{ext}} \), due to external loading, can also be applied and measured about the elbow joint. These three torques can be incorporated into the simple elbow model of Figure 4.17. In Figure 4.17, the upper arm is modeled as a mechanical ground about which the forearm rotates. The elbow is modeled as an ideal hinge joint. For non-fatiguing isometric contraction, the equation of motion which governs the arm is

\[
0 = T_F - T_E - T_{\text{ext}}
\]

\[
= \mathcal{F}_F(s_F, \theta) - \mathcal{F}_E(s_E, \theta) - T_{\text{ext}}
\]

Because co-contraction about the joint has been modeled, the flexion and extension torques can not generally be determined directly from measurement of \( T_{\text{ext}}, \theta \) and the EMG amplitudes. However, estimation techniques can be used to resolve the respective torques. The technique of linear regression (see, for example, Lyung, 1987, page 130) will be applied to the present problem. A linear regression problem is formed by constraining the relationship between joint torque and the EMG amplitude to be a sum of basis functions, each basis function being scaled by a fit parameter. The basis functions can be any linear or non-linear function of measured or known quantities. The basis functions should be independent, although such a set is not required. With these constraints, the flexor and extensor torques can be written as

\[
T_F = \mathcal{F}_F(s_F, \theta) = \sum_{i=1}^{P_F} f_{F,i} \mathcal{F}_{F,i}(s_F, \theta)
\]

\[
T_E = \mathcal{F}_E(s_E, \theta) = \sum_{i=1}^{P_E} f_{E,i} \mathcal{F}_{E,i}(s_E, \theta)
\]
Figure 4.17: Simple Model of Torques About the Elbow

The upper arm is modeled as a mechanical ground about which the forearm rotates. The three torques are: $T_E = \text{torque due to elbow extensor muscles}$, $T_F = \text{torque due to elbow flexor muscles}$, and $T_{ext} = \text{total joint torque (torque due to external loading)}$. 

$$T_{ext} = T_F - T_E$$
where

\[ P_F = \text{number of flexor basis functions}, \]
\[ P_E = \text{number of extensor basis functions}, \]
\[ f_{F,i} = \text{flexor fitting parameters}, \]
\[ f_{E,i} = \text{extensor fitting parameters}, \]
\[ \mathcal{F}_{F,i} = \text{flexor basis functions}, \]
\[ \mathcal{F}_{E,i} = \text{extensor basis functions}. \]

If activity about the elbow is measured and then compared to the model, the error associated with the \( j \)th measurement, \( \text{error}_j \), can be defined as

\[
\text{error}_j = \mathcal{T}_{F,j} - \mathcal{T}_{E,j} - \mathcal{T}_{\text{est}_j} \\
= \sum_{i=1}^{P_F} f_{F,i} \mathcal{F}_{F,i}(s_{F,i}, \theta_j) - \sum_{i=1}^{P_E} f_{E,i} \mathcal{F}_{E,i}(s_{E,i}, \theta_j) - \mathcal{T}_{\text{est}_j}
\]

The mean square error, \( MSE \), in \( N \) measurements is then

\[
MSE = \frac{1}{N} \sum_{j=1}^{N} \text{error}_j^2
\]

The \( MSE \) in \( N \) measurements will be taken as a cost function. Minimization of this cost function with respect to the parameters \( f_{F,i} \) and \( f_{E,i} \) represents a solution to this linear regression model.

The minimization problem can be formulated more compactly in vector notation. First, construct the error vector \( \text{error} \), containing the error associated with each of \( N \) measurements as

\[
\text{error} = \begin{bmatrix}
\text{error}_1 \\
\text{error}_2 \\
\text{error}_3 \\
\vdots \\
\text{error}_N
\end{bmatrix}_{N \times 1}
\]
Next, define the vector $\mathbf{z}$, the vector $\mathbf{b}$ and the matrix $A$ as

$$
\mathbf{z} = \begin{bmatrix}
  f_{F,1} \\
  f_{F,2} \\
  f_{F,3} \\
  \vdots \\
  f_{F,P_F} \\
  -f_{E,1} \\
  -f_{E,2} \\
  -f_{E,3} \\
  \vdots \\
  -f_{E,P_S}
\end{bmatrix}_{(P_F + P_S) \times 1}
$$

$$
\mathbf{b} = \begin{bmatrix}
  T_{ext_1} \\
  T_{ext_2} \\
  T_{ext_3} \\
  \vdots \\
  T_{ext_N}
\end{bmatrix}_{N \times 1}
$$

and $A$ is the partitioned matrix

$$
A = \begin{bmatrix}
  A_F & A_E
\end{bmatrix}_{N \times (P_F + P_S)}
$$

where

$$
A_F = \begin{bmatrix}
  \mathcal{F}_{F,1}(s_{F_1}, \theta_1) & \cdots & \mathcal{F}_{F,P_F}(s_{F_1}, \theta_1) \\
  \mathcal{F}_{F,1}(s_{F_2}, \theta_2) & \cdots & \mathcal{F}_{F,P_F}(s_{F_2}, \theta_2) \\
  \mathcal{F}_{F,1}(s_{F_3}, \theta_3) & \cdots & \mathcal{F}_{F,P_F}(s_{F_3}, \theta_3) \\
  \vdots & \vdots & \vdots \\
  \mathcal{F}_{F,1}(s_{F_N}, \theta_N) & \cdots & \mathcal{F}_{F,P_F}(s_{F_N}, \theta_N)
\end{bmatrix}_{N \times P_F}
$$

$$
A_E = \begin{bmatrix}
  \mathcal{F}_{E,1}(s_{E_1}, \theta_1) & \cdots & \mathcal{F}_{E,P_E}(s_{E_1}, \theta_1) \\
  \mathcal{F}_{E,1}(s_{E_2}, \theta_2) & \cdots & \mathcal{F}_{E,P_E}(s_{E_2}, \theta_2) \\
  \mathcal{F}_{E,1}(s_{E_3}, \theta_3) & \cdots & \mathcal{F}_{E,P_E}(s_{E_3}, \theta_3) \\
  \vdots & \vdots & \vdots \\
  \mathcal{F}_{E,1}(s_{E_N}, \theta_N) & \cdots & \mathcal{F}_{E,P_E}(s_{E_N}, \theta_N)
\end{bmatrix}_{N \times P_S}
$$

The above definitions allow the error vector to be written as

$$
error = A \mathbf{z} - \mathbf{b}
$$
The $MSE$ problem can now be stated as minimizing the norm of the error vector, where

$$\text{norm error} = \left[ \text{error}_1^2 + \text{error}_2^2 + \text{error}_3^2 + \cdots + \text{error}_N^2 \right]^{1/2}$$

Minimizing the above norm is equivalent to minimizing, in the least squares sense, $A \hat{\mathbf{x}} - \mathbf{b}$ with respect to the fit parameters $\hat{\mathbf{x}}$. This problem is precisely the linear least squares problem, for which robust solutions exist.

### 4.5 Discussion

#### 4.5.1 Functional Modeling and the Physiology

The models presented herein are functional models due to the complexity of the underlying physiology. However, there is some evidence which may provide a link between the physiology and these low-order functional models. Recall that the regulation of force is governed by a complex spatio-temporal summation of the activities of each MU. The temporal activation of a particular MU can be simplified by use of an average firing rate. Further, DeLuca et al. (1982a, 1982b) have asserted that the nervous system may not control the firing rates of MU’s individually, but instead may act upon a pool of motoneurons in a uniform manner. This common drive suggests that central nervous system mediation of the firing rates of all active MU’s might be captured through a single parameter. Let this single parameter be called the *global firing rate*, formally defined as the mean value of the firing rates of all active MU’s. Clearly, the global firing rate can not account for the fact that different MU’s, weighted equally in the computation of the global firing rate, will make unequal contributions to both the MSEM G waveform and joint torque. Further, rigorous determination as to
when a MU is active versus inactive, as well as rigorous definition of the MU firing rate may not be possible. Nonetheless, the global firing rate represents a single statistic which may provide a proportional measure of the activation state of all active MU’s.

Next, the number of MU’s which participate in muscular contraction — recruitment — must be accounted. Again, recruitment can be, in general, quite complex. But, for many contractions, and most physiologists would argue for all contractions, recruitment does occur in an orderly fashion (i.e. the size principle discussed previously in Chapter 2). Hence, recruitment might also be represented by a single parameter. Let this single parameter be denoted as the global recruitment fraction, formally defined as the fraction of active MU’s out of the total number of MU’s in a muscle. As before, the global recruitment fraction is a necessarily degenerate representation of the underlying physiology.

At this point, the complex excitation process has been condensed to two parameters — the global firing rate and the global recruitment fraction. Yet, even this simplification is not sufficient for the present problem. Functional modeling of the MSEM waveform has identified a single parameter, EMG amplitude, as that which is altered as a function of force. Identification of a global firing rate and a global recruitment fraction would still require decomposition of the MSEM waveform. But, it is clear that as the global firing rate increases/decreases, so does the EMG amplitude. Similarly, as the global recruitment fraction increases/decreases, so does the EMG amplitude. Hence, there is reason to suggest a single composite parameter representative of the degree to which contraction has been initiated for all MU’s within the muscle. Denote this parameter the generalized active state of the muscle. This proposed model assumes
that the generalized active state is the driving function for the MSEM waveform and joint torque. For such a model to be useful, it must be assumed that the generalized active state can be identified from the MSEM waveform.

While the above formulation is interesting, there presently exist insufficient techniques to investigate and/or verify the proposed model. Actual identification, via a single parameter, of the complex underlying physiologic events is certainly not possible. Further, there do exist techniques to investigate the relationship between estimated EMG amplitude and joint torque. Hence, the relationship between estimated EMG amplitude and torque was studied in this thesis.

4.5.2 Surface EMG Model

The optimal processors developed in this chapter are greatly reduced in complexity due to the use of inverse filters (i.e. temporal whitening and spatial uncorrelation filters). Justification for the inverse filters does not rely on the mathematic details of the stochastic process. Rather, because these filters are invertible, invertibly stable and invertibly causal, any probabilistic input to the filters will yield outputs in which no information has been lost. Further, the outputs will be uncorrelated (in either time or space). Thus, in the event that the MSEM waveform is not distributed as a Gaussian random process, inverse filtering remains beneficial. Additionally, the general form of the inverse filters allows a large range of selection in temporal whitening/spatial uncorrelation techniques. Many such methods can be represented by these optimal processors.

The major assumptions upon which the model is based are similar to assumptions made by previous investigators. Note that the CE assumption was
not needed for any of the mathematics developed thus far. This assumption was used in practice to form PSD/correlation estimates needed in the computation of whitening filters.

The theoretic development showed that SNR performance is proportional to the length of the smoothing window $N$, the statistical bandwidth of each channel, and the number of channels. Whitening serves to increase the statistical bandwidth of a channel. If $B_r$ is considered a representative bandwidth for all $L$ EMG channels, then Hogan and Mann (1980b) have shown that

$$SNR \approx \sqrt{2NB_rL}$$

The statistical bandwidth and number of channels were examined experimentally in this project. The smoothing window length was standardized to approximately 245ms, a value commonly utilized in EMG amplitude estimation.

In practice, all single channel MSEMГ waveforms were sampled rapidly enough that significant temporal correlation existed. As well, all multiple channel MSEMГ waveforms were spatially correlated. Accordingly, model cases II and VI were applied to the MSEMГ data acquired for this thesis.

### 4.5.3 Torque Generation About the Elbow

The mechanical model for the elbow assumes that all flexor (extensor) muscles can be treated as a single muscle group. This assumption reduces the model complexity in at least two important ways. First, several electrodes could span the entire muscle group, producing a single EMG amplitude estimate. The combination of multiple channels was expected to provide an inherently better amplitude estimate. Second, the number of EMG inputs associated with the relationship between estimated EMG amplitude and joint torque was reduced.
to two (flexor, extensor EMG amplitudes). Increasing the number of muscles in the model would likely have resulted in poorer EMG amplitude estimation and more difficult identification of model parameters.

Additionally, the elbow model assumed that extensor (flexor) EMG amplitude contributed only to extensor (flexor) torque about the elbow. No cross terms, i.e. $T_{Cross} = \mathcal{F}_{E,F}(s_E, s_F, \theta)$, were included in the formulation. A cross term could arise if EMG from one muscle group was recorded by electrode-amplifiers placed over another muscle group (EMG crosstalk). Including cross terms, however, would have precluded attributing specific torque contributions to each muscle group. One of the project goals was to develop a technique which would allow an estimate of individual muscle group contributions to the total joint torque. The more appropriate response to crosstalk is to develop an electrode-amplifier configuration in which no significant crosstalk exists.

The elbow model assumed that joint torque was an identifiable function of the EMG amplitude. This assumption did not provide any technique for performing identification. In general, multiple input, multiple parameter identification can be a difficult task. Thus, prior to the experimental trial which investigated identification, a short simulation study was conducted. This study evaluated some simple identification strategies as applied specifically to the EMG to torque problem.

4.6 Conclusion

This chapter has provided a model which allows investigation of the relationship between the MSEM waveforms and joint torque to be divided into two subproblems. The first subproblem is that of estimating the EMG amplitude from the
MSEMG waveform. A model of the MSEMG waveform provided a technique for developing optimal estimators of the EMG amplitude. The second subproblem is that of estimating the joint torque from the estimated EMG amplitude. A model of the relationship between the EMG amplitude and joint torque has formulated this subproblem in the form of a standard linear least squares problem. The thesis chapters which follow seek to test these models as well as investigate the practical aspects of model identification and of relating the MSEMG to joint torque.
Chapter 5

Experiment 1 — Part I: Single Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle
5.1 Introduction

The model presented in Chapter 4 formulated the MSEMg waveform to torque problem as two subproblems. The first subproblem was that of estimating the EMG amplitude from the MSEMg waveform. The second subproblem was the estimation of joint torque from the estimated EMG amplitude. This chapter will describe an experiment which examined the first subproblem for non-fatiguing, isometric, isotonic contraction at one angle. This chapter will present the results for single channel EMG amplitude estimators only. Chapter 6 will present the results for multiple channel EMG amplitude estimators. A separate experiment investigated the effect of joint angle on estimation of EMG amplitude.

Multiple channels of the MSEMg data were recorded from one muscle group (either elbow flexors or extensors) during isometric, isotonic contraction of that muscle group. For the single channel case, several factors were explored with respect to temporal whitening of single channels. Temporal whitening was achieved by modeling the whitening filter as a moving average filter. The filter order, the amount of data used to estimate the filter coefficients, the effect of contraction level, the effect of sampling rate, the effect of electrode location, and the effect of subject upon the performance of a whitening filter were investigated. Additionally, an ad hoc estimator, called the mean absolute value estimator, was compared to the optimal estimator. In all cases, performance was expressed as the signal to noise ratio (SNR) of the amplitude estimate, where the "true" amplitude was taken as the mean value of the amplitude estimate averaged over time (the rise time of the estimation filter was ignored). For reference, the performance was compared to a simple single channel MARMS estimator. All
estimators incorporated a fixed smoothing window corresponding to approximately 245ms. The effect of smoothing window length on SNR performance was not studied as it is a more appropriate topic for non-isotonic contractions. A smoothing window of approximately 245ms is in the range of commonly employed EMG amplitude estimators.

Single channel estimation was studied first for two principal reasons. First, the MSEM model has shown that single channel amplitude estimation and multiple channel amplitude estimation share the common step of temporal whitening of individual MSEM channels. Thus, studying temporal whitening first greatly reduced the scope of investigation for the complete multiple channel processor. Second, if a universal whitening filter exists (i.e. a single whitening filter which is effective for all subjects) then such a filter might be incorporated into the band-pass characteristics of future commercial electrode-amplifiers. Such a technique would automatically make temporal whitening the first step in an optimal EMG amplitude estimator.

The sections which follow describe the experimental trial. Initially, descriptions of the experimental apparatus and the experimental methods are provided. Then, the results of single channel amplitude estimation are reported.

5.2 Experimental Apparatus

All experiments were performed in the Eric P. and Evelyn E. Newman Laboratory for Biomechanics and Human Rehabilitation. Informed consent was received from the subject. The subject was seated in the Instrumented Torque Chair, shown in Figure 5.1. (Detailed description of the Instrumented Torque Chair is provided in Appendix D.) The chair was bolted to a wood platform
Figure 5.1: Instrumented Torque Chair

A straight-back chair was bolted to a wood platform. Instrumentation for measuring torque about the elbow was bolted to the chair back rest supports.
to prevent tipping. An aluminum plate was bolted to the chair, serving as the
chair back rest. Foam padding for the subject’s back was provided for comfort.
The subject was secured to the back rest plate via five quick-release belts. Two
belts criss-crossed the front of the body from shoulder to opposite hip. A belt
was oriented from shoulder to respective hip, one belt for each side of the body.
The fifth belt was worn across the lap. The belts were pulled taut to secure the
subject’s trunk to the back rest plate.

A pivot plate was mounted to the back rest plate via a crossbar. The location
of the pivot plate (always to the right of the subject) could be altered by three
adjustments. Slots in the back rest plate allowed vertical adjustment of the
crossbar and pivot plate. Slots in the crossbar allowed horizontal adjustment
of the crossbar and pivot plate. Slots on the pivot plate allowed front to back
adjustment of the pivot plate. The subject was instructed to elevate his/her
right arm so that the upper and lower arm were in a plane parallel to the plane
of the floor (shoulder abducted 90 degrees from the anatomic position), and
the upper arm was directed laterally outward from the shoulder (normal to the
sagittal plane). The apparatus was adjusted such that a pivot point, on the
pivot plate, was located at the medial epicondyle of the subject’s elbow.

An aluminum beam was mounted to the pivot plate via an angle bracket.
The subject’s right wrist, at the level of the styloid processes, was fit into a
cuff with the palm of the hand held parallel to the beam. The cuff was rigidly
attached to the beam, eliminating the need for active grasp. The location of the
cuff could be adjusted via a slot in the beam. The pivot point facilitated discrete
selections of the angle between the upper arm and the forearm.

Four strain gauges (BLH Electronics FAE-50-35S13EL), two on each side
of the beam, were mounted at the level of the pivot point. All four strain gauges sensed active strain. The four strain gauges were arranged as a Wheatstone bridge. The bridge was excited by a voltage of ±8V. The bridge output was sensed by an instrumentation amplifier (Analog Devices AD521), amplified within the amplifier circuit by a factor of approximately 150, and then low pass filtered by a passive single pole RC filter with a cutoff frequency of approximately 160 Hz.

Commercial EMG electrode-amplifiers (Liberty Mutual MYO111) were utilized to measure the surface EMG waveform. For flexion experiments, eight electrode-amplifiers were placed side by side latitudinally across the biceps brachii muscles, the electrode contacts of each electrode-amplifier being oriented longitudinally along the muscles (in the direction of conduction of the action potential). Figure 5.2 shows the arrangement of the electrode-amplifiers over the biceps muscles. The electrode-amplifiers were located approximately midway between the elbow and the midpoint of the upper arm, thereby avoiding the innervation zone. The electrode-amplifiers spanned both the short and long heads of the biceps brachii. The distance between a pair of electrode-amplifiers was approximately \( \frac{11}{16} \) of an inch. The electrode-amplifiers were sequentially numbered zero through seven, with electrode-amplifier zero being placed in the most medial location and electrode-amplifier seven being placed in the most lateral location. A single ground electrode was placed in the vicinity of the electrode-amplifiers. Electrode-amplifiers and the ground electrode were held in place with an elastic arm band (Chattanooga Corporation Nylatex\textsuperscript{R} Wrap). For extension experiments, electrode-amplifiers were oriented in a similar manner on the triceps muscles. The electrode-amplifiers were located approximately midway between
Figure 5.2: Electrode-Amplifier Locations

The arrangement of the electrode-amplifiers on the biceps brachii muscles is shown. Enlarged view is shown at bottom.
the elbow and the midpoint of the upper arm, spanning both the lateral and long heads of the triceps. Electrode-amplifier zero was placed in the most lateral location and electrode-amplifier seven was placed in the most medial location.

Each Liberty Mutual MYO111 electrode-amplifier consisted of (Greelish, 1987) a pair of 4mm diameter, stainless steel, hemi-spherical electrode contacts embedded in an injection-molded package. The electrode contacts protruded 1.3mm from the package and were separated by a distance of 15mm (center to center). This tight inter-electrode pair spacing might help reduce EMG crosstalk by excluding input from distant muscle. The electrode-amplifier package had dimensions of 17.2 x 26.3 x 5mm. A differential amplifier circuit, also embedded within the package, had an input impedance of 10⁶ ohms, a common mode rejection ratio greater than 100 dB, a gain of approximately 3600, and a second-order bandpass filter with pass band from 100 Hz. to 500 Hz. Each electrode-amplifier was powered at ±9V. After amplification on the Liberty Mutual electrode-amplifier, each EMG signal was further amplified by an inverting amplifier with a variable gain of 0-10. No further signal filtering was required.

The eight EMG signals and the strain gauge signal were sampled at a rate of 2048 Hz. on a 12-bit A/D converter (Datel DVME-612A controlled on a VME bus by a Motorola MVME133-1 single board computer) set to a ±10V input range. The sampled data were transferred to hard disk for off-line processing on a set of Sun Microsystems workstations.

5.3 Experimental Methods

Five subjects, four male and one female, ranging in age from 23 to 37 years participated in five experiments. Subjects had no known neuromuscular deficits
of the right shoulder, arm or hand. Two experiments studied flexion of the elbow, three experiments studied extension of the elbow. All trials investigated isometric, isotonic flexion/extension of the right elbow with a 90 degree angle between the upper arm and the forearm.

A subject was seated and secured into the instrumented chair, and the electrodes were mounted on the subject. During an experimental trial, the subject was instructed to lift his/her right elbow above the pivot plate and to contact the instrumented beam only via the wrist cuff. The output voltage of the strain gauge circuit was presented on an oscilloscope to the subject. A target torque level was presented to the subject as the second display of a two channel oscilloscope. The subject was instructed to begin at rest, then, over a time period comfortable to the subject (typically 0.5-1 s), gradually increase flexion/extension torque until the target torque level was achieved. The subject attempted to maintain a consistent posture during and throughout all trials, so as to present a repeatable contraction task. The subject tried to relax all muscles not directly involved in flexion/extension about the elbow. By observing the oscilloscope display, the subject maintained the target torque level until a five second segment of data was recorded at a sampling rate of 2048 Hz. The location on the oscilloscope screen of the target torque level was fixed for all trials. The null location of the strain gauge circuit output voltage could be adjusted. For all trials, the oscilloscope gain was selected as the maximum gain for which the null voltage and the target torque level could be observed simultaneously. This gain selection maximized the available sensitivity of the oscilloscope display for each contraction level. Also, for errors in maintaining the desired contraction torque, the visual angle subtended by the subject's deviation from the target torque (i.e.
the visual error) was approximately the same for all contraction levels.

Two initial contractions were used to determine the strain gauge circuit output voltage corresponding to MVC. The subject was instructed to flex/extend at his/her maximum torque level for three seconds. After a three minute rest the task was repeated. The strain gauge circuit output voltage was observed for the two trials and then averaged. This pair of trials provided a rough estimate of MVC, sufficient for the coarse gradation of contraction levels investigated in this experimental trial. All subsequent trials were calibrated as a percent of this MVC determination. A sequence of five sets of contractions was then conducted. Each set consisted of four trials, one trial each at 10, 25, 50 and 75% of MVC. Trials within a set were randomized by flipping a coin. A rest period of two minutes between trials was provided. Between contraction sets the subject was released from the wrist cuff to prevent impaired blood flow to/from the hand.

The duration of rest between trials was assigned to prevent fatigue. Muller (1965) reviewed human performance and endurance, showing that the rest allowance $RA$ between successive trials of isometric contraction of any muscle required to continue successive trials for eight hours, expressed as a percent of the trial duration, is

$$RA = 18 \left( \frac{t}{T_{\text{max}}} \right)^{1.4} \left( \frac{k}{K_{\text{max}}} - 0.15 \right)^{0.5} \cdot 100\%$$

where

$RA = \text{rest allowance in percent of } t,$

$t = \text{trial duration in minutes},$

$T_{\text{max}} = \text{maximum trial duration in minutes},$

$k = \text{developed force},$
\[ K_{max} = \text{maximum force.} \]

The maximum trial duration is also independent of the contracting muscle, and is given as
\[
T_{max} = -1.5 + \frac{2.1}{\left( \frac{k}{K_{max}} \right)} - \frac{0.6}{\left( \frac{k}{K_{max}} \right)^2} + \frac{0.1}{\left( \frac{k}{K_{max}} \right)^3}
\]

With this rest criterion, contraction at 75% of MVC for five seconds requires a rest period of approximately 6.2 seconds in order to continue trials for eight hours. The assigned rest period of two minutes is more than adequate to meet this criterion.

5.4 Methods of Analysis — Single Channel EMG Amplitude Estimation

The basic computational steps involved in optimal estimation of single channel EMG amplitude were temporal whitening followed by calculation of the MARMS. The MARMS step could be performed directly from its definition. The whitening step required knowledge of the temporal correlation of the data. Thus, formulation of the whitening filter was an estimation problem itself.

Knowledge of the temporal correlation of the data is equivalent to knowledge of the temporal shaping filter \( H_{time}(e^{j\omega}) \) in the MSEM model of Figure 4.2. Identification of a general linear filter can be a difficult task. To simplify the identification problem, it was assumed that the temporal shaping filter could be modeled as an \( n \)-th-order, rational, all-poles filter, written as;
\[
H_{time}(e^{j\omega}) = \frac{1}{d_0 + d_1 e^{-j\omega} + d_2 e^{-2j\omega} + \cdots + d_n e^{-nj\omega}} = \frac{1}{\sum_{i=0}^{n} d_i e^{-ij\omega}}
\]

This filter model is also called an infinite impulse response filter, a recursive filter, or an autoregressive filter (AR). Such a filter provided several modeling
advantages. First, modeling coefficients were utilized efficiently. Each filter coefficient made a contribution to the filter characteristic at every frequency. Second, the filter was easily inverted and its inverse was always stable and causal. In particular,

\[ H_{time}^{-1}(e^{j\omega}) = \sum_{i=0}^{I} d_i e^{-ij\omega} \]

This inverse filter was an all-zeroes filter, also called a finite impulse response filter, a non-recursive filter, or a moving average filter. Third, the data of Triolo et al. (1988) suggested that an AR model of the temporal shaping filter for EMG is appropriate. They found that the surface EMG for isometric contractions of constant force as recorded from a site intermediate to knee flexors and extensors remained exclusively AR of low order (two – six) regardless of electrode position, contraction level and limb function. Lastly, an AR model greatly simplified the filter identification problem. Robust solutions to the AR identification problem exist. Clearly, however, an all-poles model would have difficulty modeling a zero, and could not achieve a null-valued DC gain (i.e. model the zero mean description of the MSEMGG process). As a consequence of not modeling a zero mean signal, the inverse (whitening) filter would then have insufficient gain near DC. This possible deficit should have been of little significance to this project since the electrode-amplifiers attenuated the vast majority of the signal near DC. Thus, there was little signal near DC to be whitened.

In order to identify the whitening filter, consider the following. If a white sequence of unit intensity \( w_i \) is the input to \( H_{time}(e^{j\omega}) \) and the sequence \( n_i \) is the output (see Figure 4.2), then

\[ S_{nn}(e^{j\omega}) = \left| H_{time}(e^{j\omega}) \right|^2 S_{ww}(e^{j\omega}) \]
Since $S_{ww}(e^{j\omega}) = 1$, the AR model for $H_{time}(e^{j\omega})$ gives

$$S_{nn}(e^{j\omega}) = \frac{1}{\sum_{i=0}^{l} d_i e^{-ij\omega}}$$

Except for the scaling factor $s$ (representing the EMG amplitude), the above PSD models the PSD of the MSEM signal. Hence, identification of the MSEM PSD via an AR model led to specification of the whitening filter. Press et al. (Section 12.8, 1988) provide an algorithm which was utilized to estimate the AR coefficients. Their PSD model takes the equivalent form

$$S_{mm}(e^{j\omega}) = \frac{a_0}{\left|1 - \sum_{i=1}^{l} a_i e^{-ij\omega}\right|^2}$$

where $m_i$ is the sequence of surface EMG. Thus, the model of Press et al. (1988) and the whitening filter coefficients were related as:

$$d_0 = \frac{1}{\sqrt{a_0}}$$
$$d_i = \frac{-a_i}{\sqrt{a_0}} \quad 1 \leq i \leq l$$

To form the PSD estimate, Press et al. (1988) first estimate the correlation function of the MSEM signal, $C_{mm}(\tau)$, from $N + 1$ data points $m_0, m_1, m_2, \ldots, m_N$ as

$$\hat{C}_{mm}(\tau) = \frac{1}{N + 1 - \tau} \sum_{i=0}^{N-\tau} m_i m_{i+\tau}$$

Because the MSEM signal was assumed to be correlation-ergodic, the $N + 1$ data points were taken from a single time record and no ensemble averaging was necessary in order to form this correlation function estimate. The data points provided an estimate of the correlation function over the index $-N \leq \tau \leq N$. Since the correlation function and the PSD are related as

$$S_{mm}(e^{j\omega}) = \sum_{\tau=-\infty}^{\infty} C_{mm}(\tau) e^{-j\omega \tau}$$
one would expect that the correlation function should have some relation to the AR coefficients. Consider writing a PSD estimate from the estimated correlation function by truncating the Fourier Transform expression for the PSD to the index range $-l \leq \tau \leq l$, giving

$$\hat{S}_{mm}(e^{j\omega}) = \sum_{\tau=-l}^{l} \hat{C}_{mm}(\tau)e^{-j\omega\tau}$$

If the AR model PSD is written as a truncated Laurent series in $e^{-j\omega}$, truncated to the range $e^{-j\omega l}$ to $e^{j\omega l}$, then matching Laurent series coefficients to the corresponding correlation estimate in the above truncated Fourier Transform yields $l + 1$ relations between the correlation estimates and the $l + 1$ coefficients $a_0, a_1, a_2, \ldots, a_l$. These relationships are linear, and can be written as

$$\begin{bmatrix}
\hat{C}(0) & \hat{C}(1) & \hat{C}(2) & \cdots & \hat{C}(l) \\
\hat{C}(1) & \hat{C}(0) & \hat{C}(1) & \cdots & \hat{C}(l-1) \\
\hat{C}(2) & \hat{C}(1) & \hat{C}(0) & \cdots & \hat{C}(l-2) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\hat{C}(l) & \hat{C}(l-1) & \hat{C}(l-2) & \cdots & \hat{C}(0)
\end{bmatrix} \begin{bmatrix}
1 \\
a_1 \\
a_2 \\
\vdots \\
a_l
\end{bmatrix} = \begin{bmatrix}
a_0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}$$

The matrix in the above equation is a symmetric Toeplitz matrix. Press et al. (1988) implement a recursive algorithm to determine the coefficients $a_0, a_1, a_2, \ldots, a_l$ from the above matrix equation. Since a maximum of $N + 1$ distinct correlation function estimates exist, $l$ must be less than or equal to $N$. (See Press et al., 1988, for further details of this PSD estimation method.)

Once the AR estimate of the MSEMG PSD had been formed, the whitening filter $H_{time}^{-1}(e^{j\omega})$ was defined. To emulate real time processing, whitening was accomplished via a time domain filter. If the input to the filter was the sequence $m_i$ and the output of the filter was the sequence $v_i$, then the time domain whitening was performed as

$$v_i = \sum_{j=0}^{l} d_j m_{i-j}$$
In addition to AR estimation of the PSD, a technique based on the Discrete Fourier Transform (DFT) was used. The resultant PSD estimate was not utilized to perform whitening. Rather, the DFT estimation technique provided an estimate that could be compared to the AR estimate. DFT estimation of the PSD is often a more familiar estimate. To form the estimate, each 5 second record of 10240 samples was divided into 20 successive segments of 512 samples each. The DFT of the Parzen windowed data of each segment was computed. The PSD estimate for each segment was found at a chosen frequency by summing the square of the magnitude of the DFT at the positive and negative chosen frequency, and then normalizing by the square of the number of data points in the segment. The PSD estimate for the complete record was found by averaging the PSD estimates from the 20 segments. In this manner, the PSD was estimated at 256 discrete frequencies, providing an estimate every four Hz. The standard deviation of the PSD estimate at each frequency is equal to 100% of the expected value of the estimate at that frequency divided by the square root of the number of segments. Since the data was divided into 20 segments, the standard deviation of the PSD estimate at each frequency was approximately equal to 22.4% of the expected value of the estimate at that frequency. Press et al. (Section 12.7, 1988) provided an implementation of the above algorithm.
5.5 Results and Discussion —
Single Channel EMG Amplitude Estimation

5.5.1 Assessing Some Model Assumptions

Each subject performed five sets of contractions, a set consisting of one trial each at 10, 25, 50 and 75% of MVC. Each trial was five seconds in duration. For each subject, surface EMG data were recorded from eight electrode-amplifiers, yielding 160 MSEM1 data records per subject. For five subjects, then, a total of 800 MSEM1 data records were recorded. Each data record was plotted and visually inspected. It was determined that the high gain characteristic of the electrode-amplifier caused certain electrode-amplifiers, during certain experiments, to saturate during the higher contraction trials. Accordingly, all data from any electrode-amplifier which saturated during any portion of any experimental record were discarded from further analysis. Figure 5.3 lists the five subjects with the number of viable channels for each subject. In all, there were 33 viable electrode-amplifiers for a total of 660 viable MSEM1 data records. Figure 5.3 also identifies the muscle group (elbow flexors/ extensors) studied with each subject.

To confirm the zero mean assumption of the MSEM1 model, the mean value of each of the 660 MSEM1 records was examined. Except for two records from one electrode-amplifier in one subject, the mean value of the MSEM1 signal from a particular channel varied one A/D count or less throughout a complete experiment. The two records in exception varied by two A/D counts. Since the standard deviation of the MSEM1 signal was at least greater than 40-50 A/D counts (for the minimum 10% MVC level), it was concluded that the
<table>
<thead>
<tr>
<th>Subject</th>
<th>Extension/Flexion Experiment</th>
<th>Number and Identity of Viable Electrodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>Extension</td>
<td>5 (1,2,3,6,7)</td>
</tr>
<tr>
<td>CC</td>
<td>Extension</td>
<td>6 (0,2,4,5,6,7)</td>
</tr>
<tr>
<td>DA</td>
<td>Flexion</td>
<td>8 (0,1,2,3,4,5,6,7)</td>
</tr>
<tr>
<td>EA</td>
<td>Flexion</td>
<td>6 (0,3,4,5,6,7)</td>
</tr>
<tr>
<td>FA</td>
<td>Extension</td>
<td>8 (0,1,2,3,4,5,6,7)</td>
</tr>
</tbody>
</table>

**Figure 5.3:** Chart of Subjects for Experiment 1

The five subjects were denoted AD, CC, DA, EA and FA. The muscle group studied (elbow flexors or extensors) as well as the number and identity of viable electrodes for each subject is charted.
mean value of each channel was constant within a particular MSEM G channel and throughout the duration of an experiment. Mean values did vary between MSEM G channels because the signal from each MSEM G channel was amplified and filtered by analog hardware which varied slightly in performance between channels. The constant offset was removed by subtracting from each record the mean value of that record in A/D counts. Note that, more appropriately, a single offset should have been applied to all records from a particular MSEM G channel. Additionally, the actual offset need not have an integer value (an A/D count must be an integer value). The implemented method of offset removal was simpler and reduced the constant offset to less than one A/D count. Further, for both single and multiple channel amplitude estimation, the offset would have no effect on the SNR since a constant offset only alters the mean value of the estimate and not deviations about the mean. The offset adjusted (zero mean) MSEM G data were converted to double precision floating point. All subsequent analysis was performed in double precision floating point.

Figure 5.4 shows typical time plots of measured torque for the four contraction levels. These plots provide an assessment of the assumption that the subject exerted a constant torque throughout a particular contraction. Note that the data of Figure 5.4 were taken from an experiment which investigated elbow extension. Extensor torques were expressed in negative A/D units, while flexor torques were expressed in positive A/D units. The figure lists the computed SNR’s of each contraction. During the 75 and 50% MVC’s, noticeable high and low frequency periodic oscillations were superimposed on the torque record. To help evaluate the frequency content of the torque records, DFT PSD’s of these records were taken. These PSD’s are shown in Figure 5.5. At the 75% MVC
Figure 5.4: Measured Torque Versus Time at the Four Contraction Levels

Measured torque in A/D units is plotted versus time at the four contraction levels — 10, 25, 50 and 75% MVC. Subject AD produced elbow extension, whose resultant torque was expressed in negative A/D units (flexion torque was expressed in positive A/D units). The signal to noise ratio (SNR) (deviations relative to the mean) of each contraction is provided.
Figure 5.5: Power Spectra of the Measured Torque

Power spectra of the measured torque at the four contraction levels — 10, 25, 50 and 75% MVC. Each data record was normalized to a zero mean prior to estimation. Estimation resolution is 2 Hz.
level, the high frequency oscillations occurred at a frequency of approximately 14 Hz. in this subject. At 50% MVC the PSD plot is inconclusive, but the time domain plot suggests high frequency oscillations at approximately 14 Hz. To determine if the instrumented chair was influencing the high frequency oscillations, resonances of the instrumented beam were evaluated by deflecting the beam with an impulse (striking with a hammer) and observing the resultant oscillation for one cycle. This test was performed both while a subject contracted at the two high contraction levels and with no subject seated in the instrumented chair. In both cases, the resultant oscillation corresponded to a frequency greater than 30 Hz. Thus, the natural modes of the beam likely contributed little to the high frequency oscillation. Therefore, it was concluded that the high frequency oscillations were due to normal tremor activity associated with the high torque output of the muscle (Joyce and Rack, 1974). The oscillations had a noticeable impact on the SNR's of the high torque records, limiting the SNR's to values in the lower forties. These values were typical of all of the torque records. Clearly, the oscillations introduce oscillations into the generation of the joint torque. Compared to the mean level of torque, however, the oscillations are still small and of high frequency. At the two lower torque levels, no oscillations were noticed. At 25% of MVC, the computed SNR rose above 100. But, at 10% of MVC, the computed SNR rose only to 55.3. A closer inspection of the lower torque records revealed that the computed SNR was limited by the measurement technique. In particular, at the 10% contraction level, measured torque for the trial shown in Figure 5.4 produced A/D counts which only spanned the range –102 to –114. Since the SNR was the square root of the squared mean divided by variance of the signal, measurement noise with a variance of as little as two A/D counts (10
mV) was sufficient to provide a computed SNR in the middle seventies even if the applied torque was perfectly constant. Additionally, if the actual mean torque value was not an integer A/D count, considerable error could be introduced to the SNR computation at low torque levels. It is, therefore, likely that the actual SNR for 10% MVC was much higher than that which was measured. A similar argument applied at 25% of MVC. At the two high torque contraction levels, the signal span was sufficient to obscure any effect due to a few A/D counts of measurement noise.

5.5.2 A Temporal Whitening Example

In order to gain a deeper understanding of temporal whitening, the steps involved in whitening will be elucidated for an example record. Figure 5.6 shows a sequence of AR model PSD estimates used to form a sequence of whitening filters. The AR models were of order 4, 9, 14, 19 and 24 corresponding to 5, 10, 15, 20 and 25 filter coefficients, respectively. Figure 5.7 illustrates how a whitening filter is derived from the EMG waveform. Figure 5.8 shows how the MSEMG signal is altered by whitening. The whitened signal (bottom plots) clearly has a flatter PSD estimate than the original (raw) signal (top plots). Note that even if perfect whitening could be achieved, the PSD estimate would not be perfectly flat, since it is an estimate. As discussed previously, the DFT estimation technique, as utilized herein, produced PSD estimates at discrete frequencies with a standard deviation approximately equal to 22.4% of the expected value of the PSD at each frequency. Figure 5.9 shows the amplitude estimate (MARMS filter) formed from the raw and whitened MSEMG signals shown in Figure 5.8. The SNR of the amplitude estimate formed from the whitened MSEMG signal
Figure 5.6: Autoregressive Modeling of the EMG Power Spectrum

The dash line graph in each plot is the power spectral density (PSD) estimate of the record formed with a Discrete Fourier Transform (DFT) technique. The DFT technique PSD is replicated for each plot and serves as a means to visually compare separate plots. The solid line graph in each plot is the PSD estimate of the record formed with the autoregressive modeling technique. The autoregressive models were of order 4, 9, 14, 19 and 24 corresponding to 5, 10, 15, 20 and 25 filter coefficients, respectively. All PSD estimates were made from one five second record at a contraction level of 75% MVC. The area under each PSD estimate is normalized.
Figure 5.7: Whitening Filter Design

Top left plot shows a one second portion of the original (raw) EMG signal (total record length is five seconds). Jagged plot in the top right is the Discrete Fourier Transform technique power spectral density (PSD) estimate of the raw EMG. Smooth plot in the top right is the fourth-order autoregressive (AR) PSD estimate of the raw EMG. Bottom right plot is the magnitude response of the whitening filter formed from the AR PSD estimate. Bottom left plot is the sample response of the whitening filter formed from the AR PSD estimate.
Figure 5.8: Effect of Temporal Whitening on the EMG Signal

The four plots detail the effect of temporal whitening on the EMG signal. The upper left plot shows a 0.5 second portion of the original (raw) EMG signal (total record length is five seconds). The upper right plot is the Discrete Fourier Transform (DFT) technique power spectral density (PSD) estimate of the raw EMG. A five coefficient whitening filter was formed from a separate record from the same EMG channel during an identical level of contraction (75% MVC). The lower left plot is a 0.5 second portion of the whitened EMG signal. The lower right plot is the DFT technique PSD estimate of the whitened EMG. Each graph is independently normalized to its maximum value and graphed to a linear scale.
Figure 5.9: Effect of Temporal Whitening on the EMG Amplitude Estimate

The upper left plot shows a one second portion of the original (raw) EMG signal (total record length is five seconds). The upper right plot is the root mean square (MARMS) filtered raw EMG. The MARMS filter window is 500 samples (approximately 244 milliseconds). The lower left plot is the corresponding measured torque. A five coefficient whitening filter was formed from a separate record from the same EMG channel during an identical level of contraction (75% MVC). The lower right plot is the MARMS filtered whitened EMG. Whitening improved the signal to noise ratio (SNR) by 71%. Filter rise time is not included in the SNR computation. Each graph is independently normalized to its maximum value and graphed to a linear scale.
was a 71% improvement over that of the raw MSEM signal. Figure 5.9 also shows the measured torque and a portion of the raw MSEM signal for this record.

5.5.3 Study of Variations in the Construction of Temporal Whitening Filters

Several temporal whitening filters were constructed and applied to the data to determine the effectiveness of whitening as well as the relative merits of particular whitening schemes. At all times, the investigation was directed towards developing whitening filters which could be easily calibrated and implemented in real-time. Several sets of whitening filters will next be described, with SNR performance for each set shown in a separate figure. In the ensuing summary section, a complete chart of all the SNR results is provided. Performance differences between pairs of amplitude estimators were evaluated for statistical significance by paired t-tests (see Press et al., 1988, Section 13.4). For all estimators, the duration of the smoothing window corresponded to approximately 244 ms.

Order and Calibration Record Length

Initially, the effect of the number of coefficients in the AR PSD estimate along with the amount (time length) of MSEM data used to form the AR PSD estimate were examined. Two time lengths of data were initially evaluated — 5 seconds and 20 seconds. For a time length of 5 seconds, the whitening filter was formed from a separate record of the same subject from the same MSEM channel during an identical level of contraction. Since there were five records at each contraction level, four such separate records existed. If the five records at a particular torque level were sequentially denoted A, B, C, D and E, then E
calibrated the whitening filter for A, A calibrated the whitening filter for B, etc. Hence, each record calibrated the whitening filter for one other record. For a time length of 20 seconds, the whitening filter was formed from all four separate records of the same subject from the same MSEM G channel during an identical level of contraction. The four records were simply concatenated and an AR PSD estimate made from the concatenated sequence. Five different number of coefficients were initially evaluated — 5, 10, 15, 20 and 25. The performance for each whitening filter was expressed as a SNR. Results were averaged across all 660 electrodes. The mean and standard deviation SNR for each whitening filter are presented in Figure 5.10. All of the whitening filters improved the SNR performance of the simple amplitude estimator ($p < 0.000001$ for all paired comparisons). Further, all ten algorithms improved the average SNR performance from $10.7 \pm 3.3$ to approximately $17.4 \pm 6.1$, an approximate 63% increase in the average SNR. While each higher order filter did provide a statistically significant improvement in SNR performance compared to the respective five coefficient filter ($p < 0.008$ for all paired comparisons), the strength of the difference in SNR performance was small (at most 0.2). (Note that small differences could be found significant because of the large data volume.) There was no statistically significant difference between the SNR performance using 5 versus 20 seconds of data to calibrate the whitening filters ($p > 0.45$ for all paired comparisons).

The above data suggested that, at most, a five coefficient temporal whitening filter performed approximately as well as the higher order filters. For real-time applications, it was of interest to determine if yet lower order filters would perform as well. Thus, a portion of the above analysis was performed at all filter orders with less than five coefficients. Temporal whitening filters were derived
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG records, for several whitening algorithms are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars corresponding to one filter coefficient denotes the SNR when no whitening is performed. Remaining solid line error bars denote results from whitening filters calibrated from 20 seconds of EMG data from separate records of the same subject from the same EMG channel during an identical level of contraction. Dashed line error bars denote results from whitening filters calibrated with 5 seconds of EMG data from a separate record of the same subject from the same EMG channel during an identical level of contraction. Results are provided for 5, 10, 15, 20 and 25 filter coefficients. All whitening algorithms improved the average SNR performance from $10.7 \pm 3.3$ to approximately $17.4 \pm 6.1$. 

Figure 5.10: Single Channel Whitening — Effect of Number of Filter Coefficients and Length of Filter Calibration Record
only from five second data records since the above results demonstrated no performance difference between 5 and 20 second PSD estimators. Figure 5.11 shows the results. There was a statistically significant performance improvement with each added coefficient ($p < 0.000001$ for all paired comparisons). Further, the strength of each performance improvement is strong. Thus, a five coefficient (fourth-order) model seems to capture the shape of the EMG PSD. Triolo et al. (1988) concluded the same result.

The above data also suggested that, at most, five seconds of data were needed to calibrate the whitening filters. Again, for real-time applications, it was of interest to determine if yet shorter time durations would perform as well. Five coefficient temporal whitening filters were derived from portions of five second data records. Figure 5.12 shows the results for sample lengths of 128, 256, 512, 1024, 2048, 4096, 6144 and 8192 samples. (Since the sampling rate was 2048 Hz., these sample lengths correspond to time windows ranging in length from 62.5 ms to 4 s.) Compared to the five second calibration record length, a statistically significant drop in performance occurs for calibration record lengths less than 6144 samples (3 seconds) ($p < 0.0029$ for all paired comparisons). The strength of this drop is small — with a 128 sample (62.5 ms) calibration record, the drop in SNR performance is 0.7 (<5%). Although the SNR performance improvement with longer ($\geq 3$ seconds) calibration record lengths is small, the additional computation is a one-time cost incurred during calibration. If future applications require rapid calibration, then shorter calibration record lengths would be appropriate.

From all of the above results, it appears that a five coefficient filter, calibrated from at least a three second record of data, will perform approximately as well
Figure 5.11: Single Channel Whitening — Effect of Number of Filter Coefficients

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG records, for several whitening algorithms are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from whitening filters calibrated from 5 seconds of EMG data from separate records of the same subject from the same EMG channel during an identical level of contraction. Results are provided for 1 (no whitening), 2, 3, 4 and 5 filter coefficients.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG records, for several whitening algorithms are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from whitening filters calibrated from EMG data from separate records of the same subject from the same EMG channel during an identical level of contraction. Results are provided for calibration record lengths of 128, 256, 512, 1024, 2048, 4096, 6144 and 8192 samples (sampling frequency was 2048 Hz.).

Figure 5.12: Single Channel Whitening — Effect of Calibration Record Length
as the best possible combination of filter order and calibration record length. Calibration from as little as 62.5ms of data drops SNR performance by less than 5%. Future temporal whitening filters might need to modify the preferred filter order, since the AR PSD model has difficulty modeling the zero DC gain of the EMG signal. For the electrode-amplifiers used in this study, which attenuated the vast majority of the signal near DC, poor PSD modeling near DC should not have been a deficit. Future electrode-amplifiers which do admit significant power near DC may require better modeling near DC. In such a case, autoregressive moving average shaping filter models may be more appropriate.

Effect of Contraction Level

In order to investigate if the SNR performance of the above tests was influenced by the contraction level, results were segregated by contraction level for one test. Five coefficient temporal whitening filters were calibrated from five seconds of MSEMg data from one record of the same subject of the same contraction level from the same MSEMg channel. Results were separated into the four contraction levels and plotted in Figure 5.13. Both the raw and whitened performance were better at low levels of contraction. In particular, the SNR performance difference (for temporally whitened data) between each pair of contraction levels was statistically significant \((p < 0.000001)\), except for between the two highest contraction levels \((p \approx 0.53)\). Perhaps the tremor activity present at the high contraction levels contributed to this difference in performance. With tremor activity present, muscle activation deviated, in part, from an isotonic contraction. Since an isotonic contraction was assumed in calculating SNR performance, tremor activity would hinder the achieved SNR.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG recordings, are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from five coefficient whitening filters calibrated from five seconds of EMG data from one record of the same subject of the same contraction level from the same EMG channel. Dashed line error bars denote results when no whitening filter was applied. Results are separated by the four contraction levels of 10%, 25%, 50% and 75% of maximum voluntary contraction (MVC).
Effect of Sampling Rate

The effect of sampling rate was studied next. All data were decimated by a factor of two. Decimation by two was equivalent to changing the sampling frequency from 2048 Hz. to 1024 Hz. Figure 5.14 shows the DFT PSD estimate of one data record for the two sampling rates (1024 Hz. and 2048 Hz.). Although sampling at 1024 Hz. might have produced aliasing (in fact, no formal proof was made to assure that aliasing did not occur with a sampling rate of 2048 Hz.), only a standard deviation estimate—not a complete reconstruction of the input waveform—was desired. In fact, the effect of aliasing can be examined analytically. Figure 5.15 depicts the effect of aliasing upon the whitening filter. Figure 5.15 shows that all non-aliased frequencies are properly whitened, but all aliased frequencies (including frequencies of the continuous time signal which are greater than the Nyquist frequency) receive insufficient gain. However, note that frequencies higher than the Nyquist frequency are still represented in the data, and, therefore, the EMG amplitude estimate. In practice, then, it is important to experimentally evaluate the tradeoff between SNR and sampling rate in the absence of anti-aliasing filters.

The effect of the number of coefficients in the AR PSD estimate along with the time length of MSEM data used to form the AR PSD estimate (as discussed above) were reexamined in the same manner as above. (In order to maintain a smoothing window of approximately 244 ms, one half the number of discrete-time samples were used in the MARMS smoothing window.) The results are presented in Figure 5.16. In each case, temporal whitening still provided a statistically significant improvement over simple amplitude estimation \((p < 0.000001)\). All ten algorithms improved the average SNR performance.
Figure 5.14: Power Spectra at Different Sampling Rates

All plots are Discrete Fourier Transform estimates of the power spectral density of the same data record. Plots at left are linear/logarithmic plots for the data sampled at 2048 Hz. Plots at right are linear/logarithmic plots for the data sampled at 1024 Hz. (the original data were decimated by a factor of two).
Figure 5.15: Effect of Aliasing on the Temporal Whitening Filter

(A) Fourier Transform of a continuous time signal. (B) Fourier Transform of (A) after sampling at the rate $\omega_c$ (less than the Nyquist rate). (C) Whittening filter designed based on (B). Dotted lines in (B) and (C) indicate overlap in the periodic repetitions of the continuous time transform.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG records, for several whitening algorithms are presented. All data was first decimated by a factor of two, reducing the effective sampling rate from 2048 Hz. to 1024 Hz. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bar corresponding to one filter coefficient denotes the SNR when no whitening is performed. Remaining solid line error bars denote results from whitening filters calibrated from 20 seconds of EMG data from separate records of the same subject from the same EMG channel during an identical level of contraction. Dashed line error bars denote results from whitening filters calibrated with 5 seconds of EMG data from a separate record of the same subject from the same EMG channel during an identical level of contraction. Results are provided for 5, 10, 15, 20 and 25 filter coefficients. All whitening algorithms improved the average SNR performance from $10.7 \pm 3.3$ to approximately $13.5 \pm 4.6$. 

**Figure 5.16: Single Channel Whitening — Effect of Sampling Rate**
from $10.7 \pm 3.3$ to approximately $13.5 \pm 4.6$, an approximate 26% increase in the average SNR.

The unwhitened signal experienced no decrease in performance due to sampling at only 1024 Hz. Since the raw MSEM waveform has little power above 512 Hz., increasing the sampling rate to 2048 Hz. introduced essentially no new information. But, with temporal whitening, the faster sampling rate provided an improvement. Thus, whitening at a sampling rate of 2048 Hz. apparently was able to recover considerable information beyond 512 Hz. up to 1048 Hz. This recovery confirmed the contention that significant signal power (significant referring to more signal than noise — even though the absolute signal level at high frequencies was a few orders of magnitude lower than the signal level at the peak EMG frequency) existed in the MSEM waveform up through 1024 Hz. for most contraction levels. The slower sampling rate was not able to effectively capture this high frequency information. Still faster sampling rates may have produced further performance improvements so long as signal power continued to dominate noise power at higher frequencies.

One Temporal Whitening Filter Per Channel

From the above results, it was concluded that whitening filters with five coefficients, calibrated from a five second record of data, performed as well as any of the other evaluated whitening filters. Since this whitening filter was easiest to derive, it was used in all future analysis. All of the previous investigation restricted the whitening filter for a record to be calibrated from a separate record(s) at the same contraction level. This restriction was relaxed in the next investigation. A single record per subject was chosen and whitening filters found from
each channel within that record. These whitening filters were then applied to the corresponding channel within all remaining records of that subject. This method provided whitening to a total of 627 MSEMg records. This investigation was performed four times, once each by forming a whitening filter from a record corresponding to 10, 25, 50 and 75% MVC. The results are presented in Figure 5.17. Calibrating the whitening filter from a record corresponding to 50 or 75% of MVC increased the SNR performance from approximately $10.8 \pm 3.4$ to approximately $17.6 \pm 6.0$, an approximate 63% increase in the average SNR. This improvement degraded slightly when calibration was from a record corresponding to 25% of MVC, and degraded markedly when calibration was from a record corresponding to 10% of MVC. Statistically, these SNR performance differences were all significant ($p < 0.0002$ for all paired comparisons), except when comparing whitening filters formed from records corresponding to 50 and 75% MVC ($p \approx 0.070$).

Closer inspection of the whitening process was performed for a few records when calibration of the whitening filter was from a record corresponding to 10% of MVC. It was determined that insufficient gain had been applied to high frequency components of the MSEMg signal, particularly those above the bandwidth of the original MSEMg signal. It is possible to attribute this discrepancy in performance to unmodeled measurement noise as follows. With the electrode-amplifiers used herein, the measurement noise is small at all frequencies during a high contraction. However, at low contractions, significant noise was often evident at frequencies outside the bandwidth of raw MSEMg. This noise will increase the estimated PSD intensity at high frequencies. The derived whitening filter (whose gain was related inversely to the PSD) necessarily had decreased
Figure 5.17: Filter Coefficients Derived from One Record

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 627 EMG recordings, are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from whitening filters calibrated from five seconds of EMG data from one record of the same subject from the same EMG channel. Four such calibration records, one each corresponding to 10%, 25%, 50% and 75% of maximum voluntary contraction (MVC) were evaluated. Dashed line error bars denote results when no whitening filter was applied.
gain at these high frequencies. When this whitening filter was applied to data corresponding to higher contraction levels, insufficient gain at high frequencies resulted. Overall, however, these results suggested that a single whitening filter per channel could be used to whiten data from all levels of contraction, but the whitening filter should be formed from data corresponding to a high contraction level.

Composite Whitening Filter

The next comparison studied the feasibility of applying a single whitening filter, derived from one record from a subject, to all channels of all records for that subject. A “composite” whitening filter was formed by first normalizing the variance of each MSEMГ channel within the calibration record. The normalized data were then summed, forming a single five second sequence. The whitening filter was derived from the summed sequence. Note that if the channels were completely mutually uncorrelated, the PSD of the normalized sum of channels would equal the sum of the normalized PSD of each channel. Although the channels are not actually uncorrelated, the assumption equalized the relative contribution of each channel to the formation of the whitening filter. This investigation was performed four times, once each by forming a composite whitening filter from a record corresponding to 10, 25, 50 and 75% MVC. Averaged results of the 660 MSEMГ records are presented in Figure 5.18. At high torque levels, the SNR performance improved from 10.7 ± 3.3 to approximately 16.7 ± 5.7, an approximate 56% increase in the average SNR. Again, performance dipped when calibration was from a record corresponding to 10% MVC. As with the previous investigation, these SNR performance differences were all significant
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 660 EMG recordings, are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from composite whitening filters formed from all channels within a record and applied to all channels from that subject. Four composite whitening filters where constructed per subject, one each corresponding to data from 10%, 25%, 50% and 75% of maximum voluntary contraction (MVC). Dashed line error bars denote results when no whitening filter was applied.
(\(p < 0.003\) for all paired comparisons), except when comparing whitening filters formed from records corresponding to 50 and 75\% MVC \((p \approx 0.55)\). Although these improvements were not as good as the previous set, it is clear that a single composite whitening filter per subject provided considerable SNR improvement.

**Universal Whitening Filter**

The last comparison of whitening filters investigated the feasibility of a universal whitening filter. That is, a single filter which could be applied to all channels at all contraction levels for all subjects (including both flexor and extensor muscles acting about the elbow). A composite whitening filter was derived from all channels of a single record from a single subject in the same manner discussed in the previous comparison. This universal filter was applied to all channels of all records of the four remaining subjects. The investigation was performed five times, once each by forming the universal whitening filter from a record corresponding to each of the five subjects. Because previous whitening filters performed best when derived from records associated with high contraction, the records used to form universal whitening filters were all associated with contraction at 50\% of MVC. Figure 5.19 shows the results. Average SNR improvements per subject varied from 39\% to 55\%, yielding average SNR's ranging from 14.7 \(\pm\) 5.4 to 17.4 \(\pm\) 5.5. Considerable SNR improvement was facilitated with a universal temporal whitening filter.

**5.5.4 Robustness — The Problem of Uncharacteristic Data**

The optimal estimators studied in this chapter all assumed that the temporal correlation of the data did not change over time (i.e. the assumption of station-
Figure 5.19: Single Channel Whitening — Universal Whitening Filter

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window) are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote whitening from a universal filter formed from data at the 50% maximum voluntary contraction level of the identified subject. Dashed line error bars denote results when no whitening filter was applied.
arity). On close examination, it was found that a few of the MSEMGS channels displayed large amplitude, high frequency perturbations in a few, but not all, of that channel’s recordings. Figure 5.20 depicts a problem that can arise when the character of the MSEMGS signal changes over time. In the case of these data, a temporal whitening filter was calibrated from a separate record, which did not exhibit the perturbations, of the same channel at the same contraction level. The high frequency portions of the raw data were considered uncharacteristic by the whitening filter, and thus, greatly amplified by the whitening filter. The whitening filter output should have a standard deviation of 1 (using the axis scales shown in the figure), if viewed about any subinterval. But, deviations as large as 10 standard deviations or more are present. The SNR performance of this channel is actually reduced from 11.1 to 2.8.

The figure suggests that the perturbations resemble single motor unit recordings. It may be the case that a single motor unit located near the electrode-amplifier was active during some contractions, but not others. A single identifiable motor unit would likely present high frequency information to the MSEMGS recording. As well, recordings with high frequency artifact might suffer in the same manner. This resultant poor SNR performance is not surprising if one notes that the high frequency gain of the whitening filters is as large as 1000 times or more that of the low frequency gain. A small increase in the high frequency content of the MSEMGS can have a profound effect on amplitude estimation. Robust amplitude estimators may require a mechanism to continuously monitor for changes in the character of the MSEMGS. Note that the few anomalous channels should have had little effect on the previously discussed single channel results since all single channel SNR averages were made with over 600 entries.
Figure 5.20: Whitening Uncharacteristic Data

Top left graph is a five second recording of raw MSEMGE during an isometric, isotonic, non-fatiguing contraction. Top right graph is an expanded view of one segment of the raw MSEMGE. Note the large amplitude, high frequency perturbations. The bottom graphs are corresponding plots of the whitened EMG. The temporal whitening filter was calibrated from a separate record, which did not exhibit the perturbations, of the same channel at the same contraction level.
5.5.5 The Mean Absolute Value Processor

The MAV processor was studied two ways. First, simple amplitude estimation (no temporal whitening) was performed and compared to the MARMS detector. With the MAMAV detector, SNR performance improved from 10.7 ± 3.3 to 11.4 ± 3.0, a 6.5% improvement. Second, each data record was temporally whitened. One whitening filter per channel was derived from a single 50% MVC of that channel, and applied to all recordings from that channel. This method whitened all 660 MSEMGl records. Amplitude estimation of the whitened data was performed comparing the MARMS detector to the MAMAV detector. Again, the MAMAV detector improved upon the MARMS detector, increasing the SNR performance form 17.4 ± 5.9 to 17.8 ± 5.1 (an increase in performance of 2.2%). In both of the above cases, the performance difference due to the MAMAV detector was statistically significant ($p < 0.000002$).

These results are clearly not in agreement with those predicted by the Gaussian EMG model. The Gaussian EMG model predicted an approximate 6-7% SNR performance decrement due to the MAMAV detector. Inspection of the EMG waveform data suggested that outlier samples might occur more frequently than predicted by the Gaussian model. The first order MAMAV detector is influenced less by outliers than the second order MARMS detector. Hence, the MAMAV amplitude estimator de-emphasizes the outliers as compared to the MARMS amplitude estimator. Perhaps EMG data are more appropriately described by the Laplacian PDF. (Recall from Chapter 4 that the MAMAV detector is optimal if the data are Laplacian distributed and that uncorrelation is equivalent to independence.)
5.5.6 Predicted Versus Achieved SNR Performance

If perfect whitening could be achieved and the Gaussian EMG model accurately described the surface EMG, then SNR performance for whitened data would depend only on the length of the smoothing window, $N$. Since $N = 500$, the SNR performance should be approximately 31.6. Perfect whitening cannot be achieved, however, and the Gaussian EMG model only approximates the surface EMG phenomenon.

Two studies investigated the relationship between predicted and achieved performance. First, the simple amplitude estimate formed from the raw EMG waveform was studied. Second, the amplitude estimate formed from whitened EMG data was studied. To whiten the data, one whitening filter per channel was derived from a single 50% MVC of that channel, and applied to all recordings from that channel. Predicted performance was computed in both cases by evaluating the number of degrees of freedom in the data. (Clearly, the whitened data had more degrees of freedom than the raw data.) The number of degrees of freedom in the data is dependent on the statistical bandwidth, $B_s$, of the data. $B_s$ was estimated for each record by first estimating the PSD of each record using the DFT technique. $B_s$ was then estimated from the PSD estimate, $\hat{S}_{mm}(e^{j\omega})$, as

$$\hat{B}_s = \frac{\left[\sum_{k=0}^{K} \hat{S}_{mm}(k)\right]^2}{\sum_{k=0}^{K} \hat{S}_{mm}^2(k)}$$

where $K=256=$ the number of discrete frequencies at which the PSD was estimated. Results of predicted versus achieved performance are shown in the scatter plots of Figure 5.21.

The figures demonstrate a weak relationship between predicted and actual performance. The whitened data, in particular, seem clustered around a pre-
Figure 5.21: Scatter Plots of Predicted Vs. Achieved SNR Performance

The plot on the left shows SNR performance of simple amplitude estimates formed from the raw EMG waveforms. The plot on the right shows SNR performance of amplitude estimates formed from whitened EMG waveforms. One whitening filter per channel was derived from a single 50% MVC of that channel, and applied to all recordings from that channel. Each scatter plot has 660 entries.
dicted SNR of approximately 14, but achieved performance varies widely. In practice, there seems to be no advantage to using the predicted SNR over the achieved SNR since both measures must be computed from the data. Using the achieved SNR performance provides a definitive measure of performance.

5.6 Summary and Conclusion —
Single Channel EMG Amplitude Estimation

Figure 5.22 summarizes the results of the various single channel temporal whitening filters. The use of five or more filter coefficients worked approximately equally well, improving the SNR from approximately 10.7±3.3 to 17.4±6.1. Performance dropped progressively when less than five filter coefficients were used. Whitening filters could be calibrated from three or more seconds of EMG data. Only a small performance loss (<5%) was incurred with calibration record lengths as short as 62.5ms. Decreasing the sampling rate did not influence the SNR of the raw MSEM, but did diminish the SNR of the temporally whitened MSEM to approximately 13.5±4.6. If a single temporal whitening filter was formed from a high contraction trial from each electrode-amplifier, then SNR performance was high at 17.6 ± 6.0. If a single composite temporal whitening filter was formed from a high contraction trial, most of the performance improvement was realized, yielding an average ± standard deviation SNR of 16.7 ± 5.7. SNR performance using universal composite whitening filters ranged from 14.7 ± 5.4 to 17.4 ± 5.5, depending on the subject. For all temporal whitening filters, SNR performance was best at lower levels of contraction. The robustness of all temporal whitening filters must address the problem of uncharacteristic high frequency noise.
The MAMAV detector was a modest (2-7%) improvement over the MARMS detector.
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Raw EMG SNR ±σ</th>
<th>Whitened EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 20 seconds of data from same channel, same contraction level, same subject, with; 5 filter coefficients,</td>
<td>10.7 ± 3.3</td>
<td>17.4 ± 6.1</td>
<td>63</td>
</tr>
<tr>
<td>10 filter coefficients,</td>
<td>10.7 ± 3.3</td>
<td>17.5 ± 6.2</td>
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</tr>
<tr>
<td>15 filter coefficients,</td>
<td>10.7 ± 3.3</td>
<td>17.6 ± 6.2</td>
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<td>20 filter coefficients,</td>
<td>10.7 ± 3.3</td>
<td>17.6 ± 6.2</td>
<td>64</td>
</tr>
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<td>25 filter coefficients.</td>
<td>10.7 ± 3.3</td>
<td>17.6 ± 6.3</td>
<td>64</td>
</tr>
<tr>
<td>2) 5 seconds of data from same channel, same contraction level, same subject, with; 1 filter coefficient,</td>
<td>10.7 ± 3.3</td>
<td>12.9 ± 5.0</td>
<td>21</td>
</tr>
<tr>
<td>2 filter coefficients,</td>
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<td>15.8 ± 5.8</td>
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<td>16.9 ± 6.1</td>
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<tr>
<td>15 filter coefficients,</td>
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<td>17.6 ± 6.2</td>
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<td>25 filter coefficients.</td>
<td>10.7 ± 3.3</td>
<td>17.6 ± 6.2</td>
<td>64</td>
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Figure 5.22: Tabulated Results of Single Channel Amplitude Estimation

Single channel amplitude estimation results shown graphically in previous figures are provided in one chart. This figure is continued on ensuing pages.
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Raw EMG SNR ±σ</th>
<th>Whiten EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
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<tr>
<td>3) Portion of one record from same channel, same contraction level, same subject, with;</td>
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<td>128 samples (62.5ms),</td>
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<td>8192 samples (4s),</td>
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<td>4) 5 seconds of data from same channel, same contraction level, same subject, with, 5 filter coefficients, for;</td>
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<tr>
<td>10% MVC,</td>
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Figure 5.22 (Continued)
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<th>Method of Determining Whitening Filter</th>
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<th>Whitened EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
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<td>5) Decimate data by 2:</td>
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<tr>
<td>20 seconds of data from same channel,</td>
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<td>6) Decimate data by 2:</td>
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<tr>
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<tr>
<td>25 filter coefficients.</td>
<td>10.7 ± 3.3</td>
<td>13.8 ± 4.7</td>
<td>29</td>
</tr>
<tr>
<td>7) One 5 coefficient filter per channel,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>per subject, derived from data at;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>10.7 ± 3.3</td>
<td>15.6 ± 6.3</td>
<td>46</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>10.7 ± 3.4</td>
<td>17.0 ± 6.5</td>
<td>59</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>10.8 ± 3.4</td>
<td>17.6 ± 6.0</td>
<td>63</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>10.7 ± 3.4</td>
<td>17.5 ± 5.7</td>
<td>64</td>
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</table>

Figure 5.22 (Continued)
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Raw EMG SNR $\pm \sigma$</th>
<th>Whitened EMG SNR $\pm \sigma$</th>
<th>Percent Increase in Average SNR</th>
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</thead>
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<tr>
<td>8) One 5 coefficient composite filter per subject derived from data at; 10% MVC,</td>
<td>10.7 $\pm$ 3.3</td>
<td>15.7 $\pm$ 5.9</td>
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<tr>
<td>25% MVC,</td>
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<td>16.5 $\pm$ 5.9</td>
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<tr>
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<td>16.7 $\pm$ 5.7</td>
<td>55</td>
</tr>
<tr>
<td>75% MVC.</td>
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<td>16.7 $\pm$ 5.6</td>
<td>56</td>
</tr>
<tr>
<td>9) Universal 5 coefficient filter derived from 50% MVC of subject; AD,</td>
<td>10.6 $\pm$ 3.0</td>
<td>16.4 $\pm$ 5.8</td>
<td>55</td>
</tr>
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<td>CC,</td>
<td>10.6 $\pm$ 3.2</td>
<td>14.7 $\pm$ 5.4</td>
<td>39</td>
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<td>DA,</td>
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<td>15.7 $\pm$ 5.3</td>
<td>51</td>
</tr>
<tr>
<td>EA,</td>
<td>10.7 $\pm$ 3.4</td>
<td>16.3 $\pm$ 5.5</td>
<td>52</td>
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<tr>
<td>FA.</td>
<td>11.3 $\pm$ 3.5</td>
<td>17.4 $\pm$ 5.5</td>
<td>54</td>
</tr>
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</table>

Figure 5.22 (Final Sheet)
Chapter 6

Experiment 1 — Part II: Multiple Channel Optimal Estimation of Constant Torque EMG Amplitude at One Angle
6.1 Introduction

The previous chapter described a set of experiments which investigated the performance of EMG amplitude estimators, and presented the results for single channel EMG estimation. This chapter reports the results of multiple channel EMG amplitude estimators applied to the same data. All of the multiple channel estimators incorporate temporal whitening filters.

Multiple channels of the MSEMG data were recorded from one muscle group (either elbow flexors or extensors) during isometric, isotonic contraction of that muscle group. Several factors were explored with respect to the spatial whitening filter. The amount of data used to estimate the filter coefficients, the number of electrode-amplifiers participating in the estimate, the effect of contraction level, the effect of sampling rate, and the effect of channel correlation upon the performance of the spatial whitening filter were investigated. Additionally, the MAV processor was compared to the optimal estimator. As before, all estimators incorporated a fixed smoothing window corresponding to approximately 245ms. The experimental apparatus and experimental methods were detailed in Chapter 5.

6.2 Methods of Analysis — Multiple Channel EMG Amplitude Estimation

The basic computational steps involved in optimal estimation of multiple channel EMG amplitude were temporal whitening of individual MSEMG channels, followed by spatial uncorrelation of the whitened channels, followed by MARMS calculation. Temporal whitening was discussed in detail in the previous chap-
ter. For multiple channel analysis, a common temporal whitening technique was utilized. A single five coefficient whitening filter was constructed for each electrode-amplifier for each subject from one five second record (at the 50% MVC level) per subject, and applied to all trials recorded by that electrode-amplifier. All MSEM data were temporally whitened before any channels were spatially combined. Spatial uncorrelation required computation of spatial uncorrelation filters. Since this computation implied knowledge of the spatial correlation of the data, it was an estimation problem itself. Once estimated, spatial uncorrelation filters could be applied through a direct linear transformation of the MSEM channels.

In order to form a spatial uncorrelation filter for $L$ channels of data, the ensemble random vector $\mathbf{m}_{*,i}$ formed from the $L$ channels of MSEM activity corresponding to time $i$ as

$$
\mathbf{m}_{*,i} = \begin{bmatrix}
m_{1,i} \\
m_{2,i} \\
m_{3,i} \\
\vdots \\
m_{L,i}
\end{bmatrix}
$$

was considered. As discussed in Chapter 4 and Appendix C, the eigenvalues and eigenvectors of the covariance matrix $K_{\mathbf{m}_{*,i}\mathbf{m}_{*,i}}$ (evaluated at a reference value for the EMG amplitude $s$) define the spatial uncorrelation filter. Thus, first an estimate of $K_{\mathbf{m}_{*,i}\mathbf{m}_{*,i}}$ was formed. Since all MSEM channels were assumed to be JWSS processes, $K_{\mathbf{m}_{*,i}\mathbf{m}_{*,i}}$ was the same for all time $i$. Thus, the $j, k$ element of the ensemble covariance matrix (or, equivalently, the ensemble correlation matrix since both processes were zero mean) was estimated from the respective
channels of MSEMG data as

\[ \hat{k}_{m_j, m_k, i} = \frac{1}{N} \sum_{i=0}^{N-1} m_{j, i} m_{k, i} \]

where \( N \) is the number of time samples per channel.

Next, the eigenvalues and eigenvectors of the estimated covariance matrix were computed. An algorithm described by Press et al. (1988, Section 11.1) was utilized. This algorithm uses the fact that a real symmetric matrix has a complete set of real linearly independent eigenvectors. Consider choosing these eigenvectors to be orthonormal. If the columns of a matrix, \( D^T \), are formed from the \( L \) eigenvectors, then the \( L \) eigenvalue equations can be written in one equation as

\[ K_{m_m, m_m, i} \cdot D^T = D^T \text{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_L) \]
\[ = D^T \Lambda \]

where \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_L \) are the \( L \) eigenvalues. Since \( D^T \) was constructed such that its columns are orthonormal, \( D^{-1} = D^T \), and thus

\[ D \cdot K_{m_m, m_m, i} \cdot D^T = \Lambda \]

The above shows that any real symmetric matrix can be diagonalized by a similarity transformation, that the columns of the transformation matrix \( D^T \) are the eigenvectors, and that the diagonal matrix \( \Lambda \) contains the eigenvalues. The strategy of the algorithm presented by Press et al. (1988) is to perform a series of similarity transforms on the matrix \( K_{m_m, m_m, i} \), with each transformation moving the resultant matrix towards a diagonal structure. For example, the first transformation can be denoted

\[ \Lambda_1 = P_1^{-1} K_{m_m, m_m, i} P_1 \]
and the second transformation can be denoted

\[
\Lambda_2 = P_2^{-1} \Lambda_1 P_2 \\
= P_2^{-1} P_1^{-1} K_{m_{*,i}m_{*,i}} P_1 P_2
\]

etc., where the \( P_i \) are the transformation matrices. By selecting the transformation matrices via the cyclic Jacobi method, Press et al. (1988) show that the transformations converge to a matrix which is diagonal. This iterative procedure was terminated when the matrix \( \Lambda_i \) was diagonal to machine precision (using double precision mathematics). The eigenvectors are taken as the columns of the matrix

\[
D^T = P_1 \cdot P_2 \cdot P_3 \cdots
\]

Once the eigenvectors and eigenvalues were known, spatial uncorrelation of the ensemble random vector \( m_{*,i} \) was performed as

\[
u_{*,i} = \Lambda^{-\frac{1}{2}} D m_{*,i}
\]

where

\[
\Lambda^{-\frac{1}{2}} = \text{diag} \left( \frac{1}{\sqrt{\lambda_1}}, \frac{1}{\sqrt{\lambda_2}}, \frac{1}{\sqrt{\lambda_3}}, \ldots, \frac{1}{\sqrt{\lambda_L}} \right)
\]

and \( \nu_{*,i} \) was the transformed, uncorrelated ensemble random vector.

In addition to the above computations, an estimate of the correlation coefficient matrix of the ensemble random vector \( m_{*,i} \) was desired. Since all MSEMG channels were assumed to be JWSS processes, the second-order statistics of \( m_{*,i} \) were the same for all time \( i \). Thus, the \( j, k \) element of the ensemble correlation coefficient matrix was estimated by first estimating each element of the ensemble covariance matrix (or, equivalently, the ensemble correlation matrix since all
processes were zero mean) as

\[ \hat{k}_{m_j,m_k,i} = \frac{1}{N} \sum_{i=0}^{N-1} m_{j,i} m_{k,i} \]

where \( N \) is the number of time samples per channel. Then,

\[ r_{m_j,m_k,i} = \frac{\hat{k}_{m_j,m_k,i}}{\sqrt{\hat{k}_{m_j,m_j,i} \hat{k}_{m_k,m_k,i}}} \]

### 6.3 Results and Discussion — Multiple Channel EMG Amplitude Estimation

#### 6.3.1 A Spatial Uncorrelation Example

As in the single channel case, prior to presenting performance results which were collated across all records in all subjects, it is informative to follow multiple channel processing in detail for one record. For this example, four MSEM channels from a 50% MVC level are presented. Figure 6.1 shows a one half second segment of the four MSEM channels. The four channels were temporally whitened and optimally combined. The multidimensional spatial filter was calibrated from one five second record from another contraction trial from the same subject. The four resultant optimally filtered channels are shown in Figure 6.2. Note that although the optimal channels are labeled in correspondence with the raw channels, each optimal channel is actually a linear combination of all raw channels. The lowest numbered optimal channel corresponds to the largest eigenvalue in the spatial filter, the second lowest numbered optimal channel corresponds to the second largest eigenvalue, etc. Figures 6.3 and 6.4 show the DFT technique PSD of these raw and optimal signals, respectively. The optimal channels are clearly whiter than the original signals. Figure 6.5 shows
Figure 6.1: Four Channels of Raw EMG

A one half second segment of four measured surface EMG's from a single 50% maximum voluntary contraction are shown. Each graph is independently normalized to its maximum value and graphed to a linear scale.
Figure 6.2: Four Channels of Temporally Whitened, Spatially Uncorrelated EMG

A one half second segment of four temporally whitened, spatially uncorrelated EMG's from a single 50% maximum voluntary contraction are shown. Note that although the optimal channels are labeled with channel numbers, each optimal channel is a linear combination of all raw channels. The lowest numbered optimal channel corresponds to the largest eigenvalue in the spatial uncorrelation filter, etc. Each graph is independently normalized to its maximum value and graphed to a linear scale. (Same raw data as Figure 6.1.)

190
Figure 6.3: Power Spectrum of Four Channels of Raw EMG

Power spectrum of four measured surface EMG's from a single 50% maximum voluntary contraction. Each graph is independently normalized to its maximum value and graphed to a linear scale. (Same raw data as Figure 6.1.)
Figure 6.4: Power Spectrum of Four Channels of Temporally Whitened, Spatially Uncorrelated EMG

Power spectrum of four temporally whitened, spatially uncorrelated EMG's from a single 50% maximum voluntary contraction. Each graph is independently normalized to its maximum value and graphed to a linear scale. (Same raw data as Figure 6.1.)
\[
\hat{R}_{\text{temporally whitened}} = \begin{bmatrix}
1.000 & 0.693 & 0.577 & 0.538 \\
0.693 & 1.000 & 0.834 & 0.693 \\
0.577 & 0.834 & 1.000 & 0.890 \\
0.538 & 0.693 & 0.890 & 1.000 \\
\end{bmatrix}
\]

\[
\hat{R}_{\text{optimal}} = \begin{bmatrix}
1.000 & -0.090 & 0.141 & 0.036 \\
-0.090 & 1.000 & 0.022 & 0.085 \\
0.141 & 0.022 & 1.000 & -0.019 \\
0.036 & 0.085 & -0.019 & 1.000 \\
\end{bmatrix}
\]

**Figure 6.5:** Ensemble (Spatial) Correlation Coefficient Matrices

The estimated ensemble correlation coefficient matrices of the temporally whitened and optimal (temporally whitened and spatially uncorrelated) signals are shown. These matrices express the degree of correlation between EMG channels. (Same data as Figure 6.1.)

the estimated ensemble (spatial) correlation coefficient matrices for the temporally whitened and optimal signals. Estimates were formed from the five second, 10240 sample waveforms. The spatial correlation in the channels was essentially eliminated by the spatial filter. Figure 6.6 shows the multiple channel amplitude estimate, which has a SNR of 32.4. Figure 6.7, which charts the SNR of each of the four raw and temporally whitened channels, shows that this four channel amplitude estimate improved the average raw SNR by approximately 226% and the average temporally whitened SNR by approximately 75%.
Figure 6.6: Multiple Channel Amplitude Estimate

Top trace is the EMG amplitude estimate based on four temporally whitened, spatially uncorrelated channels. Bottom trace is the corresponding measured joint torque. Filter rise time is not included in the SNR computation. Each graph is independently normalized to its maximum value and graphed to a linear scale. (Same raw data as Figure 6.1.)
<table>
<thead>
<tr>
<th>EMG Channel</th>
<th>Raw EMG SNR</th>
<th>Whitened EMG SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10.0</td>
<td>16.5</td>
</tr>
<tr>
<td>5</td>
<td>9.4</td>
<td>14.7</td>
</tr>
<tr>
<td>6</td>
<td>9.0</td>
<td>19.7</td>
</tr>
<tr>
<td>7</td>
<td>11.3</td>
<td>23.0</td>
</tr>
</tbody>
</table>

**Figure 6.7:** Chart of SNR Performance of Four EMG Channels

SNR performance of four raw and temporally whitened EMG channels. Data from subject EA, Record 10. (Same data as Figure 6.1.)

### 6.3.2 Study of Variations in the Construction of Spatial Uncorrelation Filters

Several spatial uncorrelation filters were constructed and applied to the data to determine the effectiveness of spatial uncorrelation as well as the relative merits of particular techniques for calibrating uncorrelation filters. This study focused on different techniques for estimating the ensemble covariance matrix, since the spatial uncorrelation filters followed from the estimates of the ensemble covariance matrix. In general, the amount of computation involved in spatial filtering was a function only of the number of channels of data. As with the study of temporal whitening, each set of comparisons will be shown in a separate figure with a complete chart of all results provided in the ensuing summary section. Performance differences between pairs of amplitude estimators were evaluated for statistical significance by paired t-tests (Press et al., 1988, Section...
13.4). All MSEM G data were temporally whitened (as described above) before any channels were spatially combined. For all estimators, the duration of the smoothing window corresponded to approximately 244 ms.

**Number of Channels and Calibration Record Length**

Initially, the effect of the number of channels participating in the amplitude estimate along with the amount (time length) of MSEM G data used to form the ensemble covariance matrix estimate were examined. Two time lengths of data were initially evaluated — 5 seconds and 20 seconds. For a time length of 5 seconds, the ensemble covariance matrix was estimated from a separate record of the same subject from an identical level of contraction. Each record was used to estimate the ensemble covariance matrix for one other record, in the same manner as the 5 second calibrations for temporal whitening. For a time length of 20 seconds, the ensemble covariance matrix was estimated from the concatenation of all four separate records of the same subject from an identical level of contraction. Figure 6.8 shows the mean and standard deviation SNR for multiple channel amplitude estimation with 1, 2, 4 and 6 MSEM G channels. Since there was freedom in deciding which channels out of the total would participate in a particular estimate, whenever possible data from non-adjacent electrode-amplifiers were selected. All subjects yielded at least four viable channels (100 contraction trials), four of five subjects yielded at least six viable channels (80 contraction trials), and two of five subjects yielded seven or more viable channels (40 contraction trials). There was no statistical difference in performance between using 5 or 20 seconds of data to estimate the ensemble covariance matrix. (p < 0.36 for all paired comparisons of 2, 4, 6 and 8 channel estimators). The fig-
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 80 or more contraction trials, for several multiple channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from spatial uncorrelation filters calibrated from 20 seconds of MSEMg data from separate records from the same subject during an identical level of contraction. Dashed line error bars denote results from spatial uncorrelation filters calibrated with 5 seconds of MSEMg data from a separate record from the same subject during an identical level of contraction. All subjects produced at least four viable MSEMg channels. Four of five subjects produced six viable MSEMg channels. Non-adjacent MSEMg channels were selected as available.
ure shows that the SNR increased with the number of channels. Caution must be exercised in making this statement, because the particular channels which contributed to an amplitude estimate necessarily changed with the total number of channels participating in the amplitude estimate. Thus, the “baseline” SNR (i.e. the average non-optimized single channel SNR) of the channels participating in the amplitude estimate did change with the number of channels. However, this change is small (on average less than 1 SNR). With the above caution in mind, using both 5 and 20 seconds of data to estimate the ensemble covariance matrix improved the approximate SNR from 10.9 ± 4.2 to 22.2 ± 7.9 (an approximate 104% increase in the average SNR) for two channels, from 11.0 ± 3.6 to 26.8 ± 10.0 (an approximate 144% increase in the average SNR) for four channels, and from 10.7 ± 3.0 to 29.0 ± 11.9 (an approximate 171% increase in the average SNR) for six channels. Each increment in the number of EMG channels described above provided a statistically significant increase in SNR performance ($p < 0.002$ for all paired comparisons). Additional results from the two subjects which yielded more than six viable MSEM channels will be presented later.

The above data suggest that, at most, five seconds of data are needed to calibrate the spatial uncorrelation filters. For real-time applications, it was of interest to determine if yet shorter time durations would perform as well. Figure 6.9 shows the SNR performance results when four channel spatial uncorrelation filters were calibrated from sample lengths of 32, 64, 128, 256, 512, 1024, 2048, 4096, 6144 and 8192 samples. (Since the sampling rate was 2048 Hz., these sample lengths corresponded to time windows ranging in length from 15.625 ms to 4 s.) Compared to the five second calibration record length, none of the shorter calibration record lengths significantly altered the SNR performance
Figure 6.9: Multiple Channel Uncorrelating — Effect of Calibration Record Length

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 100 contraction trials, for several four channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from spatial uncorrelation filters calibrated from denoted samples of MSEM data from separate records from the same subject during an identical level of contraction. Non-adjacent MSEM channels were selected as available.
(0.09 < \(p\) < 0.85 for the various paired comparisons).

Effect of Contraction Level

In order to investigate if the SNR performance of the above tests was influenced by the contraction level, results were segregated by contraction level for one test. Amplitude estimates were formed from four channels with spatial uncorrelation filters calibrated with 5 seconds of MSEM data from a separate record from the same subject during an identical level of contraction. Results were separated into the four contraction levels and plotted in Figure 6.10. The figure shows that there was a marked difference in performance as a function of contraction level. Performance at 10% MVC was almost twice that at 75% MVC. The performance differences between each pair of contraction levels were statistically significant (\(p < 0.007\) for all paired comparisons). Again, it is likely that the tremor activity present at the large contraction levels contributed to this difference in performance.

Effect of Sampling Rate

The effect of sampling rate was next studied. All data were decimated by a factor of two, reducing the effective sampling frequency from 2048 Hz. to 1024 Hz. Amplitude estimates were formed using 5 and 20 seconds of data to estimate the ensemble covariance matrix for two, four and six channels. (In order to maintain a smoothing window of approximately 244ms, one half the number of discrete-time samples were used in the MARMS smoothing window.) The results are presented in Figure 6.11. Again, there was no statistically significant difference between using 5 or 20 seconds of data to estimate the ensemble covariance matrix.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 100 contraction trials, are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from four channel spatial uncorrelation filters calibrated with 5 seconds of MSEMGS data from a separate record from the same subject during an identical level of contraction. Dashed line error bars denote single channel results when no whitening filter was applied. Results are separated by the four contraction levels of 10%, 25%, 50% and 75% of maximum voluntary contraction (MVC).

Figure 6.10: Multiple Channel Whitening at Each Contraction Level
Figure 6.11: Multiple Channel Whitening — Effect of Sampling Rate

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 80 or more contraction trials, for several multiple channel amplitude estimators are presented. All data were first decimated by a factor of two, reducing the effective sampling rate from 2048 Hz. to 1024 Hz. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from spatial uncorrelation filters calibrated from 20 seconds of MSEM data from separate records from the same subject during an identical level of contraction. Dashed line error bars denote results from spatial uncorrelation filters calibrated with 5 seconds of MSEM data from a separate record from the same subject during an identical level of contraction. All subjects produced at least four viable MSEM channels. Four of five subjects produced six viable MSEM channels. Non-adjacent MSEM channels were selected as available.
(p > 0.11 for 2, 4 and 6 channel comparisons). Also, as the number of channels increased, so did the SNR performance. Each increment in the number of EMG channels provided a statistically significant increase in the SNR performance (p < 0.002 for all paired comparisons). The overall results, however, were well below those when the data were sampled at a frequency of 2048 Hz. With the slower sampling frequency, the SNR improved from approximately 10.9 ± 4.2 to 14.2 ± 3.2 (an approximate 30% increase in the average SNR) for two channels, from 11.0 ± 3.6 to 17.9 ± 4.6 (an approximate 63% increase in the average SNR) for four channels, and from 10.7 ± 3.0 to 20.0 ± 6.2 (an approximate 87% increase in the average SNR) for six channels. These performance decrements (compared to sampling at 2048 Hz.) were similar to those of the single channel results.

For comparison, one optimized channel sampled at 2048 Hz. performed about as well as four optimized channels sampled at 1024 Hz. As before, these results suggested that sampling rates above 2048 Hz. might have further improved the SNR performance.

One Spatial Uncorrelation Filter Per Subject

The next set of comparisons studied the feasibility of calibrating a single spatial uncorrelation filter from one record from a subject, and applying that spatial uncorrelation filter to all records for that subject. This investigation was performed four times with a two channel processor, once each by forming a spatial uncorrelation filter from a record corresponding to 10, 25, 50 and 75% MVC. The entire process was then repeated with a four channel processor, a six channel processor (four out of five subjects), and an eight channel processor (subject DA only). In addition, two, four and eight channel processors were also formed
in which no spatial uncorrelation was performed. Rather, the data from each channel were normalized (based on calibration from the data which determine the temporal whitening filters) and then simply combined. Figure 6.12 shows all of these results, as well as the earlier 5 and 20 second calibration results, for the two channel processors. Figures 6.13, 6.14, and 6.15 show the results for the four, six and eight channel processors, respectively.

In general, all seven multiple channel combination techniques provided considerable performance improvement for all multiple channel filters. Spatial uncorrelation filters calibrated from data at an identical contraction level (either 5 or 20 seconds of data) performed marginally better than the other techniques, however, the statistical significance of this conclusion was weak. With four, six or eight channels, calibration by simply normalizing the temporally whitened channels was statistically poorer, on average, than the other techniques ($p < 0.002$ for all paired comparisons). Both of the above differences, however, were small in strength. Thus, for most applications, the equal variance channel combiner, which uses no spatial uncorrelation filter, is justified.

**Seven and Eight Channel Estimators**

The multiple channel analysis was applied next to greater than six channels. Thus, seven channels of data from subject FA were combined and all eight channels from subject DA were combined. A single spatial filter per subject was calibrated from a single record corresponding to 50% MVC. Amplitude estimation was also performed for two, four and six MSEM channels. The results are shown in Figure 6.16. With seven channels in subject FA, the average SNR improved from approximately $8.8 \pm 1.7$ to $27.8 \pm 8.7$ (an approximate 216% increase
Figure 6.12: Multiple Channel Whitening with Two Channels

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 100 contraction trials, for several two channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Non-adjacent MSEM.G channels were selected as available.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 100 contraction trials, for several four channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Non-adjacent MSEM channels were selected as available.
Figure 6.14: Multiple Channel Whitening with Six Channels

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 80 contraction trials, for several six channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Non-adjacent MSEM3G channels were selected as available. Results are from four of five subjects.
Figure 6.15: Multiple Channel Whitening with Eight Channels

The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 20 contraction trials, for several eight channel amplitude estimators are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Non-adjacent MSEMNG channels were selected as available. Results are from subject DA only.
The mean and standard deviation signal to noise ratio (SNR) (244ms smoothing window), averaged across 20 contraction trials, for multiple channel amplitude estimators for subjects DA and FA are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid line error bars denote results from subject DA. Dashed line error bars denote results from subject FA. A single spatial filter per subject was calibrated from a single record corresponding to 50% of maximum voluntary contraction. For Subject DA, 1, 2, 4, 6 and 8 channel results are shown. For Subject FA, 1, 2, 4, 6 and 7 channel results are shown.
in the average SNR). With eight channels in subject DA, the SNR improved from approximately 11.8±2.5 to 35.0±13.4 (an approximate 197% increase in the average SNR). From these and previous results it is likely that increasing the number of electrode-amplifiers beyond eight may continue to provide an improvement in SNR performance.

**Effect of Channel Correlation**

All of the previously discussed multiple channel analysis paid little attention to the effect of inherent channel correlation on the resultant SNR performance. As discussed earlier, when possible the electrode-amplifiers for a particular multiple channel processor where chosen from non-adjacent locations on the muscle group. The next examination contrasted non-adjacent channel selection with adjacent channel selection. Presumably, adjacent channels were inherently more correlated than non-adjacent channels. Two sets of two channel processors were evaluated. The first set was comprised of non-adjacent channels, the second set was comprised of adjacent channels (electrode-amplifiers 6 and 7 for all subjects). The correlation coefficient matrix was estimated for each adjacent and non-adjacent processor. The average ± standard deviation (averaged across all 20 trials in all five subjects) of each element in these matrices was computed and is shown in Figure 6.17. As assumed, the adjacent channels were more linearly correlated than the non-adjacent channels. The SNR performance of the adjacent and non-adjacent channels was compared for three different methods of determining the uncorrelation filter. The three filters and the resultant SNR performance are also described in Figure 6.17. The entire investigation was then repeated with four adjacent/ non-adjacent electrode-amplifiers. The average ±
\begin{align*}
|\hat{R}_{\text{Non-Adjacent}}| &= \begin{bmatrix}
1.00 & 0.15 \pm 0.13 \\
0.15 \pm 0.13 & 1.00
\end{bmatrix} \\
|\hat{R}_{\text{Adjacent}}| &= \begin{bmatrix}
1.00 & 0.61 \pm 0.19 \\
0.61 \pm 0.19 & 1.00
\end{bmatrix}
\end{align*}

<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>SNR ±σ for 2 Non-Adjacent Channels</th>
<th>SNR ±σ for 2 Adjacent Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 5 seconds of data from same contraction level, same subject.</td>
<td>22.2 ± 7.9</td>
<td>22.7 ± 8.4</td>
</tr>
<tr>
<td>2) One uncorrelation filter from 50% MVC trial.</td>
<td>21.7 ± 6.9</td>
<td>21.2 ± 7.2</td>
</tr>
<tr>
<td>3) Equalized variance.</td>
<td>21.7 ± 7.0</td>
<td>20.4 ± 6.5</td>
</tr>
<tr>
<td>4) No spatial uncorrelation or temporal whitening.</td>
<td>10.9 ± 4.2</td>
<td>10.3 ± 3.1</td>
</tr>
</tbody>
</table>

Figure 6.17: Two Channel Processor with Adjacent and Non-Adjacent Channels

The average magnitude ± standard deviation of each element of the correlation coefficient matrix for two adjacent and non-adjacent channels after temporal whitening, averaged across all trials in all subjects, is shown at the top. The bottom gives the SNR performance of the adjacent and non-adjacent processors for three different combination techniques, as well as the results when no optimization was utilized.
standard deviation correlation coefficient matrix and the SNR performance are shown in Figure 6.18. The results show little difference in performance between adjacent/ non-adjacent channel selection for both the two and four channel investigations. Statistical tests suggest that the poorer performance of the simple equalized variance method may be significant ($p < 0.011$ for both paired comparisons). However, even if these differences are significant, their strengths are weak. Hence, it appears that channel correlation coefficients as high as 0.5-0.6 perform about as well as more weakly correlated (correlation coefficients of 0.15-0.4) channels. Thus, the tight spacing of the set of electrode-amplifiers used in these experiments was an effective and spatially efficient manner to record multiple MSEM channels.

Local Perturbations in EMG Amplitude Estimates

The data of Figure 6.6 suggest that local perturbations in the EMG amplitude estimates may track local perturbations in the measured torque. To visually examine this observation, the two graphs of Figure 6.6 were overlaid on a single plot, shown in Figure 6.19. Clearly, local perturbations in this EMG amplitude estimate appear correlated to the measured torque. While such a claim is promising, aligning the operating points of the two signals is not a trivial matter. The third experiment addresses this problem.

However, if the two signals are assumed to have equal mean values, the correlation assumption can be tested. Two tests were performed on the data. The first test used a single channel estimator. One temporal whitening filter per channel was constructed from one record at 50% MVC. The second test used a four channel estimator. Temporal whitening and spatial uncorrelation filters
\[
\begin{align*}
|\hat{R}_{\text{Non-Adjacent}}| &= \\
&= \begin{bmatrix}
1.00 & 0.25 \pm 0.15 & 0.17 \pm 0.10 & 0.15 \pm 0.10 \\
0.25 \pm 0.15 & 1.00 & 0.27 \pm 0.22 & 0.16 \pm 0.11 \\
0.17 \pm 0.10 & 0.27 \pm 0.22 & 1.00 & 0.41 \pm 0.16 \\
0.15 \pm 0.10 & 0.16 \pm 0.11 & 0.41 \pm 0.16 & 1.00
\end{bmatrix} \\
|\hat{R}_{\text{Adjacent}}| &= \\
&= \begin{bmatrix}
1.00 & 0.48 \pm 0.21 & 0.40 \pm 0.12 & 0.28 \pm 0.16 \\
0.48 \pm 0.21 & 1.00 & 0.51 \pm 0.21 & 0.35 \pm 0.20 \\
0.40 \pm 0.12 & 0.51 \pm 0.21 & 1.00 & 0.57 \pm 0.22 \\
0.28 \pm 0.16 & 0.35 \pm 0.20 & 0.57 \pm 0.22 & 1.00
\end{bmatrix}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>SNR ±σ for 4 Non-Adjacent Channels</th>
<th>SNR ±σ for 4 Adjacent Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 5 seconds of data from same contraction level, same subject.</td>
<td>26.8 ± 10.0</td>
<td>26.6 ± 11.9</td>
</tr>
<tr>
<td>2) One uncorrelation filter from 50% MVC trial.</td>
<td>26.1 ± 9.3</td>
<td>25.5 ± 11.3</td>
</tr>
<tr>
<td>3) Equalized variance.</td>
<td>25.1 ± 9.0</td>
<td>23.1 ± 9.3</td>
</tr>
<tr>
<td>4) No spatial uncorrelation or temporal whitening.</td>
<td>11.0 ± 3.6</td>
<td>10.4 ± 3.6</td>
</tr>
</tbody>
</table>

Figure 6.18: Four Channel Processor with Adjacent and Non-Adjacent Channels

The average magnitude ± standard deviation of each element of the correlation coefficient matrix for four adjacent and non-adjacent channels after temporal whitening, averaged across all trials in all subjects, is shown at the top. The bottom gives the SNR performance of the adjacent and non-adjacent processors for three different combination techniques, as well as the results when no optimization was utilized.
Figure 6.19: Overlay of Multiple Channel Amplitude Estimate

The torque and EMG amplitude data of Figure 6.6 are shown on one plot. Solid line is the measured torque, dashed line is the EMG amplitude estimate based on four temporally whitened, spatially uncorrelated channels.

EMG SNR = 32.4
Torque SNR = 47.1
were constructed from one record at 50% MVC. For each test, EMG amplitude estimation was performed and SNR performance determined. Then, a "true" SNR, termed $SNR_T$, was computed by considering the measured torque for each record as the true signal, i.e.

$$SNR_T = \sqrt{\frac{\sum_i T_{\text{ext},i}^2}{\sum_i (T_{\text{ext},i} - \delta_i)^2}}$$

The $SNR_T$ performance for the single channel test was $16.6 \pm 5.4$ and for the four channel test was $23.9 \pm 7.9$. These results were 6% and 9%, respectively, lower than the results with the standard SNR definition. Obviously, the problem of relating local perturbations in the EMG amplitude estimate to local perturbations in joint torque is more complicated than that which was evaluated by these tests.

Robustness of the Optimal Multiple Channel Estimator

Recall that with seven EMG channels, the SNR performance for subject FA was $27.8 \pm 8.7$. When the eighth channel was added, the average SNR of the 20 trials drastically reduced to $16.0 \pm 8.0$. On close examination, it was found that four to six raw MSEM data from the added channel (channel 2) had large amplitude, high frequency perturbations superimposed on the normal MSEM signal. As discussed in the previous chapter, when temporally whitened, the uncharacteristic perturbations were amplified by the whitening filters. Since the spatial uncorrelation filter (calibrated from temporally whitened data which did not have the high frequency perturbations) normalized the contribution of each channel towards the amplitude estimate, the large amplitude perturbations dominated the entire multiple channel processor. Hence, multiple channel SNR performance was reduced tremendously (to about 4 to 8) in these trials. In turn,
these poor trials drastically reduced the average SNR (averaged over only 20 trials) for this subject. Thus, a small number of single channel recordings with large amplitude perturbations had a large impact on an entire experiment. Clearly, a more robust amplitude estimator which utilizes several electrode-amplifiers must be prepared to correct for the problem of anomalous inputs from one or more electrode-amplifiers. Within this thesis study, the anomalous MSEM G channel was not included in any multiple channel processor except the one mentioned above.

6.3.3 The Mean Absolute Value Processor

The MAV processor was studied two ways. First, one spatial uncorrelation filter per subject was calibrated from one record at 50% MVC, and the MARMS detector compared to the MAMAV detector. The MAMAV detector significantly improved the SNR performance from 25.5 ± 11.3 to 26.4 ± 9.8 (p < 0.005). Second, a multiple channel processor was formed via the equal variance technique, followed by detection. The MAMAV detector provided a significant (p < 0.009) improvement in SNR performance from 23.1 ± 9.3 to 24.1 ± 8.0. Overall, MAMAV detection provided an approximate 3–5% performance improvement. Since the MAMAV detector performed better than the MARMS detector, a final investigation determined its performance with seven and eight channels (subjects FA and DA, respectively). First, a single spatial uncorrelation filter per subject was calibrated from a record at 50% MVC. Second, the equal variance technique was utilized. The results of this investigation are shown in Figure 6.20. As before, results are best at the 10% MVC level, with the performance of the eight channel processor equal to 50.8 ± 4.2, and the performance of the seven channel processor
<table>
<thead>
<tr>
<th>Method of Optimizing</th>
<th>SNR: 10% MVC</th>
<th>SNR: 25% MVC</th>
<th>SNR: 50% MVC</th>
<th>SNR: 75% MVC</th>
<th>SNR: Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject FA, 7 Channels: One 50% MVC trial.</td>
<td>39.6 ± 5.0</td>
<td>33.3 ± 5.6</td>
<td>23.6 ± 3.7</td>
<td>19.9 ± 2.5</td>
<td>29.1 ± 8.9</td>
</tr>
<tr>
<td>Equalized variance.</td>
<td>38.0 ± 3.2</td>
<td>25.2 ± 3.2</td>
<td>18.6 ± 3.0</td>
<td>17.8 ± 1.0</td>
<td>24.9 ± 8.6</td>
</tr>
<tr>
<td>Subject DA, 8 Channels: One 50% MVC trial.</td>
<td>50.8 ± 4.2</td>
<td>46.5 ± 9.9</td>
<td>30.4 ± 7.0</td>
<td>26.0 ± 3.1</td>
<td>38.4 ± 12.4</td>
</tr>
<tr>
<td>Equalized variance.</td>
<td>47.7 ± 6.2</td>
<td>41.1 ± 5.9</td>
<td>25.3 ± 4.0</td>
<td>24.7 ± 3.2</td>
<td>34.7 ± 11.2</td>
</tr>
</tbody>
</table>

Figure 6.20: MAV Processor Performance

SNR performance for a seven and an eight channel EMG amplitude estimator.
equal to 39.6 ± 5.0. These results, combined with those of the previous chapter, demonstrate that MAMAV detection performs better than MARMS detection. Thus, MAMAV detection is preferred via both the experimental results and its ease of implementation. Also, as discussed in Chapter 5, these results suggest that EMG data may be more appropriately described by the Laplacian PDF than by the Gaussian PDF.

6.3.4 Predicted Versus Achieved SNR Performance

If perfect temporal whitening and spatial uncorrelation could be achieved and the Gaussian EMG model accurately described the surface EMG, then SNR performance would depend only on the length of the smoothing window and the number of channels. With a window length of \( N = 500 \), ideal SNR performance for 1–8 channels would approximately be 31.6, 44.7, 54.8, 63.2, 70.7, 77.5, 83.7 and 89.4, respectively. However, neither perfect temporal whitening nor perfect spatial uncorrelation can be achieved.

For multiple channels, a single study investigated the relationship between predicted and achieved performance. A four channel amplitude estimator was calibrated for each record from a separate record at the same contraction level. Predicted performance was computed by evaluating the total number of degrees of freedom in the data. The number of degrees of freedom in the data was taken as the sum of the number of degrees of freedom in each channel, calculated as described for the single channel results. Results of predicted versus achieved performance are shown in the scatter plot of Figure 6.21.

Note that computation of the predicted performance did not account for the spatial correlation in the data. The theoretic discussion of Chapter 4 (and
Figure 6.21: Multiple Channel Predicted Vs. Achieved SNR Performance

A four channel channel amplitude estimator was calibrated for each record from a separate record at the same contraction level. Predicted and achieved signal to noise ratios (SNR's) are shown in the scatter plot. The scatter plot has 100 entries.
Appendix C) did not provide a mechanism for considering the spatial correlation. Thus, all of the predicted SNR's should be larger than the achieved SNR's. The figure affirms this relationship. As argued in the previous chapter, there seems to be no advantage to using this (or even a more robust) predicted SNR over the achieved SNR. In practice, both measures must be computed from the data, but using the achieved SNR performance provides a definitive measure of performance.

6.4 Summary and Conclusion —
Multiple Channel EMG Amplitude Estimation

Figure 6.22 summarizes the results of the various multiple channel amplitude estimators. The results show that as the number of EMG channels increased, so did the SNR, for all of the spatial uncorrelation techniques. Spatial uncorrelation filters could be derived from 15.625ms or more of data. As in the single channel temporal whitening case, performance was severely degraded when the sampling frequency was reduced from 2048 Hz. to 1024 Hz. All of the spatial uncorrelation techniques provided an equitable performance improvement in the SNR, except for the equal variance filter which performed slightly poorer for four or more channels. Detailed study of channel correlations showed little difference between processors based on adjacent or non-adjacent channels. For all spatial uncorrelation filters, SNR performance was best at lower levels of contraction, the 10% MVC level performing almost twice as well as the 75% MVC level. The multiple channel estimates were heavily influenced by the robustness of single channel temporal whitening. The MAMAV processor performed 3–5% better
than the MARMS processor.
<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 20 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>10.7 ± 3.3</td>
<td>17.4 ± 6.1</td>
<td>63</td>
</tr>
<tr>
<td>2 channels,</td>
<td>10.9 ± 4.2</td>
<td>22.2 ± 7.9</td>
<td>104</td>
</tr>
<tr>
<td>4 channels,</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 9.9</td>
<td>144</td>
</tr>
<tr>
<td>6 channels*</td>
<td>10.7 ± 3.0</td>
<td>29.3 ± 11.8</td>
<td>174</td>
</tr>
<tr>
<td>2) 5 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>10.7 ± 3.3</td>
<td>17.4 ± 6.1</td>
<td>63</td>
</tr>
<tr>
<td>2 channels,</td>
<td>10.9 ± 4.2</td>
<td>22.2 ± 7.9</td>
<td>104</td>
</tr>
<tr>
<td>4 channels,</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 10.0</td>
<td>144</td>
</tr>
<tr>
<td>6 channels*</td>
<td>10.7 ± 3.0</td>
<td>29.0 ± 11.9</td>
<td>171</td>
</tr>
</tbody>
</table>

Figure 6.22: Tabulated Results of Multiple Channel Amplitude Estimation

Multiple channel amplitude estimation results shown graphically in previous figures are provided in one chart. This figure is continued on ensuing pages.

*Denotes data from four of five subjects.
<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>3) Four MSEM channels:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portion of one record</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>from same contraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>level, same</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>subject, with;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32 samples (15.625ms),</td>
<td>11.0 ± 3.6</td>
<td>25.8 ± 10.0</td>
<td>235</td>
</tr>
<tr>
<td>64 samples (31.25ms),</td>
<td>11.0 ± 3.6</td>
<td>26.3 ± 10.1</td>
<td>239</td>
</tr>
<tr>
<td>128 samples (62.5ms),</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 10.3</td>
<td>244</td>
</tr>
<tr>
<td>256 samples (125ms),</td>
<td>11.0 ± 3.6</td>
<td>26.9 ± 10.4</td>
<td>245</td>
</tr>
<tr>
<td>512 samples (250ms),</td>
<td>11.0 ± 3.6</td>
<td>26.9 ± 10.3</td>
<td>245</td>
</tr>
<tr>
<td>1024 samples (500ms),</td>
<td>11.0 ± 3.6</td>
<td>26.9 ± 10.1</td>
<td>245</td>
</tr>
<tr>
<td>2048 samples (1s),</td>
<td>11.0 ± 3.6</td>
<td>26.9 ± 10.1</td>
<td>245</td>
</tr>
<tr>
<td>4096 samples (2s),</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 10.0</td>
<td>244</td>
</tr>
<tr>
<td>6144 samples (3s),</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 10.0</td>
<td>244</td>
</tr>
<tr>
<td>8192 samples (4s).</td>
<td>11.0 ± 3.6</td>
<td>26.8 ± 10.0</td>
<td>244</td>
</tr>
<tr>
<td>4) 5 seconds of data from</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>same contraction level,</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>same subject,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>four MSEM channels,</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>12.4 ± 4.7</td>
<td>35.8 ± 8.6</td>
<td>189</td>
</tr>
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<td>25% MVC,</td>
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<td>30.5 ± 8.8</td>
<td>168</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>10.2 ± 2.9</td>
<td>22.8 ± 6.2</td>
<td>124</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>9.8 ± 2.3</td>
<td>18.0 ± 4.9</td>
<td>84</td>
</tr>
</tbody>
</table>

Figure 6.22 (Continued)
<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>Unwhitened Single Channel EMG SNR $\pm\sigma$</th>
<th>Whitened Multiple Channel EMG SNR $\pm\sigma$</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>5) Decimate data by 2:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>$10.7 \pm 3.3$</td>
<td>$13.5 \pm 4.5$</td>
<td>$26$</td>
</tr>
<tr>
<td>2 channels,</td>
<td>$10.9 \pm 4.2$</td>
<td>$14.2 \pm 3.2$</td>
<td>$30$</td>
</tr>
<tr>
<td>4 channels,</td>
<td>$11.0 \pm 3.6$</td>
<td>$18.0 \pm 4.6$</td>
<td>$64$</td>
</tr>
<tr>
<td>6 channels*</td>
<td>$10.7 \pm 3.0$</td>
<td>$20.2 \pm 6.2$</td>
<td>$89$</td>
</tr>
<tr>
<td>6) Decimate data by 2:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>$10.7 \pm 3.3$</td>
<td>$13.5 \pm 4.6$</td>
<td>$26$</td>
</tr>
<tr>
<td>2 channels,</td>
<td>$10.9 \pm 4.2$</td>
<td>$14.2 \pm 3.2$</td>
<td>$30$</td>
</tr>
<tr>
<td>4 channels,</td>
<td>$11.0 \pm 3.6$</td>
<td>$17.9 \pm 4.6$</td>
<td>$63$</td>
</tr>
<tr>
<td>6 channels*</td>
<td>$10.7 \pm 3.0$</td>
<td>$20.0 \pm 6.2$</td>
<td>$87$</td>
</tr>
<tr>
<td>7) Two MSEMG channels:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from trial at;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10%$ MVC,</td>
<td>$10.9 \pm 4.2$</td>
<td>$20.7 \pm 7.6$</td>
<td>$90$</td>
</tr>
<tr>
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<td>$96$</td>
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<tr>
<td>$50%$ MVC,</td>
<td>$10.9 \pm 4.2$</td>
<td>$21.7 \pm 6.9$</td>
<td>$99$</td>
</tr>
<tr>
<td>$75%$ MVC,</td>
<td>$10.9 \pm 4.2$</td>
<td>$21.4 \pm 6.7$</td>
<td>$96$</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>$10.9 \pm 4.2$</td>
<td>$21.7 \pm 7.0$</td>
<td>$99$</td>
</tr>
</tbody>
</table>

Figure 6.22 (Continued)

*Denotes data from four of five subjects.
<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) Four MSEM channels:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>filter from trial at;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.0 ± 3.6</td>
<td>24.3 ± 10.4</td>
<td>121</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.0 ± 3.6</td>
<td>26.1 ± 10.6</td>
<td>137</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.0 ± 3.6</td>
<td>26.1 ± 9.3</td>
<td>137</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.0 ± 3.6</td>
<td>25.6 ± 8.4</td>
<td>133</td>
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<tr>
<td>Equalized variances.</td>
<td>11.0 ± 3.6</td>
<td>25.1 ± 9.0</td>
<td>128</td>
</tr>
<tr>
<td>9) Six MSEM channels:*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>filter from trial at;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>10.7 ± 3.0</td>
<td>26.6 ± 11.8</td>
<td>151</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>10.7 ± 3.0</td>
<td>27.9 ± 12.9</td>
<td>161</td>
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<tr>
<td>50% MVC,</td>
<td>10.7 ± 3.0</td>
<td>28.2 ± 10.3</td>
<td>164</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>10.7 ± 3.0</td>
<td>27.3 ± 9.2</td>
<td>155</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>10.7 ± 3.0</td>
<td>25.5 ± 8.8</td>
<td>138</td>
</tr>
<tr>
<td>10) Eight MSEM channels:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Subject DA:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>filter from trial at;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.8 ± 2.5</td>
<td>35.9 ± 17.4</td>
<td>204</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.8 ± 2.5</td>
<td>37.5 ± 19.1</td>
<td>218</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.8 ± 2.5</td>
<td>35.0 ± 13.4</td>
<td>197</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.8 ± 2.5</td>
<td>33.5 ± 10.6</td>
<td>184</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>11.8 ± 2.5</td>
<td>31.8 ± 10.9</td>
<td>169</td>
</tr>
</tbody>
</table>

Figure 6.22 (Continued)

*Denotes data from four of five subjects.
<table>
<thead>
<tr>
<th>Method of Determining Uncorrelation Filter</th>
<th>Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
<th>Percent Increase in Average SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>11) For Subject DA:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from 50% MVC trial, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>11.8 ± 2.5</td>
<td>20.5 ± 5.6</td>
<td>74</td>
</tr>
<tr>
<td>2 channels,</td>
<td>11.9 ± 2.8</td>
<td>25.4 ± 7.1</td>
<td>113</td>
</tr>
<tr>
<td>4 channels,</td>
<td>11.8 ± 2.7</td>
<td>30.6 ± 9.2</td>
<td>159</td>
</tr>
<tr>
<td>6 channels,</td>
<td>11.8 ± 2.4</td>
<td>32.1 ± 10.6</td>
<td>172</td>
</tr>
<tr>
<td>8 channels.</td>
<td>11.8 ± 2.5</td>
<td>35.0 ± 13.4</td>
<td>197</td>
</tr>
<tr>
<td>12) For Subject FA:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from 50% MVC trial, for;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>8.8 ± 1.7</td>
<td>15.3 ± 4.6</td>
<td>74</td>
</tr>
<tr>
<td>2 channels,</td>
<td>8.5 ± 1.2</td>
<td>18.8 ± 3.8</td>
<td>121</td>
</tr>
<tr>
<td>4 channels,</td>
<td>8.8 ± 1.6</td>
<td>22.8 ± 5.9</td>
<td>159</td>
</tr>
<tr>
<td>6 channels,</td>
<td>8.9 ± 1.7</td>
<td>26.0 ± 7.9</td>
<td>192</td>
</tr>
<tr>
<td>7 channels.</td>
<td>8.8 ± 1.7</td>
<td>27.8 ± 8.7</td>
<td>216</td>
</tr>
</tbody>
</table>

Figure 6.22 (Final Sheet)
Chapter 7

Experiment 2 —
Influence of Joint Angle on the
Construction and Performance
of Optimized Single and
Multiple Channel Constant
Torque EMG Amplitude
Estimators
7.1 Introduction

The previous two chapters described a first set of experiments which investigated optimal single and multiple channel constant torque EMG amplitude estimators. This chapter describes a second experimental set which investigated the influence of joint angle on the construction and performance of these estimators. The results of the previous chapters were incorporated into this study.

Multiple channels of the MSEM data were recorded from one muscle group (either elbow flexors or extensors) during isometric, isotonic contraction of that muscle group. Data were recorded at five different angles of the elbow joint. Optimized single and multiple channel EMG amplitude estimators were evaluated. The emphasis of this study was to determine if temporal whitening filters and spatial uncorrelation filters calibrated at one joint angle could be applied effectively to data acquired at other joint angles. In all cases, performance was expressed as the SNR of the amplitude estimate, where the "true" amplitude was taken as the mean value of the amplitude estimate averaged over time (the rise time of the estimation filter was ignored). As before, all estimators incorporated a fixed smoothing window corresponding to approximately 245ms. The sections which follow describe the experimental trial. The experimental apparatus has been detailed in Chapter 5.

7.2 Experimental Methods

Four subjects, three male and one female, ranging in age from 23 to 29 years, participated in four experiments. Subjects had no known neuromuscular deficits of the right shoulder, arm or hand. Two experiments studied flexion of the
elbow, two experiments studied extension of the elbow. All trials investigated isometric, isotonic flexion/extension of the right elbow at five different angles between the upper arm and the forearm.

A subject was seated and secured into the instrumented chair, and eight electrode-amplifiers were mounted on the subject (oriented as described in Chapter 5). During an experimental trial, the subject was instructed to lift his/her right elbow above the pivot plate and to contact the instrumented beam only via the wrist cuff. The output voltage of the strain gauge circuit was presented on an oscilloscope to the subject. A target torque level was presented to the subject as the second display of a two channel oscilloscope. The subject was instructed to begin at rest, then, over a time period comfortable to the subject (typically 0.5-1s), gradually increase flexion/extension torque until the target torque level was achieved. The subject attempted to maintain a consistent posture during and throughout all trials, so as to present a repeatable contraction task. The subject tried to relax all muscles not directly involved in flexion/extension about the elbow. By observing the oscilloscope display, the subject maintained the target torque level until a five second segment of data was recorded at a sampling rate of 2048 Hz. The location on the oscilloscope screen of the target torque level was fixed for all trials. The null location of the strain gauge circuit output voltage could be adjusted. For all trials, the oscilloscope gain was selected as the maximum gain for which the null voltage and the target torque level could be observed simultaneously. This gain selection maximized the available sensitivity of the oscilloscope display for each contraction level. Also, for errors in maintaining the desired contraction torque, the visual angle subtended by the subject's deviation from the target torque (i.e. the visual error) was approximately the
same for all contraction levels.

A sequence of five sets of contractions was conducted, one set at each of five joint angles. Each set consisted of one maximal contraction followed by three contractions at 50% MVC. The maximal contraction trials provided a rough estimate of MVC at each angle, sufficient for a coarse gradation of contraction levels. A rest period of three minutes after maximal contractions and two minutes after 50% MVC trials was provided. Chapter 5 discussed the method by which rest periods were assigned. Between contraction sets, the subject was released from the wrist cuff to prevent impaired blood flow to/from the hand. The five different joint angles (angle between the upper arm and forearm) were 45°, 75°, 90°, 105° and 135°. The order of presentation of these angles was randomized by flipping a coin.

7.3 Methods of Analysis

Both single channel and multiple channel amplitude estimators were studied for data at the 50% MVC level. For single channel amplitude estimators, five coefficient temporal whitening filters were constructed from one five second recording at the 50% MVC level. Separate temporal whitening filters were constructed for each electrode-amplifier. For multiple channel amplitude estimators, the data from four electrode-amplifiers were combined. Initially, the data from each of the four channels were whitened as in the single channel analysis. The channels were then combined either by the simple equal variance technique or by forming an optimal spatial uncorrelation filter from one five second recording at the 50% MVC level. For both the single and multiple channel cases, the studies investigated if temporal whitening filters and spatial uncorrelation filters calibrated at
one joint angle could be applied effectively to data acquired at other joint angles. Performance differences between pairs of amplitude estimators were evaluated for statistical significance by paired t-tests and unequal variance t-tests, as appropriate (Press et al., 1988, Section 13.4).

7.4 Results

7.4.1 Volume of Usable Data Collected

For each subject, surface EMG data were recorded from eight electrode-amplifiers during 15 50% MVC trials, yielding 120 MSEMG data records per subject. For the four subjects, a total of 480 MSEMG data records were recorded. Each data record was plotted and inspected. As before, data from any electrode-amplifier which saturated during any portion of any experimental record were discarded from further analysis. Figure 7.1 lists the four subjects with the number of viable channels for each subject. In all, there were 27 viable electrode-amplifiers for a total of 406 viable MSEMG data records. Figure 7.1 also identifies the muscle group (elbow flexors/ extensors) studied for each subject.

7.4.2 Single Channel Amplitude Estimation

Initially, temporal whitening filters were formed for each recording from a separate record from the same subject from the same MSEMG channel during a contraction at the same joint angle. Since there were three records at each joint angle, two such separate records existed. If the three records at a particular joint angle were sequentially denoted A, B and C, then C calibrated the whitening filter for A, A calibrated the whitening filter for B, and B calibrated the whitening filter for C. The mean and standard deviation SNR performances from this
<table>
<thead>
<tr>
<th>Subject</th>
<th>Extension/Flexion Experiment</th>
<th>Number and Identity of Viable Electrodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>Flexion</td>
<td>7 (0,1,2,3,5,6,7)</td>
</tr>
<tr>
<td>HA</td>
<td>Extension</td>
<td>6 (1,2,3,4,6,7)</td>
</tr>
<tr>
<td>IA</td>
<td>Flexion</td>
<td>6 (0,1,2,5,6,7)</td>
</tr>
<tr>
<td>JA</td>
<td>Extension</td>
<td>8 (0,1,2,3,4,5,6,7)</td>
</tr>
</tbody>
</table>

**Figure 7.1:** Chart of Subjects for Experiment 2

The four subjects were denoted GA, HA, IA and JA. The muscle group studied (elbow flexors or extensors) as well as the number and identity of viable electrodes for each subject is charted.
study are shown in the first row of Figure 7.2.

Next, a single temporal whitening filter per channel was formed from one five second recording at the 45° joint angle, and applied to all remaining records from that subject. This study was then repeated a joint angles of 75°, 90°, 105° and 135°. All of these results are also shown in Figure 7.2.

All six of the whitening algorithms improved the SNR performance from $9.6 \pm 2.8$ to approximately $16.0 \pm 6.0$. Whitening filters calibrated from respective joint angles did provide a statistically significant improvement in SNR performance compared to the corresponding results for a single whitening filter per channel per subject when calibrated from any of the joint angles ($p < 0.001$ for all paired comparisons). However, the strength of this significance was small (at most 0.8).

In order to investigate if the SNR performance of the above tests was influenced by the joint angle, results were segregated by joint angle for the first study above. Results for both the raw and temporally whitened data are presented in Figure 7.3. In general, there was either no significant difference in performance as a function of joint angle or the difference was small. The 90° joint angle was slightly superior in both cases. Statistical tests (unequal variance t-tests) found the 90° joint angle to be significantly superior when compared to 135° for temporally whitened data, and when compared to all except 135° for raw data ($p < 0.007$).

### 7.4.3 Multiple Channel Amplitude Estimation

All multiple channel amplitude estimators were formed from four adjacent channels. Initially, temporal whitening filters (one per channel) and optimal spatial uncorrelation filters were formed for each record from a separate record from the
<table>
<thead>
<tr>
<th>Calibrate From Record at Angle:</th>
<th>Subject GA SNR ± σ</th>
<th>Subject HA SNR ± σ</th>
<th>Subject IA SNR ± σ</th>
<th>Subject JA SNR ± σ</th>
<th>Combined SNR ± σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Respective</td>
<td>13.9 ± 5.7</td>
<td>14.2 ± 3.8</td>
<td>18.0 ± 6.3</td>
<td>18.1 ± 6.0</td>
<td>16.1 ± 5.9</td>
</tr>
<tr>
<td>2) 45°</td>
<td>12.9 ± 5.8</td>
<td>13.4 ± 3.7</td>
<td>18.5 ± 7.3</td>
<td>17.0 ± 5.5</td>
<td>15.5 ± 6.1</td>
</tr>
<tr>
<td>3) 75°</td>
<td>13.2 ± 5.7</td>
<td>13.9 ± 4.0</td>
<td>17.0 ± 7.0</td>
<td>17.6 ± 5.8</td>
<td>15.5 ± 6.0</td>
</tr>
<tr>
<td>4) 90°</td>
<td>13.4 ± 5.9</td>
<td>13.7 ± 3.8</td>
<td>16.0 ± 6.0</td>
<td>17.8 ± 6.1</td>
<td>15.3 ± 5.9</td>
</tr>
<tr>
<td>5) 105°</td>
<td>13.2 ± 5.3</td>
<td>13.8 ± 3.7</td>
<td>17.3 ± 6.2</td>
<td>18.4 ± 6.0</td>
<td>15.8 ± 5.9</td>
</tr>
<tr>
<td>6) 135°</td>
<td>13.7 ± 5.3</td>
<td>13.6 ± 3.5</td>
<td>19.4 ± 7.5</td>
<td>16.6 ± 6.0</td>
<td>15.8 ± 6.2</td>
</tr>
<tr>
<td>No Whitening</td>
<td>9.5 ± 2.7</td>
<td>9.7 ± 2.3</td>
<td>7.9 ± 2.2</td>
<td>10.9 ± 3.1</td>
<td>9.6 ± 2.8</td>
</tr>
</tbody>
</table>

**Figure 7.2:** Tabulated Results of Single Channel Amplitude Estimation at Different Angles
<table>
<thead>
<tr>
<th>Joint Angle</th>
<th>Raw Single Channel EMG SNR ±σ</th>
<th>Optimal Single Channel EMG SNR ±σ</th>
<th>Optimal Four Channel EMG SNR ±σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>9.1 ± 2.4</td>
<td>16.0 ± 5.6</td>
<td>23.2 ± 5.7</td>
</tr>
<tr>
<td>75°</td>
<td>9.1 ± 2.8</td>
<td>16.3 ± 5.1</td>
<td>21.5 ± 5.7</td>
</tr>
<tr>
<td>90°</td>
<td>10.6 ± 2.5</td>
<td>17.9 ± 6.0</td>
<td>21.5 ± 6.7</td>
</tr>
<tr>
<td>105°</td>
<td>9.4 ± 2.7</td>
<td>16.1 ± 5.9</td>
<td>21.1 ± 5.8</td>
</tr>
<tr>
<td>135°</td>
<td>9.8 ± 3.3</td>
<td>14.3 ± 6.3</td>
<td>19.4 ± 9.1</td>
</tr>
</tbody>
</table>

**Figure 7.3: SNR Performance Versus Joint Angle**

For the optimal amplitude estimators, temporal whitening filters and spatial uncorrelation filters were calibrated from a separate record from the same subject at the same joint angle.
same subject during a contraction at the same joint angle. Each record from a particular joint angle was used to calibrate the filters of one other record from that joint angle. The mean and standard deviation SNR performance of this study is shown in the first row of Figure 7.4. Next, for each subject, a single set of temporal whitening filters (one per channel) and an optimal spatial uncorrelation filter were formed from one five second recording at the 45° joint angle, and applied to all records from that subject. This study was then repeated at joint angles of 75°, 90°, 105° and 135°. Finally, all of the above investigations were repeated, with the optimal spatial uncorrelation filter replaced with the simple equal variance technique. All of these results are also shown in Figure 7.4.

All of the multiple channel algorithms improved the SNR performance from $9.7 \pm 2.8$ to approximately $21.0 \pm 6.5$. There was no statistical difference in performance, for either the optimal or equal variance technique, between filters calibrated at respective joint angles and a single set of filters calibrated at any of the joint angles ($p > 0.07$ for all paired comparisons). For these four channel amplitude estimators, there was no significant difference in SNR performance between the optimal and equal variance channel combiners ($p > 0.5$ for all paired comparisons).

A set of comparisons was made to investigate the robustness in performance of using a single set of filters for an entire subject. For each subject, a single set of temporal whitening filters (one per channel) and an optimal spatial uncorrelation filter were formed from one five second trial at the 90° joint angle, and applied to all records from that subject. Then, the study was repeated twice, using for calibration each of the two remaining trials at the 90° joint angle. This entire investigation was repeated, with the optimal spatial uncorrelation filter
<table>
<thead>
<tr>
<th>Method of Determining Optimal Filters:</th>
<th>Subject GA SNR ± σ</th>
<th>Subject HA SNR ± σ</th>
<th>Subject IA SNR ± σ</th>
<th>Subject JA SNR ± σ</th>
<th>Combined SNR ± σ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal at Angle:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1) Respective</td>
<td>19.6 ± 6.0</td>
<td>18.6 ± 5.3</td>
<td>19.6 ± 4.9</td>
<td>27.6 ± 7.0</td>
<td>21.3 ± 6.9</td>
</tr>
<tr>
<td>2) 45°</td>
<td>16.0 ± 6.4</td>
<td>18.3 ± 5.7</td>
<td>22.9 ± 7.3</td>
<td>24.0 ± 5.2</td>
<td>20.3 ± 7.0</td>
</tr>
<tr>
<td>3) 75°</td>
<td>16.9 ± 6.6</td>
<td>17.9 ± 4.8</td>
<td>19.1 ± 6.5</td>
<td>22.4 ± 5.9</td>
<td>19.1 ± 6.3</td>
</tr>
<tr>
<td>4) 90°</td>
<td>20.3 ± 8.7</td>
<td>17.2 ± 5.0</td>
<td>19.9 ± 5.7</td>
<td>23.6 ± 7.3</td>
<td>20.3 ± 7.1</td>
</tr>
<tr>
<td>5) 105°</td>
<td>16.2 ± 6.3</td>
<td>17.1 ± 4.6</td>
<td>21.9 ± 6.3</td>
<td>24.7 ± 6.8</td>
<td>20.0 ± 6.9</td>
</tr>
<tr>
<td>6) 135°</td>
<td>20.0 ± 6.3</td>
<td>17.2 ± 3.9</td>
<td>22.5 ± 5.9</td>
<td>23.1 ± 9.5</td>
<td>20.7 ± 7.1</td>
</tr>
<tr>
<td><strong>Equalize Variances at Angle:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) Respective</td>
<td>18.6 ± 5.3</td>
<td>18.4 ± 4.8</td>
<td>21.8 ± 6.1</td>
<td>25.5 ± 5.5</td>
<td>21.1 ± 6.1</td>
</tr>
<tr>
<td>8) 45°</td>
<td>14.7 ± 5.9</td>
<td>17.9 ± 5.2</td>
<td>22.3 ± 6.9</td>
<td>23.1 ± 4.9</td>
<td>19.5 ± 6.7</td>
</tr>
<tr>
<td>9) 75°</td>
<td>16.1 ± 6.0</td>
<td>18.1 ± 4.9</td>
<td>20.7 ± 6.5</td>
<td>22.1 ± 5.0</td>
<td>19.2 ± 6.1</td>
</tr>
<tr>
<td>10) 90°</td>
<td>19.2 ± 7.3</td>
<td>17.7 ± 4.8</td>
<td>21.0 ± 6.5</td>
<td>22.9 ± 6.1</td>
<td>20.2 ± 6.5</td>
</tr>
<tr>
<td>11) 105°</td>
<td>15.8 ± 5.9</td>
<td>17.9 ± 4.7</td>
<td>22.9 ± 5.1</td>
<td>24.3 ± 6.2</td>
<td>20.2 ± 6.5</td>
</tr>
<tr>
<td>12) 135°</td>
<td>19.6 ± 5.9</td>
<td>17.7 ± 4.0</td>
<td>22.9 ± 5.6</td>
<td>21.6 ± 5.9</td>
<td>20.5 ± 5.8</td>
</tr>
<tr>
<td><strong>No Whitening</strong></td>
<td>9.6 ± 2.7</td>
<td>9.8 ± 2.2</td>
<td>8.0 ± 2.1</td>
<td>11.3 ± 3.0</td>
<td>9.7 ± 2.8</td>
</tr>
</tbody>
</table>

**Figure 7.4:** Tabulated Results of Four Channel Amplitude Estimation at Different Angles
replaced with the simple equal variance technique. All of the results are shown in Figure 7.5. All of the algorithms provided a nearly identical increase in SNR performance from $9.7 \pm 2.8$ to approximately $20.2 \pm 6.5$. None of the differences in performance was statistically significant ($p > 0.85$ for all paired comparisons).

As in the single channel case, the influence of joint angle on estimator performance was investigated by segregating the results of the first multiple channel study as a function of joint angle. The results are presented in Figure 7.3. Note that each average incorporates 12 SNR’s. There was no statistically significant difference in performance between any two of the joint angles ($p > 0.29$, unequal variance t-tests).

Lastly, the correlation between the data channels was evaluated. All of the data were temporally whitened from a separate calibration record from the same subject during a contraction trial at the same joint angle. The correlation coefficient matrix was then computed. The average magnitude $\pm$ standard deviation of each element of the correlation coefficient matrix is shown in Figure 7.6.

### 7.5 Discussion and Conclusion

The results of this experimental trial, combined with those of the previous experimental trial, suggest that optimal multiple channel EMG amplitude estimation can be successfully accomplished over the 10–75% MVC range and over the joint angles $45^\circ$–$135^\circ$ from a single three second or greater calibration recording at 50% MVC and at a $90^\circ$ joint angle. The similarities across angle for the multiple channel amplitude estimator results further suggest that one estimator might be appropriate for all joint angles. The single channel results, however,
<table>
<thead>
<tr>
<th>Method of Determining Optimal Filters:</th>
<th>Subject GA SNR ± σ</th>
<th>Subject HA SNR ± σ</th>
<th>Subject IA SNR ± σ</th>
<th>Subject JA SNR ± σ</th>
<th>Combined SNR ± σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal at Angle:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1) 90° Trial 14</td>
<td>20.3 ± 8.7</td>
<td>17.2 ± 5.0</td>
<td>19.9 ± 5.7</td>
<td>23.6 ± 7.3</td>
<td>20.3 ± 7.1</td>
</tr>
<tr>
<td>2) 90° Trial 15</td>
<td>20.1 ± 8.7</td>
<td>17.9 ± 5.3</td>
<td>16.9 ± 4.4</td>
<td>25.2 ± 6.6</td>
<td>20.0 ± 7.2</td>
</tr>
<tr>
<td>3) 90° Trial 16</td>
<td>19.4 ± 8.2</td>
<td>17.5 ± 5.4</td>
<td>20.5 ± 5.8</td>
<td>23.3 ± 6.7</td>
<td>20.2 ± 6.9</td>
</tr>
<tr>
<td>Equalize Variances at Angle:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) 90° Trial 14</td>
<td>19.2 ± 7.3</td>
<td>17.7 ± 4.8</td>
<td>21.0 ± 6.5</td>
<td>22.9 ± 6.1</td>
<td>20.2 ± 6.5</td>
</tr>
<tr>
<td>5) 90° Trial 15</td>
<td>19.0 ± 7.2</td>
<td>18.1 ± 5.0</td>
<td>18.7 ± 5.4</td>
<td>24.3 ± 6.4</td>
<td>20.0 ± 6.5</td>
</tr>
<tr>
<td>6) 90° Trial 16</td>
<td>18.2 ± 6.9</td>
<td>17.9 ± 5.0</td>
<td>21.5 ± 6.4</td>
<td>22.7 ± 5.6</td>
<td>20.1 ± 6.3</td>
</tr>
<tr>
<td>No Whitening</td>
<td>9.6 ± 2.7</td>
<td>9.8 ± 2.2</td>
<td>8.0 ± 2.1</td>
<td>11.3 ± 3.0</td>
<td>9.7 ± 2.8</td>
</tr>
</tbody>
</table>

Figure 7.5: Tabulated Results of Four Channel Amplitude Estimation at One Angle
\[ |\hat{R}| = \begin{bmatrix}
1.00 & 0.65 \pm 0.09 & 0.32 \pm 0.17 & 0.20 \pm 0.19 \\
0.65 \pm 0.09 & 1.00 & 0.50 \pm 0.18 & 0.28 \pm 0.18 \\
0.32 \pm 0.17 & 0.50 \pm 0.18 & 1.00 & 0.52 \pm 0.22 \\
0.20 \pm 0.19 & 0.28 \pm 0.18 & 0.52 \pm 0.22 & 1.00
\end{bmatrix} \]

**Figure 7.6: Four Channel Processor Channel Correlations**

The average magnitude ± standard deviation of each element of the correlation coefficient matrix for the four channel amplitude estimators (after temporal whitening), averaged across all trials in all subjects, of Experiment 2. Temporal whitening filters were formed for each record from a separate record from the same subject during a contraction at the same joint angle.

Hint at small performance differences which might need to be considered if amplitude estimation is performed across a wider selection of joint angles. Over the joint angles 45°–135°, there is little or no improvement realized by altering either temporal whitening filters or spatial uncorrelation filters as a function of either contraction level or joint angle. Further, the equal variance combiner is much simpler and provides all, or almost all, of the performance improvement provided by the optimal combiner. The results indicate that changes (if any), as a function of joint angle, in the temporal and spatial correlation of the data need not be considered while performing amplitude estimation. Further, there are only small performance differences in amplitude estimation as a function of joint angle. (No differences were demonstrated for multiple channel estimators.) Hence, a relatively simple procedure exists for performing amplitude estimation over a wide range of contraction levels and joint angles: For any set of electrode-amplifiers, a fixed set of linear temporal whitening filters and a fixed
linear spatial uncorrelation filter define the complete optimal multiple channel amplitude estimator. These filters can both be calibrated from a single three second or greater calibration recording at 50% MVC and at a 90° joint angle.
Chapter 8

Simulation Studies of EMG Amplitude Estimation
8.1 Introduction

A pair of simulation studies — the first based on the functional models presented in Chapter 4, and the second based, in part, on the physiology — investigated EMG amplitude estimation. The functional simulations provided a controlled noise-free environment to study the EMG models and solutions presented in Chapter 4. The physiologic simulations provided data which are reminiscent of the manner by which actual surface EMG is generated. Both of these simulations served two purposes. First, they provided noise-free data which necessarily fit their respective model. Comparison of simulation results to the analytically predicted results provided insight into the expected performance, in practice, of the amplitude estimation algorithms. These expectations did vary from the analytically predicted results. Second, the simulations provided a controlled data set for testing the software algorithms used in EMG amplitude estimation.

In the following sections, the functional model, then the physiologic model, will be described. Initially, the design of each model is presented. Then, simulation trials and their results are given. Finally, a discussion of the results is provided.

8.2 Functional Simulations

8.2.1 Design — Functional Simulations

Multiple channels of EMG data, correlated both temporally and spatially, were simulated according to the multiple channel model of Figure 4.3. To create the EMG processes, initially $L$ independent, JWSS, CE, white, uniformly distributed processes were created in double-precision format with UNIX-style commands
“random” and “srandom” (on a Sun Microsystems, Inc. workstation with operating system SunOS Release 4.0.3). These processes were then transformed to jointly Gaussian processes (Press et al., 1988, Section 7.2) and denoted $w_{j,i}$ where $j$ is the channel index and $i$ is the time index.

Next, these channels were spatially correlated by a spatial correlation filter. Spatial correlation filters, one per simulation trial, were derived to match the statistics of experimental data. In order to derive a particular spatial correlation filter, $L$ channels from a subject record were temporally whitened (a single five coefficient temporal whitening filter per electrode per subject was calibrated from one five second record at the 50% MVC level) to form the $L$ channels $v_1$, $v_2$, $v_3$ ... $v_L$. The ensemble random vector

$$v_{*,i} = \begin{bmatrix} v_{1,i} \\ v_{2,i} \\ v_{3,i} \\ \vdots \\ v_{L,i} \end{bmatrix}_{L \times 1}$$

has the covariance matrix $K_{v_{*,i}v_{*,i}}$. Recall that to spatially uncorrelate these experimental data, the ensemble random vector was pre-multiplied by a matrix formed from the eigenvalues and eigenvectors of $K_{v_{*,i}v_{*,i}}$ (see Appendix C). If the pre-multiplying matrix is denoted $A$ (where from Appendix C, $A = \Lambda^{1/2} D$), then the spatially uncorrelated ensemble vector

$$w_{*,i} = \begin{bmatrix} w_{1,i} \\ w_{2,i} \\ w_{3,i} \\ \vdots \\ w_{L,i} \end{bmatrix}_{L \times 1}$$

was formed as

$$w_{*,i} = A \cdot v_{*,i}$$
Equating the covariance matrices of the terms on both sides of the above equation yields the identity

\[ I = A K_{\mathbf{u}_{*,i} \mathbf{u}_{*,i}} A^T \]

since

\[ K_{\mathbf{w}_{*,i} \mathbf{w}_{*,i}} = I \]

Now, to spatially correlate the simulated data, the inverse of the experimentally derived matrix \( A \) was applied to the ensemble simulated spatially uncorrelated channels, denoted \( \mathbf{w}'_{*,i} \), as

\[ \mathbf{u}'_{*,i} = A^{-1} \mathbf{w}'_{*,i} \]

The covariance matrix for \( \mathbf{u}'_{*,i} \) was

\[ K_{\mathbf{u}'_{*,i} \mathbf{u}'_{*,i}} = A^{-1} K_{\mathbf{w}'_{*,i} \mathbf{w}'_{*,i}} A^{-1T} \]
\[ = A^{-1} I A^{-1T} \]

Now, substituting the above identity for \( I \),

\[ K_{\mathbf{u}'_{*,i} \mathbf{u}'_{*,i}} = A^{-1} (A K_{\mathbf{u}_{*,i} \mathbf{u}_{*,i}} A^T) A^{-1T} \]
\[ = K_{\mathbf{u}_{*,i} \mathbf{u}_{*,i}} \]

Thus, the simulated data \( \mathbf{u}'_{*,i} \) were spatially correlated according to the corresponding experimental data.

The \( L \) spatially correlated simulated channels were then individually temporally correlated. Temporal correlation filters, one per simulation channel, were also derived to match the statistics of experimental data. In order to derive a particular temporal correlation filter, the data of one subject from one channel were extracted. The subject data were then overlapped and reduced into a 512
sample (250 ms, since the data were sampled at 2048 Hz.) buffer by adding samples 0–511 to samples 512–1023, respectively, and adding this result to samples 1024–1535, respectively, etc. In this manner, 20 data segments (total of five seconds) were overlapped in a buffer. The buffer represented an impulse response which, when applied to the simulated data $y_{j,i}$, as a 512 sample FIR filter, temporally correlated the simulation data according to the temporal correlation in the experimental data. The first 512 outputs of the FIR filter (the startup time of the filter) were discarded and the ensuing 10240 outputs (representing five seconds of data sampled at 2048 Hz.) were saved. The EMG amplitude was implicitly contained in the filters (both spatial and temporal) formed from the experimental data. The data were lastly scaled by the factor 0.05 and converted into a short integer (two byte) format. The scaling was necessary to fit the data into the short integer format. The short integer format allowed the simulated data to be evaluated by the same software as the experimental data.

The $L$ channels for a given simulation trial had their temporal correlation derived from $L$ distinct channels from a given subject record. The corresponding spatial correlation for that simulation trial was derived from these same experimental channels. Distinct simulation trials corresponded to distinct subject records. In this manner, a simulation experiment could emulate the spatial and temporal correlation in each trial of an actual experiment.

### 8.2.2 Methods — Functional Simulations

A complete simulation experiment was created corresponding to a complete actual experiment. Since subject DA yielded eight viable channels for all 20 trials, this subject's data were used to derive spatial and temporal correlation filters.
The simulated data, therefore, were comprised of 20 trials with eight channels per trial. Each channel of each trial simulated five seconds of EMG data sampled at 2048 Hz. Since the simulation data corresponded precisely to the format of the data from Experiment 1. An analysis identical to that of Experiment 1 was conducted.

8.2.3 Results — Functional Simulations

Figure 8.1 shows a sample white Gaussian process, and its corresponding PSD, as created by the computer random number generator. The figure also shows a simulated EMG signal and its corresponding PSD. Figure 8.2 shows typical EMG amplitude estimation results for the simulation data. The single channel simulation results are compared with the single channel results of Subject DA in Figure 8.3, for studies of variations in the construction of temporal whitening filters. Similarly, a comparison of multiple channel results, for studies of variations in the construction of spatial uncorrelation filters, is shown in Figure 8.4. Results which compare the MARMS and MAMAV detectors are shown in Figure 8.5. For the MARMS versus MAMAV study, temporal whitening filters were derived for each channel from a single 50% MVC of that channel, and applied to all recordings from that channel. A single spatial uncorrelation filter per subject (simulation or real) was calibrated from a record at 50% MVC. The figure also compares the MARMS and MAMAV detector for computer generated white Gaussian noise. Figure 8.6 shows the predicted versus achieved SNR performance for the raw single channel and whitened single channel estimators. Temporal whitening filters were the same as those used in the MARMS versus MAMAV study.
Figure 8.1: Functional Simulation Data

The upper left plot shows a typical 0.5 second portion of a simulated white Gaussian process (total record length is five seconds). The upper right plot is the Discrete Fourier Transform (DFT) technique power spectral density (PSD) of the white Gaussian process. The lower left plot is a 0.5 second portion of a simulated EMG signal. The lower right plot is the DFT technique PSD estimate of the simulated EMG. Each graph is independently normalized to its maximum value and graphed to a linear scale.
Figure 8.2: Simulation EMG Amplitude Estimates

Top left plot is the simulated torque. Top right plot is the moving average root mean square (MARMS) estimate (245ms window) of the amplitude of simulated EMG channel 3. Bottom left plot is the temporally whitened MARMS estimate of the amplitude of simulated EMG channel 3. Bottom right is the optimal eight channel estimate of the EMG amplitude. Beginning portion of each amplitude estimate depicts the rise time of the estimator. All data are from the same simulation trial.
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th>Subject DA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw EMG SNR ±σ</td>
<td>Whitened EMG SNR ±σ</td>
</tr>
<tr>
<td>1) 20 seconds of data from same channel, same contraction level, same subject, with;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>25.1 ± 4.2</td>
</tr>
<tr>
<td>10 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>25.1 ± 4.0</td>
</tr>
<tr>
<td>15 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>25.3 ± 4.2</td>
</tr>
<tr>
<td>20 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>25.4 ± 4.2</td>
</tr>
<tr>
<td>25 filter coefficients.</td>
<td>11.7 ± 2.3</td>
<td>25.4 ± 4.1</td>
</tr>
<tr>
<td>2) 5 seconds of data from same channel, same contraction level, same subject, with;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 filter coefficient,</td>
<td>11.7 ± 2.3</td>
<td>17.0 ± 3.1</td>
</tr>
<tr>
<td>2 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>22.2 ± 4.1</td>
</tr>
<tr>
<td>3 filter coefficients,</td>
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<td>24.3 ± 4.0</td>
</tr>
<tr>
<td>4 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>24.6 ± 4.1</td>
</tr>
<tr>
<td>5 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>24.4 ± 3.9</td>
</tr>
<tr>
<td>10 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>24.5 ± 4.0</td>
</tr>
<tr>
<td>15 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>24.5 ± 3.9</td>
</tr>
<tr>
<td>20 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>24.5 ± 3.9</td>
</tr>
</tbody>
</table>

**Figure 8.3:** Tabulated Single Channel Results for Functional Simulations and Subject DA
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th></th>
<th>Subject DA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw EMG SNR ±σ</td>
<td>Whitened EMG SNR ±σ</td>
<td>Raw EMG SNR ±σ</td>
<td>Whitened EMG SNR ±σ</td>
</tr>
<tr>
<td>3) Portion of one record from same channel, same contraction level, same subject, with;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>128 samples (62.5ms),</td>
<td>11.7 ± 2.3</td>
<td>23.4 ± 4.0</td>
<td>11.8 ± 2.5</td>
<td>19.8 ± 6.2</td>
</tr>
<tr>
<td>256 samples (125ms),</td>
<td>11.7 ± 2.3</td>
<td>23.9 ± 4.0</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 6.1</td>
</tr>
<tr>
<td>512 samples (250ms),</td>
<td>11.7 ± 2.3</td>
<td>24.2 ± 4.0</td>
<td>11.8 ± 2.5</td>
<td>20.2 ± 6.0</td>
</tr>
<tr>
<td>1024 samples (500ms),</td>
<td>11.7 ± 2.3</td>
<td>24.4 ± 4.0</td>
<td>11.8 ± 2.5</td>
<td>20.2 ± 6.0</td>
</tr>
<tr>
<td>2048 samples (1s),</td>
<td>11.7 ± 2.3</td>
<td>24.4 ± 4.1</td>
<td>11.8 ± 2.5</td>
<td>20.3 ± 6.0</td>
</tr>
<tr>
<td>4096 samples (2s),</td>
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<td>24.5 ± 4.1</td>
<td>11.8 ± 2.5</td>
<td>20.2 ± 6.0</td>
</tr>
<tr>
<td>6144 samples (3s),</td>
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<td>24.5 ± 4.1</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 6.0</td>
</tr>
<tr>
<td>8192 samples (4s),</td>
<td>11.7 ± 2.3</td>
<td>24.6 ± 4.1</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 6.0</td>
</tr>
<tr>
<td>4) 5 seconds of data from same channel, same contraction level, same subject, with, 5 filter coefficients, for;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>12.1 ± 2.6</td>
<td>24.4 ± 3.8</td>
<td>11.8 ± 2.8</td>
<td>24.9 ± 5.2</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.3 ± 2.2</td>
<td>25.5 ± 4.4</td>
<td>12.0 ± 2.5</td>
<td>22.7 ± 4.7</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.8 ± 1.9</td>
<td>23.7 ± 3.4</td>
<td>11.3 ± 3.1</td>
<td>14.8 ± 4.7</td>
</tr>
<tr>
<td>75% MVC.</td>
<td>11.5 ± 2.2</td>
<td>24.8 ± 4.6</td>
<td>12.2 ± 1.6</td>
<td>17.9 ± 3.9</td>
</tr>
</tbody>
</table>

Figure 8.3 (Continued)
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th>Subject DA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw EMG SNR ±σ</td>
<td>Whitened EMG SNR ±σ</td>
</tr>
<tr>
<td>5) Decimate data by 2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 seconds of data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from same channel,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>same contraction level,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>same subject, with;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>17.8 ± 2.6</td>
</tr>
<tr>
<td>10 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>18.1 ± 2.5</td>
</tr>
<tr>
<td>15 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>18.6 ± 2.7</td>
</tr>
<tr>
<td>20 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>18.7 ± 2.8</td>
</tr>
<tr>
<td>25 filter coefficients.</td>
<td>11.7 ± 2.3</td>
<td>18.6 ± 2.8</td>
</tr>
<tr>
<td>6) Decimate data by 2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 seconds of data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from same channel,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>same contraction level,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>same subject, with;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>17.5 ± 2.7</td>
</tr>
<tr>
<td>10 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>17.5 ± 2.6</td>
</tr>
<tr>
<td>15 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>17.8 ± 2.8</td>
</tr>
<tr>
<td>20 filter coefficients,</td>
<td>11.7 ± 2.3</td>
<td>17.7 ± 2.9</td>
</tr>
<tr>
<td>25 filter coefficients.</td>
<td>11.7 ± 2.3</td>
<td>17.5 ± 2.8</td>
</tr>
<tr>
<td>7) One 5 coefficient filter per channel, per subject, derived from data at;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.7 ± 2.3</td>
<td>23.0 ± 4.8</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.7 ± 2.3</td>
<td>24.0 ± 4.3</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.6 ± 2.2</td>
<td>24.2 ± 4.2</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.7 ± 2.3</td>
<td>24.0 ± 4.2</td>
</tr>
</tbody>
</table>

Figure 8.3 (Continued)
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th>Subject DA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw EMG SNR ±σ</td>
<td>Whitened EMG SNR ±σ</td>
</tr>
<tr>
<td>8) One 5 coefficient composite filter per subject derived from data at;</td>
<td>11.7 ± 2.3 23.0 ± 4.0</td>
<td>11.8 ± 2.5 19.1 ± 5.9</td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.7 ± 2.3 24.0 ± 4.1</td>
<td>11.8 ± 2.5 19.8 ± 5.9</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.7 ± 2.3 24.2 ± 4.1</td>
<td>11.8 ± 2.5 19.8 ± 5.7</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.7 ± 2.3 23.7 ± 4.1</td>
<td>11.8 ± 2.5 19.6 ± 5.7</td>
</tr>
<tr>
<td>75% MVC.</td>
<td>11.7 ± 2.3 23.7 ± 4.2</td>
<td>11.8 ± 2.5 19.8 ± 5.8</td>
</tr>
<tr>
<td>9) Universal 5 coefficient filter derived from 50% MVC of subject;</td>
<td>11.7 ± 2.3 23.7 ± 4.5</td>
<td>11.8 ± 2.5 16.9 ± 5.2</td>
</tr>
<tr>
<td>AD,</td>
<td>11.7 ± 2.3 23.4 ± 4.0</td>
<td>11.8 ± 2.5 19.4 ± 5.6</td>
</tr>
<tr>
<td>CC,</td>
<td>11.7 ± 2.3 22.4 ± 3.9</td>
<td>11.8 ± 2.5 19.3 ± 5.4</td>
</tr>
<tr>
<td>EA,</td>
<td>11.7 ± 2.3 22.4 ± 3.9</td>
<td>11.8 ± 2.5 19.3 ± 5.4</td>
</tr>
<tr>
<td>FA.</td>
<td>11.7 ± 2.3 22.4 ± 3.9</td>
<td>11.8 ± 2.5 19.3 ± 5.4</td>
</tr>
</tbody>
</table>

Figure 8.3 (Final Sheet)
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation Unwhitened Channel EMG SNR ±σ</th>
<th>Simulation Whitened Multiple Channel EMG SNR ±σ</th>
<th>Subject DA Unwhitened Channel EMG SNR ±σ</th>
<th>Subject DA Whitened Multiple Channel EMG SNR ±σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 20 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>11.7 ± 2.3</td>
<td>25.1 ± 4.2</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 6.0</td>
</tr>
<tr>
<td>2 channels,</td>
<td>12.2 ± 1.7</td>
<td>34.2 ± 5.2</td>
<td>11.9 ± 2.8</td>
<td>25.2 ± 7.3</td>
</tr>
<tr>
<td>4 channels,</td>
<td>11.7 ± 2.6</td>
<td>46.7 ± 7.4</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.6</td>
</tr>
<tr>
<td>6 channels,</td>
<td>11.5 ± 2.4</td>
<td>52.3 ± 8.3</td>
<td>11.8 ± 2.4</td>
<td>36.0 ± 15.0</td>
</tr>
<tr>
<td>8 channels.</td>
<td>11.7 ± 2.3</td>
<td>58.0 ± 8.1</td>
<td>11.8 ± 2.5</td>
<td>39.2 ± 17.6</td>
</tr>
<tr>
<td>2) 5 seconds of data from same contraction level, same subject, for;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 channel,</td>
<td>11.7 ± 2.3</td>
<td>24.6 ± 4.1</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 6.1</td>
</tr>
<tr>
<td>2 channels,</td>
<td>12.2 ± 1.7</td>
<td>34.3 ± 5.1</td>
<td>11.9 ± 2.8</td>
<td>25.2 ± 7.3</td>
</tr>
<tr>
<td>4 channels,</td>
<td>11.7 ± 2.6</td>
<td>46.0 ± 6.6</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.7</td>
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<td>6 channels,</td>
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<td>51.9 ± 7.1</td>
<td>11.8 ± 2.4</td>
<td>35.8 ± 15.0</td>
</tr>
<tr>
<td>8 channels.</td>
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<td>57.5 ± 7.1</td>
<td>11.8 ± 2.5</td>
<td>38.9 ± 17.5</td>
</tr>
</tbody>
</table>

**Figure 8.4:** Tabulated Multiple Channel Results for Functional Simulations and Subject DA
### Method of Determining Whitening Filter

#### 3) Four MSEMG channels:
- Portion of one record from same contraction level, same subject, with:
  - 32 samples (15.6ms),
  - 64 samples (31.3ms),
  - 128 samples (62.5ms),
  - 256 samples (125ms),
  - 512 samples (250ms),
  - 1024 samples (500ms),
  - 2048 samples (1s),
  - 4096 samples (2s),
  - 6144 samples (3s),
  - 8192 samples (4s),

<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th></th>
<th></th>
<th>Subject DA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unwhitened</td>
<td>Whitened</td>
<td>Unwhitened</td>
<td>Whitened</td>
</tr>
<tr>
<td></td>
<td>Single</td>
<td>Multiple</td>
<td>Single</td>
<td>Multiple</td>
</tr>
<tr>
<td></td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
</tr>
<tr>
<td>32 samples (15.6ms),</td>
<td>11.7 ± 2.6</td>
<td>42.4 ± 7.9</td>
<td>11.8 ± 2.7</td>
<td>30.6 ± 10.0</td>
</tr>
<tr>
<td>64 samples (31.3ms),</td>
<td>11.7 ± 2.6</td>
<td>43.6 ± 7.3</td>
<td>11.8 ± 2.7</td>
<td>31.4 ± 10.0</td>
</tr>
<tr>
<td>128 samples (62.5ms),</td>
<td>11.7 ± 2.6</td>
<td>44.7 ± 6.7</td>
<td>11.8 ± 2.7</td>
<td>32.0 ± 10.5</td>
</tr>
<tr>
<td>256 samples (125ms),</td>
<td>11.7 ± 2.6</td>
<td>45.3 ± 6.4</td>
<td>11.8 ± 2.7</td>
<td>32.4 ± 10.7</td>
</tr>
<tr>
<td>512 samples (250ms),</td>
<td>11.7 ± 2.6</td>
<td>45.6 ± 6.7</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.6</td>
</tr>
<tr>
<td>1024 samples (500ms),</td>
<td>11.7 ± 2.6</td>
<td>45.7 ± 6.7</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.6</td>
</tr>
<tr>
<td>2048 samples (1s),</td>
<td>11.7 ± 2.6</td>
<td>45.9 ± 6.6</td>
<td>11.8 ± 2.7</td>
<td>32.4 ± 10.5</td>
</tr>
<tr>
<td>4096 samples (2s),</td>
<td>11.7 ± 2.6</td>
<td>45.9 ± 6.4</td>
<td>11.8 ± 2.7</td>
<td>32.4 ± 10.6</td>
</tr>
<tr>
<td>6144 samples (3s),</td>
<td>11.7 ± 2.6</td>
<td>45.9 ± 6.5</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.7</td>
</tr>
<tr>
<td>8192 samples (4s),</td>
<td>11.7 ± 2.6</td>
<td>45.9 ± 6.6</td>
<td>11.8 ± 2.7</td>
<td>32.3 ± 10.7</td>
</tr>
</tbody>
</table>

#### 4) 5 seconds of data from same contraction level, same subject, four MSEMG channels, for:
- 10% MVC,
- 25% MVC,
- 50% MVC,
- 75% MVC.

<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation</th>
<th></th>
<th></th>
<th>Subject DA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unwhitened</td>
<td>Whitened</td>
<td>Unwhitened</td>
<td>Whitened</td>
</tr>
<tr>
<td></td>
<td>Single</td>
<td>Multiple</td>
<td>Single</td>
<td>Multiple</td>
</tr>
<tr>
<td></td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
<td>Channel EMG SNR ±σ</td>
</tr>
<tr>
<td>10% MVC,</td>
<td>12.7 ± 3.2</td>
<td>41.4 ± 4.9</td>
<td>11.5 ± 2.9</td>
<td>42.8 ± 4.7</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.1 ± 2.6</td>
<td>48.3 ± 4.4</td>
<td>11.9 ± 3.0</td>
<td>34.7 ± 13.7</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.7 ± 1.8</td>
<td>50.2 ± 7.9</td>
<td>11.4 ± 3.1</td>
<td>27.6 ± 6.4</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.3 ± 2.4</td>
<td>44.0 ± 4.9</td>
<td>12.3 ± 1.7</td>
<td>24.1 ± 3.0</td>
</tr>
</tbody>
</table>

Figure 8.4 (Continued)
| Method of Determining Whitening Filter | Simulation | | Subject DA | | |
|---------------------------------------|------------| |--------------------------------| | |
|                                       | Unwhitened Single Channel EMG SNR ±σ | Whitened Multiple Channel EMG SNR ±σ | Unwhitened Single Channel EMG SNR ±σ | Whitened Multiple Channel EMG SNR ±σ |
| 5) Decimate data by 2: 20 seconds of data from same contraction level, same subject, for; 1 channel, | 11.7 ± 2.3 | 17.8 ± 2.6 | 11.8 ± 2.5 | 15.2 ± 3.3 |
| 2 channels, | 12.2 ± 1.7 | 20.2 ± 2.9 | 11.9 ± 2.8 | 16.5 ± 3.4 |
| 4 channels, | 11.7 ± 2.6 | 27.4 ± 3.7 | 11.8 ± 2.7 | 21.0 ± 5.0 |
| 6 channels, | 11.5 ± 2.4 | 33.5 ± 4.9 | 11.8 ± 2.4 | 24.1 ± 7.1 |
| 8 channels. | 11.7 ± 2.3 | 38.3 ± 5.8 | 11.8 ± 2.5 | 24.8 ± 8.1 |
| 6) Decimate data by 2: 5 seconds of data from same contraction level, same subject, for; 1 channel, | 11.7 ± 2.3 | 17.5 ± 2.7 | 11.8 ± 2.5 | 15.2 ± 3.4 |
| 2 channels, | 12.2 ± 1.7 | 20.2 ± 2.9 | 11.9 ± 2.8 | 16.4 ± 3.5 |
| 4 channels, | 11.7 ± 2.6 | 27.3 ± 4.0 | 11.8 ± 2.7 | 20.9 ± 5.0 |
| 6 channels, | 11.5 ± 2.4 | 32.8 ± 4.7 | 11.8 ± 2.4 | 23.8 ± 6.9 |
| 8 channels. | 11.7 ± 2.3 | 37.6 ± 5.5 | 11.8 ± 2.5 | 24.3 ± 7.7 |
| 7) Two MSEM channels: One uncorrelation filter from trial at; 10% MVC, | 12.2 ± 1.7 | 34.1 ± 5.5 | 11.9 ± 2.8 | 25.6 ± 6.9 |
| 25% MVC, | 12.2 ± 1.7 | 34.1 ± 5.6 | 11.9 ± 2.8 | 25.5 ± 7.1 |
| 50% MVC, | 12.2 ± 1.7 | 34.2 ± 5.6 | 11.9 ± 2.8 | 25.4 ± 7.1 |
| 75% MVC, | 12.2 ± 1.7 | 33.0 ± 6.2 | 11.9 ± 2.8 | 25.5 ± 6.5 |
| Equalized variances. | 12.2 ± 1.7 | 34.1 ± 5.6 | 11.9 ± 2.8 | 25.5 ± 7.2 |

Figure 8.4 (Continued)
<table>
<thead>
<tr>
<th>Method of Determining Whitening Filter</th>
<th>Simulation Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
<th>Subject DA Unwhitened Single Channel EMG SNR ±σ</th>
<th>Whitened Multiple Channel EMG SNR ±σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) Four MSEM channels:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from trial at;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.7 ± 2.6</td>
<td>41.1 ± 6.1</td>
<td>11.8 ± 2.7</td>
<td>30.4 ± 10.5</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.7 ± 2.6</td>
<td>41.2 ± 7.0</td>
<td>11.8 ± 2.7</td>
<td>31.3 ± 11.2</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.7 ± 2.6</td>
<td>39.8 ± 10.6</td>
<td>11.8 ± 2.7</td>
<td>30.6 ± 9.2</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.7 ± 2.6</td>
<td>39.4 ± 10.5</td>
<td>11.8 ± 2.7</td>
<td>30.4 ± 8.3</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>11.7 ± 2.6</td>
<td>39.0 ± 9.9</td>
<td>11.8 ± 2.7</td>
<td>30.1 ± 9.3</td>
</tr>
<tr>
<td>9) Six MSEM channels:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from trial at;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.5 ± 2.4</td>
<td>46.8 ± 8.4</td>
<td>11.8 ± 2.4</td>
<td>34.0 ± 15.4</td>
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<td>25% MVC,</td>
<td>11.5 ± 2.4</td>
<td>47.1 ± 9.7</td>
<td>11.8 ± 2.4</td>
<td>34.3 ± 15.6</td>
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<tr>
<td>50% MVC,</td>
<td>11.5 ± 2.4</td>
<td>44.3 ± 11.4</td>
<td>11.8 ± 2.4</td>
<td>32.1 ± 10.6</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.5 ± 2.4</td>
<td>44.6 ± 11.6</td>
<td>11.8 ± 2.4</td>
<td>31.0 ± 8.5</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>11.5 ± 2.4</td>
<td>41.9 ± 8.5</td>
<td>11.8 ± 2.4</td>
<td>31.2 ± 10.3</td>
</tr>
<tr>
<td>10) Eight MSEM channels:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One uncorrelation filter from trial at;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% MVC,</td>
<td>11.7 ± 2.3</td>
<td>50.2 ± 8.0</td>
<td>11.8 ± 2.5</td>
<td>35.9 ± 17.4</td>
</tr>
<tr>
<td>25% MVC,</td>
<td>11.7 ± 2.3</td>
<td>50.8 ± 9.2</td>
<td>11.8 ± 2.5</td>
<td>37.5 ± 19.1</td>
</tr>
<tr>
<td>50% MVC,</td>
<td>11.7 ± 2.3</td>
<td>50.5 ± 11.9</td>
<td>11.8 ± 2.5</td>
<td>35.0 ± 13.4</td>
</tr>
<tr>
<td>75% MVC,</td>
<td>11.7 ± 2.3</td>
<td>51.1 ± 11.9</td>
<td>11.8 ± 2.5</td>
<td>33.5 ± 10.6</td>
</tr>
<tr>
<td>Equalized variances.</td>
<td>11.7 ± 2.3</td>
<td>44.9 ± 9.4</td>
<td>11.8 ± 2.5</td>
<td>31.8 ± 10.9</td>
</tr>
</tbody>
</table>

Figure 8.4 (Final Sheet)
<table>
<thead>
<tr>
<th>Study: Detector</th>
<th>Raw Single Channel EMG SNR ±σ</th>
<th>Whitened Single Channel EMG SNR ±σ</th>
<th>Optimal Four Channel EMG SNR ±σ</th>
<th>Equal Variance Four Channel EMG SNR ±σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functional Simulations: RMS</td>
<td>11.7 ± 2.3</td>
<td>24.3 ± 4.1</td>
<td>40.4 ± 6.4</td>
<td>36.2 ± 7.1</td>
</tr>
<tr>
<td>MAV</td>
<td>11.3 ± 2.2</td>
<td>23.4 ± 4.0</td>
<td>40.0 ± 7.1</td>
<td>35.8 ± 5.9</td>
</tr>
<tr>
<td>Subject DA: RMS</td>
<td>11.8 ± 2.5</td>
<td>20.1 ± 5.7</td>
<td>28.2 ± 12.9</td>
<td>24.8 ± 9.8</td>
</tr>
<tr>
<td>MAV</td>
<td>12.1 ± 2.4</td>
<td>20.7 ± 4.9</td>
<td>30.2 ± 11.4</td>
<td>26.7 ± 8.9</td>
</tr>
<tr>
<td>Computer White Gaussian Noise: RMS</td>
<td>33.1 ± 4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAV</td>
<td>31.2 ± 4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 8.5:** MAV Versus RMS Results for Functional Simulations and Subject DA

Temporal whitening filters were derived for each channel from a single 50% MVC of that channel, and applied to all recordings from that channel. For optimally combined data, a single spatial uncorrelation filter per subject (simulation or real) was calibrated from a record at 50% MVC.
**Figure 8.6: Simulation Predicted Vs. Achieved SNR Performance**

The left plot shows SNR performance of simple single channel amplitude estimates formed from the simulated data. The right plot shows SNR performance of single channel amplitude estimates formed from whitened simulation data. Temporal whitening filters were derived for each channel from a single 50% MVC of that channel, and applied to all recordings from that channel. Each of the scatter plots has 160 entries.
8.2.4 Discussion — Functional Simulations

In general, the results of the functional simulations followed the same trends as the results of actual Subject DA, except that the SNR performance of the simulations was superior. For temporal whitening filters, the use of five or more coefficients worked approximately equally well. Performance dropped progressively when less than five filter coefficients were used. There was little difference in performance found when whitening filters were calibrated from as little as 62.5ms of data to as much as 20s of data. Reducing the sampling rate to 1024 Hz. decreased the SNR performance 29% (compared to a 24% decrement for Subject DA). The use of one temporal whitening filter per channel, a composite filter per subject, or a universal filter performed comparably to Subject DA. In all, only one difference in trend was detected for single channel whitening filters — when the performance results were segregated from trials at different contraction levels, no consistent difference in results was found. With experimental data from Subject DA, performance results at the higher contraction levels were poorer than the lower contraction levels. Presumably, muscular tremor, present during the higher contraction trials, degraded the assumption of a constant force input (or constant EMG amplitude) during an experimental trial. This presumption is consistent with the simulation results, since a constant EMG amplitude was definitively maintained during a simulation trial.

The single channel MARMS versus MAMAV detector results differed from those of the experiments. In the experiments, the MAMAV detector was better than the MARMS detector. In the simulations, the MARMS detector prevailed. Analytically, the Gaussian model predicts that the MARMS detector should out-perform the MAMAV detector by approximately 7%, assuming the data
are temporally uncorrelated and the smoothing window length is 500 samples. When computer white Gaussian noise was investigated (the most representative temporally uncorrelated data), indeed the performance of the MARMS detector exceeded that of the MAMAV detector by 6% (see Figure 8.5). These results again suggest (see the MAMAV detector discussions in Chapters 5 and 6) that, for the purposes of EMG amplitude estimation, experimental EMG data may be more appropriately described by the Laplacian PDF than the Gaussian PDF.

The last reported single channel simulation results are the predicted versus achieved SNR performances. Compared to the actual experiment results (shown in Chapter 5), the achieved simulation results are more closely clustered about the predicted results. Again, it is likely that simulations benefit from being derived from the designed model, a condition which need not be true of the experimental data.

Two questions about the single channel simulation results remain; 1) Why were the simulation results superior to the experimental results? and 2) Why didn’t the simulation results achieve the analytic prediction of SNR=31.6 (based on a smoothing window length of 500 samples)? Addressing the former question; first, contrary to the experimental data, the simulation data were derived from a white noise source, thus assuring that all possible degrees of freedom in the data were originally present. Second, the simulated EMG amplitude was perfectly constant. There was no loss of performance due to a subject’s ability to maintain a constant force. And, third, the simulated data were necessarily Gaussian distributed, thus the “optimal” estimators were optimal (in the ML sense). Addressing the latter question; first, this simulation model did not require the PSD of the data to fit an AR model of the orders studied. In fact, the
temporal correlation filter was not even technically specified to have an inverse which existed or was stable and causal. Second, any whitening algorithm will have limitations in its abilities to identify the proper whitening filter and then perform the requisite filtering function. And, third, signal resolution, particularly at high frequencies where simulation amplitudes are very small, may cost estimation performance due to roundoff errors.

For multiple channel simulations, the general trends of the results again followed those of the actual experiments, except that the simulation SNR performance was superior. As the number of EMG channels increased, so did the SNR performance, for all of the spatial uncorrelation techniques. Optimal spatial uncorrelation filters calibrated from between 15.6ms of data and 20s of data performed approximately equally, the very short calibration lengths showing a small drop in performance. All uncorrelation techniques performed approximately equally, except for the equal variance technique which performed poorer for four or more channels. When the sampling rate was cut in half to 1024 Hz. performance was degraded similar to the results of the actual experiments.

As with the single channel simulation results, no difference in performance was found when the simulation results were segregated by contraction level. Additionally, the MARMS detector performed slightly better than the MAMAV detector. The overall better performance of the simulation data (compared to the actual experimental data) was likely due first to the better single channel results, and second because all channels of the simulated data each originally contained all possible degrees of freedom.
8.3 Physiologic Simulations

8.3.1 Design — Physiologic Simulations

For the physiologic simulations, only single channel EMG waveforms were simulated. The overall simulation technique was to create motor unit action potential trains (MUAPT's) by randomized firings of randomly shaped motor unit action potentials (MUAP's). The contributions from several MUAPT's were superimposed to create the surface EMG. This technique was based on the model for the surface electromyogram developed by DeLuca (1979) and Basmajian and DeLuca (1985), discussed in Chapter 3. The details of the simulation follow.

Monopolar action potentials (AP’s) were simulated according to the model shown in Figure 8.7. A monopolar AP had a value equal to the inverse of the distance from the electrode to the excitation site, scaled by the value “Scale”, or

\[ AP(x) = \frac{\text{Scale}}{\sqrt{x^2 + y^2}} \]

The distance \( y \) was fixed as one distance unit. The scaling was randomly assigned from a doubly truncated normal distribution (Patel and Read, 1982) with a mean value of 1.0, a variance of 0.0625, and truncated over the range 0.1 to 10.0. The distance \( x \) was incremented from a large negative distance (-250 distance units) to a large positive distance (250 distance units) by one distance unit for each increment in unit time. In this manner, the excitation site changed location, simulating AP propagation. Beyond the large negative/positive distances, the AP value was approximately zero. The time at which \( x \) equaled zero was considered the firing time of the MU represented by that particular AP waveform. For this simulation, one time unit was equal to \( \frac{1}{2048} \) seconds (the sampling period of the actual experimental trials).
**Figure 8.7:** Model for Developing Monopolar AP

The monopolar action potential (AP) has a value equal to the inverse of the distance from the electrode to the excitation site, scaled by the value "Scale".

\[
AP(x) = \frac{\text{Scale}}{\sqrt{x^2 + y^2}}
\]
A bipolar MUAP was formed by adding one randomly scaled monopolar AP to a second, time-delayed, negated, randomly scaled monopolar AP. Geometrically, this addition oriented the two bipolar leads of a simulated electrode pair along the direction of AP propagation. The time delay was eight time units, or approximately 3.9ms. Figure 8.8 shows a sample bipolar MUAP. An MUAPT was created by assigning random successive inter-pulse intervals (IPI's) to the firing times of a MUAP. The initial firing time and all subsequent IPI's were randomly assigned from a doubly truncated normal distribution with a mean value equal to the mean firing interval, a variance equal to one quarter of the square root of the mean firing interval, and truncated over the range 50 to 400 time units. (See Basmajian and DeLuca, 1985, for some sample IPI distributions.) The mean firing rate (inverse of the mean firing interval) was selectable for each simulation trial. Any particular MUAP maintained a fixed shape throughout a simulation trial. Distinct MUAP's had distinct shapes.

The simulated surface EMG was created as the zero mean sum (zero mean linear superposition) of several MUAPT's. An initial startup time (400 time units) was discarded, and the ensuing 10240 time units (five simulated seconds) was saved. The number of MU's and the mean firing rate were selectable for each simulation trial. All simulated EMG data were created and analyzed in computer double format.

8.3.2 Methods — Physiologic Simulations

The physiologic simulations were allowed to select both the MU firing rate and the number of MU's for each simulation. Initially, the number of MU's was fixed at 100 and the firing rate was ranged from 5 to 20 pulses/sec in increments of
Figure 8.8: Simulated Bipolar Motor Unit Action Potential
1 pulse/sec. Then, the firing rate was fixed at 15 pulses/sec and the number of MU's ranged from 50 to 200 in increments of 10. Thus, a total of 32 physiologic simulations were performed.

### 8.3.3 Results — Physiologic Simulations

Figure 8.9 shows a sampled simulated EMG waveform, and its PSD, for a mean firing rate of 15 pulses/sec and 100 simulated MU's. Amplitude estimation was performed on the 32 simulation trials with the simple amplitude estimator (no temporal whitening), yielding a SNR performance of 13.9 ± 1.6. A single whitening filter was derived from a five second data set, each sample in this set being formed as the sum of the respective sample from all 32 simulations. Using this whitening filter, an optimal estimator had a SNR performance of 24.7±3.2, and approximate 78% improvement over the simple estimator. Finally, in Figure 8.10 the EMG amplitude estimates, for both the simple and optimal estimators, were plotted versus the mean firing rate and the number of simulated MU's.

### 8.3.4 Discussion — Physiologic Simulations

The physiologically simulated EMG waveform, and its PSD, look similar to that of experimental data, even though this simulation model is extremely simple. Only a most rudimentary model of MU activation and AP propagation has been used.\(^1\) The SNR performance results, both for the raw and whitened data, are similar to the actual data and the functional simulations.

\(^1\)DeLuca and van Dyk (1975) present a more complete physiologically based model. They derive analytic expressions for the MAV and RMS values of the EMG, as well as the PSD. Papoulias (1984) discusses the more general topic of shot noise and the conditions by which shot noise can approximate Gaussian noise.
Figure 8.9: Physiologic Simulation EMG Waveform and its Power Spectrum

The upper left plot displays five seconds of the physiologic simulation surface EMG waveform. The simulation model utilized 100 motor units and a mean firing rate of 15 pulses/second. The upper right plot shows the first 0.5 seconds of the same data. The bottom left plot is the Discrete Fourier Transform (DFT) technique power spectral density (PSD) of these data, plotted to a linear scale. The bottom right is the DFT technique PSD of these data, plotted to a logarithmic scale. The top plots are graphed to a linear scale.
Figure 8.10: EMG Amplitude Versus Physiologic Model Parameters

The top plot shows the EMG amplitude versus the average firing rate for the simple (raw) and optimal (whitened) estimators. The bottom plot shows the EMG amplitude versus number of simulated motor units for the simple (raw) and optimal (whitened) estimators. A single whitening filter was derived from a five second data set, each sample in this set being formed as the sum of the respective sample from all 32 simulations. Each graph is independently normalized to its maximum value and graphed to a linear scale.
The logarithmic PSD plot of Figure 8.9 shows three dips in the spectrum of the data. These dips, or cancellation frequencies, are due to the differential electrode filter effect, discussed previously in Chapter 3. The cancellation frequencies occur at multiples of the muscle fiber conduction velocity, \( v \), divided by the electrode separation distance, \( d \), which are related as

\[
v \cdot \text{delay} = d
\]

where "delay" is the time for an AP to propagate from one electrode to the other. This time delay was eight time units, or 3.9ms. Thus, the cancellation frequencies occur at 256, 512 and 768 Hz., as shown by the locations of the dips in the PSD of the simulated data.

Figure 8.10 suggests that EMG amplitude estimation from either raw or whitened data produces an estimate that follows an identical contour with respect to both average firing rate and number of simulated MU's. That is, for changes in either average firing rate or number of simulated MU's, the whitened estimate was essentially a scaled version of the raw estimate — except, the whitened estimate exhibited superior SNR performance. Thus, the whitened estimate is a higher quality result which does not alter (compared to the simple raw estimate) the form of the relationship between the EMG amplitude and torque.

Previous investigators (Moore, 1967; Person and Libkind, 1970; see the discussion in Chapter 3) found that at high recruitment/ firing rate levels an increase in EMG activity led to a larger recruitment/ firing rate increment than at lower recruitment/ firing rate levels. The results of this study seem to indicate a linear relationship between EMG amplitude and recruitment/ firing rate. It is difficult to compare this study directly to the previous studies, however, since
the earlier works simulated a smaller number of MU’s contracting at faster rates.

8.4 Conclusion

Both of the simulation studies produced results which were consistent with the results of the actual experiments. In general, the SNR performance results for the simulations were better, reflecting that simulated data fit the proposed model exactly as well as the noise-free nature of the simulated data. The consistency of the results also serves to verify the software algorithms used in EMG amplitude estimation.
Chapter 9

Estimation of Joint Torque from the EMG Amplitude: A Short Simulation Study
9.1 Introduction

The previous chapters have discussed in detail multiple channel techniques for estimating the amplitude of the surface EMG waveform, for contraction levels between 10%–75% MVC. This chapter and the next will attempt to relate the EMG amplitude to joint torque. This chapter will discuss a computer simulation study, while the next chapter will report experimental work.

As discussed in Chapter 4, it was assumed that joint torque was an identifiable function of the EMG amplitude. With this assumption, estimation of individual (extensor/ flexor) torque contributions and the total torque about the joint could be organized as the method shown in Figure 9.1. The surface EMG waveforms, from each of the extensor and flexor muscle groups, were used to estimate the respective EMG amplitudes (the topic of the previous chapters). Then, the EMG amplitude estimates were related to their respective joint torque contributions and the total torque about the joint. A method for identifying the relationship between the EMG amplitude and joint torque was presented in Chapter 4. For each of the extensor and flexor muscle groups, the relationship between joint torque contribution and the EMG amplitude was constrained to be a sum of basis functions which had a linear dependence of a set of fit parameters. With these constraints, the identification problem was reduced to a linear least squares problem, for which robust solutions exist.

This short simulation study investigated two aspects of the identification problem. First, prior to acquiring experimental data, it was important to determine what tasks a subject should perform — i.e. it was important to determine what experimental data to actually acquire. Second, a study evaluated how well
Figure 9.1: Method of Estimating Joint Torque From Surface EMG Waveforms

Surface EMG waveforms, from the extensor and flexor muscle groups, were used to estimate the respective EMG amplitudes ($\hat{s}_E$ and $\hat{s}_F$). The EMG amplitude estimates were then used to provide an estimate of joint torque due to each of the two antagonistic muscle groups. The two contributions to joint torque were combined to estimate the total torque about the joint.
the identification technique performed, based on a known EMG to torque model. These results aided in the interpretation of the experimental results, wherein the model was not known.

9.2 EMG to Torque Basis Functions

Two sets of basis functions, standard and orthogonal polynomials, were used to relate the EMG amplitude to torque. The standard polynomials were \( s, s^2, s^3, \ldots \), where \( s \) is the EMG amplitude. Note that no offset, or constant, term was included. Since both agonist and antagonist muscle groups were included, a constant term for each would have yielded redundant constant terms in the model. For the orthogonal polynomial basis, well known orthogonal polynomials such as Legendre, Chebyshev, Laguerre or Hermite polynomials could not be used since each of these polynomials include an offset term. (See Hamming, 1962 and Jain et al., 1985 for definitions and examples of these polynomials.) Thus, a set of orthogonal polynomials, with no offset term, was derived. A pair of polynomials, \( f_1(s) \) and \( f_2(s) \), in the EMG amplitude \( s \) were defined to be orthogonal over the interval \([a, b]\) if

\[
\int_{a}^{b} f_1(s) f_2(s) \, ds = 0
\]

A set of polynomials were orthogonal if all pairs were mutually orthogonal. Since optimal EMG amplitude estimates were typically normalized to values from 0 to 1, the interval over which orthogonal polynomials were derived was \([0, 1]\). Up to sixth order polynomials, denoted \( c_i(s) \), were derived sequentially from order one to order six. Note that any multiple of an orthogonal polynomial was also an orthogonal polynomial. The coefficients of the orthogonal polynomials,
except polynomial $c_4(s)$,\footnote{All coefficients of polynomial $c_4(s)$ contain the factor three. Not selecting the lowest possible whole numbers for this polynomial was as oversight.} were chosen as the lowest values possible in which all coefficients were whole numbers. The first order term was always assigned a positive value. The orthogonal polynomials were

\[
\begin{align*}
    c_1(s) &= s \\
    c_2(s) &= 3s - 4s^2 \\
    c_3(s) &= 6s - 20s^2 + 15s^3 \\
    c_4(s) &= 30s - 180s^2 + 315s^3 - 168s^4 \\
    c_5(s) &= 15s - 140s^2 + 420s^3 - 504s^4 + 210s^5 \\
    c_6(s) &= 21s - 280s^2 + 1260s^3 - 2520s^4 + 2310s^5 - 792s^6
\end{align*}
\]

### 9.3 Investigation of Inputs to Identification Technique

#### 9.3.1 Evaluation of Appropriate Inputs

Three principal areas were addressed in investigating appropriate EMG amplitude inputs for the EMG to torque identification technique; 1) the influence of muscular co-contraction, 2) the time course of the EMG amplitude data, and 3) the scaling of the EMG amplitude data. As discussed in Chapter 4, the EMG to torque identification technique was shown to be equivalent to minimizing, in the least squares sense, $A \mathbf{x} - \mathbf{b}$ with respect to the fit parameters $\mathbf{x}$, where $A$ was a matrix of basis functions evaluated at the EMG amplitudes, and $\mathbf{b}$ was a vector of measured total joint torque. Each row of $A$ and $\mathbf{b}$ corresponded to a different discrete time.

To investigate appropriate EMG amplitude inputs for the EMG to torque
identification problem, the condition number of the data matrix $A$ was evaluated for several input schemes. The condition number of $A$, defined as the ratio of the largest to the smallest singular value of $A$, was a measure of the sensitivity of the fit parameters to small changes (including computational, or roundoff, errors and measurement errors) in $A$ and $b$. A small condition number indicated a well-conditioned problem, and thus fit parameters that were not sensitive to the data. A large condition number indicated an ill-conditioned problem, and thus fit parameters that were sensitive to the data. (Press et al., 1988; Strang, 1980) Obviously, a well-conditioned problem was desired.

9.3.2 Simulation Methods and Results

Five investigations were conducted. Each trial record within an investigation simulated EMG amplitudes (extensor/ flexor) from one, two or three contractions. When multiple contractions were simulated, the extensor (flexor) data from the contractions were concatenated to form the trial record. To simulate one contraction, extension EMG amplitude was linearly ramped in time from zero to a maximum value, while flexion EMG amplitude was linearly ramped in time from zero to another maximum value. EMG amplitude ramps were simulated as five seconds in duration, sampled at 2048 Hz. Thus, trials with one contraction were five simulated seconds in duration, trials with two contractions were ten simulated seconds in duration, and trials with three contractions were fifteen simulated seconds in duration. First through sixth order standard and orthogonal polynomials were investigated. (Note that order refers to the number of fit parameters per muscle group. The total number of fit parameters, and thus the total model order, was found by doubling the referenced order.) After
computing the $A$ matrix, the condition number was evaluated from the singular value decomposition algorithm of Press et al. (1988, Section 2.9).

First, EMG amplitude input comprised of a single simulated contraction was investigated. Extension (flexion) EMG amplitude was ramped from 0 to 0.8, while flexion (extension) EMG amplitude was ramped from zero to a maximum antagonist EMG amplitude. Results were evaluated for maximum antagonist EMG amplitudes of 0.0, 0.1, 0.2, ..., 0.8 (representing various levels of co-contraction). In all of the 108 above comparisons, the condition number for the matrix $A$ was found to be $> 10^5$.

Second, EMG amplitude input comprised of two simulated contractions was investigated. In the first contraction, extensor EMG amplitude was ramped from 0 to 0.8, while flexor EMG amplitude was ramped from zero to a maximum antagonist EMG amplitude. In the second contraction, flexor EMG amplitude was ramped in time from 0 to 0.8, while extensor EMG amplitude was ramped from zero to a maximum antagonist EMG amplitude. Results were evaluated for maximum antagonist EMG amplitudes as in the single contraction case, and are presented in Figure 9.2.

Third, EMG amplitude input comprised of three simulated contractions was investigated. The first and second contraction were the same as the two contraction case. For the third contraction, both extensor and flexor EMG amplitudes were ramped from zero to the maximum agonist EMG amplitude. Results were again evaluated for maximum antagonist EMG amplitudes as in the single contraction case, and are presented in Figure 9.3.

The final pair of investigations varied the scaling of the EMG amplitude inputs. In all cases, the ratio of the maximum agonist EMG amplitude to the
Table 9.2: Effect of Co-Contraction on Condition Number — Two Contractions

<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Standard Polynomial of Order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.0</td>
<td>1.0 8.8 66.0 466 3212 21922</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1</td>
<td>1.3 10.2 72.7 506 3474 23675</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>1.7 12.1 82.3 559 3794 25722</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>2.2 15.1 96.5 633 4215 28266</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
<td>3.0 19.6 119 748 4841 31847</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>4.3 27.3 157 947 5913 37824</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>7.0 42.9 236 1360 8145 50264</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>15.0 90.4 478 2628 15077 89273</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>&gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Orthogonal Polynomial of Order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.0</td>
<td>1.0 2.4 5.2 19.3 46.2 113</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1</td>
<td>1.3 3.3 7.5 31.7 77.3 190</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>1.7 3.9 9.6 34.6 86.4 208</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>2.2 4.2 11.4 32.8 83.1 199</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
<td>3.0 6.2 13.7 33.1 81.6 188</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>4.3 9.5 17.9 35.9 87.8 196</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>7.0 16.0 27.1 48.8 103 215</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>15.0 34.7 55.4 110 169 236</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>&gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5 &gt;10^5</td>
</tr>
</tbody>
</table>

The condition number for the matrix $A$ is shown for several sets of EMG amplitudes. Each set was formed from the concatenation of two simulated contractions. In the first contraction, extensor EMG amplitude was linearly ramped in time from 0 to 0.8, while flexor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude shown. In the second contraction, flexor EMG amplitude was linearly ramped in time from 0 to 0.8, while extensor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude. EMG amplitude ramps were simulated as five seconds in duration, sampled at 2048 Hz. Results for both standard and orthogonal polynomials are provided.
<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Standard Polynomial of Order:</th>
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</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.0</td>
<td>1.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1</td>
<td>2.1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>2.5</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>3.2</td>
</tr>
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<td>0.4</td>
<td>4.1</td>
</tr>
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<td>0.8</td>
<td>0.5</td>
<td>5.7</td>
</tr>
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<td>0.6</td>
<td>9.0</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>18.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>&gt; 10^5</td>
</tr>
</tbody>
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<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Orthogonal Polynomial of Order:</th>
</tr>
</thead>
<tbody>
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<td>0.8</td>
<td>0.0</td>
<td>1.7</td>
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<tr>
<td>0.8</td>
<td>0.1</td>
<td>2.1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>2.5</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>3.2</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
<td>4.1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>5.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>9.0</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>18.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>&gt; 10^5</td>
</tr>
</tbody>
</table>

Figure 9.3: Effect of Co-Contraction on Condition Number — Three Contractions

The condition number for the matrix A is shown for several sets of EMG amplitudes. Each set was formed from the concatenation of three simulated contractions. In the first contraction, extensor EMG amplitude was linearly ramped in time from 0 to 0.8, while flexor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude shown. In the second contraction, flexor EMG amplitude was linearly ramped in time from 0 to 0.8, while extensor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude. In the third contraction, both extensor and flexor EMG amplitudes were linearly ramped in time from zero to the maximum agonist EMG amplitude. EMG amplitude ramps were simulated as five seconds in duration, sampled at 2048 Hz. Results for both standard and orthogonal polynomials are provided.
maximum antagonist EMG amplitude was fixed as 4:1. The fourth investigation used EMG amplitude input comprised of two simulated contractions. The scheme of contraction was the same as the previous two contraction investigation, except different maximum EMG amplitudes were chosen. The results are shown in Figure 9.4. The fifth investigation used EMG amplitude input comprised of the three simulated contractions, also in the scheme of the previous three contraction investigation. Results using the same maximum EMG amplitudes as the fourth investigation are shown in Figure 9.5.
<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Standard Polynomial of Order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.02</td>
<td>1.7  86.5  4585  $&gt; 10^5$  $&gt; 10^5$  $&gt; 10^5$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>1.7  19.0  217  2505  29127  $&gt; 10^5$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>1.7  12.1  82.3  559  3794  25722</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.7  19.3  175  1571  14500  $&gt; 10^5$</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>1.7  36.0  599  9069  $&gt; 10^5$  $&gt; 10^5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Orthogonal Polynomial of Order:</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.02</td>
<td>1.7  184  16124  $&gt; 10^6$  $&gt; 10^5$  $&gt; 10^5$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>1.7  18.9  192  2927  22463  $&gt; 10^5$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>1.7  3.9   9.6   34.6  86.4   208</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.7  42.0  854  42659  $&gt; 10^5$  $&gt; 10^5$</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>1.7  110  5460  $&gt; 10^5$  $&gt; 10^5$  $&gt; 10^5$</td>
</tr>
</tbody>
</table>

**Figure 9.4:** Effect of EMG Amplitude Scale on Condition Number — Two Contractions

The condition number for the matrix $A$ is shown for several scales of EMG amplitudes. Each input set was formed from the concatenation of two simulated contractions. In the first contraction, extensor EMG amplitude was linearly ramped in time from zero to the maximum agonist EMG amplitude shown, while flexor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude shown. In the second contraction, flexor EMG amplitude was linearly ramped in time from zero to the maximum agonist EMG amplitude shown, while extensor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude. EMG amplitude ramps were simulated as five seconds in duration, sampled at 2048 Hz. Results for both standard and orthogonal polynomials are provided.
<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Standard Polynomial of Order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.02</td>
<td>2.5 131 6925 &gt; 10^5 &gt; 10^5 &gt; 10^5</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>2.5 28.9 330 3809 44300 &gt; 10^5</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>2.5 18.7 128 873 5940 40314</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2.5 31.8 299 2712 25094 &gt; 10^5</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2.5 59.7 1026 15664 &gt; 10^5 &gt; 10^5</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Maximum Agonist EMG Amplitude</th>
<th>Maximum Antagonist EMG Amplitude</th>
<th>Condition Number for Orthogonal Polynomial of Order:</th>
</tr>
</thead>
<tbody>
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<td>0.08</td>
<td>0.02</td>
<td>2.5 276 23973 &gt; 10^5 &gt; 10^5 &gt; 10^5</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>2.5 25.6 264 3722 28485 &gt; 10^5</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>2.5 4.3 12.5 42.9 106 253</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2.5 72.3 1478 73887 &gt; 10^5 &gt; 10^5 &gt; 10^5</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2.5 186 9414 &gt; 10^5 &gt; 10^5 &gt; 10^5</td>
</tr>
</tbody>
</table>

**Figure 9.5:** Effect of EMG Amplitude Scale on Condition Number — Three Contractions

The condition number for the matrix $A$ is shown for several scales of EMG amplitudes. Each input set was formed from the concatenation of three simulated contractions. In the first contraction, extensor EMG amplitude was linearly ramped in time from zero to the maximum agonist EMG amplitude shown, while flexor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude shown. In the second contraction, flexor EMG amplitude was linearly ramped in time from zero to the maximum agonist EMG amplitude shown, while extensor EMG amplitude was ramped from zero to the maximum antagonist EMG amplitude. In the third contraction, both extensor and flexor EMG amplitudes were linearly ramped in time from zero to the maximum agonist EMG amplitude. EMG amplitude ramps were simulated as five seconds in duration, sampled at 2048 Hz. Results for both standard and orthogonal polynomials are provided.
9.3.3 Discussion of Inputs to Identification Technique

Although the studies presented above were small in scope, they did provide insight as to the influence of co-contraction, the time course of the inputs, and the scaling of the inputs upon the conditioning of the EMG to torque identification problem. Figures 9.2 and 9.3 indicate that the condition number of the matrix $A$ rose progressively with the degree of co-contraction. In fact, when there was complete co-contraction (maximum agonist EMG amplitude equaled maximum antagonist EMG amplitude), the problem was ill-conditioned. When there was no co-contraction, the problem was well-conditioned. Experimentally, the elimination of co-contraction may not be possible. (If the elimination of co-contraction were possible, then identification of the EMG to torque relationship could be performed individually for the extensor and flexor muscle groups.)

The results of the first three investigations suggested that two contractions should be concatenated to form an input data set. Any single contraction did not seem to sufficiently span the two-dimensional input space of extensor and flexor EMG amplitude. A third contraction which explicitly modeled co-contraction did not improve, and even slightly degraded, the conditioning of the problem. The two contractions should be comprised of one extension dominant contraction and one flexion dominant contraction, with a minimum of co-contraction. Requiring both extension and flexion dominant contractions can be elucidated by the following counter-example. Imagine that identification was attempted from contraction in which only a net extension torque was measured about the elbow (only an extension dominant contraction). Since no flexion torque was measured about the elbow, the minimum norm identification technique would assign the flexion torque a value of zero, regardless of the time course of flexion EMG am-
plitude. A minimum norm identification technique would, accordingly, assign all flexor fit parameters a value of zero. Clearly, the identification technique would have failed.

The final two investigations probed the scaling of EMG amplitude data. Because the matrix A was comprised of non-linear functions of the EMG amplitude, scaling the data could alter the conditioning of the problem. Scaling was investigated assuming that the two contraction scheme was adopted, and that co-contraction could only be minimized such that the maximum agonist EMG amplitude was four times the maximum antagonist EMG amplitude. The results show that, over the short range investigated, the conditioning was best for EMG amplitude between approximately zero and one. Fortunately, this was the existing scaling for optimal estimators whose filters were calibrated from high contraction levels.

These simulation results provided two additional insights. First, the orthogonal polynomials provided for a more conditioned problem than did the standard polynomials. This result was expected, since this basis was designed to be orthogonal over the interval \([0, 1]\), while the standard polynomial basis was not orthogonal over this interval. Second, the conditioning of the problem deteriorated as the order of the model increased. The extra degrees of freedom in the model added to the sensitivity of the identification procedure.
9.4 Performance of EMG to Torque Identification Technique

9.4.1 Simulation Design

Since an evaluation of the complete EMG to torque identification technique was desired, the EMG waveform and the measured joint torque were simulated from a model of the EMG to torque relationship and assumed EMG amplitude inputs. The simulation model is shown in Figure 9.6. Each surface EMG waveform was formed by multiplying an EMG amplitude signal by a zero mean, unit intensity, WSS, Gaussian noise process. These processes were formed via the same technique described in Chapter 8. Note that although the EMG amplitude did change in time, the change was slow, and thus, all signals were considered quasi-stationary. The total moment about the elbow could be determined from the EMG amplitudes and the assigned EMG to torque relationship. Extensor and flexor EMG amplitude, and the EMG to torque relationship were the inputs to this simulation. The outputs were the extensor and flexor EMG waveforms, and the total joint torque.

9.4.2 Simulation Methods

All trials were created as the concatenation of two contractions. Each contraction was simulated as five seconds in duration, sampled at 2048 Hz. As recommended by the results of the previously reported simulation, one contraction was dominant in extensor EMG amplitude, the other dominant in flexor EMG amplitude. Co-contraction was assumed to exist. With these restrictions, two different schemes of EMG amplitude input were used — linear change in time of EMG amplitude and non-linear change in time of EMG amplitude. For
Surface EMG waveforms \((m_E, m_F)\) were formed from an EMG amplitude \((s_E, s_F)\) multiplied by Gaussian noise. The measured joint torque \((T_{\text{ext}})\) was a function of the EMG amplitudes and the relationship between EMG and torque.
the linear change in EMG amplitude, the first contraction consisted of extensor EMG amplitude ramping linearly in time from 0.8 to 0, while flexor EMG amplitude ramped linearly in time from 0.2 to 0. The second contraction consisted of extensor EMG amplitude ramping linearly in time from 0 to 0.2, while flexor EMG amplitude ramped linearly in time from 0 to 0.8. A sample set of EMG amplitudes derived from this specification is shown in Figure 9.7. For the non-linear change in EMG amplitude, the first contraction consisted of extensor EMG amplitude ramping in a cubic fashion in time from 0.8 to 0, while flexor EMG amplitude ramped in a cubic fashion in time from 0.2 to 0. The second contraction consisted of extensor EMG amplitude ramping in a cubic fashion in time from 0 to 0.2, while flexor EMG amplitude ramped in a cubic fashion in time from 0 to 0.8. A sample set of EMG amplitudes derived from this specification is also shown in Figure 9.7.

Total joint torque was computed from the EMG amplitudes and an EMG to torque relationship. All EMG to torque relationships were constructed in the form of a standard polynomial. Polynomials of order 1–3 were investigated. For the first order standard polynomials, five sets of fit coefficients were used for simulations. The extensor and flexor coefficients, respectively, were (1000, 1000), (800, 1200), (1200, 800), (1000, 600), and (600, 1000). These coefficients produced torques typically in the range 0–600. If the unit of torque measure was an A/D count, then the numeric range of simulated torque was similar to the expected range of experimental data measured from the A/D converter. For the second order EMG to torque relationship, a quadratic (no linear term) relationship was used. The same extensor and flexor coefficients listed above were applied to the quadratic terms. For the third order EMG to torque relationship,
Figure 9.7: Simulation EMG Waveforms

The top two plots are simulated extension and flexion EMG waveforms in which the EMG amplitude changed in a linear fashion in time. The bottom two plots are simulated extension and flexion EMG waveforms in which the EMG amplitude changed in a non-linear fashion in time.
a cubic (no linear or quadratic terms) relationship was used. Again, the same extensor and flexor coefficients listed above were applied to the cubic terms. In all, then, 15 simulations per ramp style, or a total of 30 simulations, were performed.

Once the simulated total joint torque and EMG waveforms were created, identification of the underlying EMG to torque relationship was attempted. EMG amplitude estimates were formed by simply using a moving average RMS estimator (each single channel of EMG was already white), with the standard window length of 500 samples (approximately 244ms, simulated). This window length was short enough, compared to the rates of change of EMG amplitude, for the EMG estimators to be considered quasi-stationary. The EMG amplitude estimates were fitted to the simulated total joint torque with both standard and orthogonal polynomial basis functions, of order 1–6. Least squares estimation of the fit parameters, using the method of singular value decomposition, was accomplished via the algorithm of Press et al. (1988, Section 14.3). Within the singular value decomposition method, any linear combination of basis functions whose corresponding singular value was greater than $10^5$ was not allowed to contribute to the estimate of the fit parameters.

Finally, a performance measure for the EMG amplitude to torque estimation was needed. To construct a measure, estimated extensor and flexor EMG amplitudes, combined with an EMG to torque relationship estimate, could be used to estimate the total joint torque, $T_{ext}$, at each instant in time. From the known (or in the experimental case, measured) total joint torque, a standard error (SE) performance measure was computed as

$$SE = \sqrt{\frac{\sum_{i=1}^{N} (\hat{T}_{ext_i} - T_{ext_i})^2}{N}}$$
where $N$ was the total number of simulated samples. A large SE denoted a large estimator error, while a small SE denoted a small estimation error.

Additionally, these simulations could investigate how accurately the individual (extensor, flexor) EMG amplitude to torque relationships had been identified. (No such measure was possible for the experimental data.) To do so, the true and estimated extensor and flexor torques were each evaluated at 10240 evenly spaced values between 0 and 0.8 (representing the complete range of input EMG amplitudes). A combined mutual SE could then be computed as

$$
\text{Combined Mutual SE} = \sqrt{\frac{\sum_{i=1}^{10240} (\hat{t}_{E_i} - t_{E_i})^2 + \sum_{i=1}^{10240} (\hat{t}_{F_i} - t_{F_i})^2}{2 \cdot 10240}}
$$

### 9.4.3 Simulation Results

Figure 9.8 shows identification results for one of the quadratic model EMG to torque simulations. Extension and flexion EMG to torque relationships (true and identified) are shown for second, fourth and sixth order standard polynomial fits. Figure 9.9 shows similar identification results for one of the cubic model EMG to torque simulations. Figure 9.10 provides the average ± standard deviation (A±SD) of the SE's between the true and estimated total joint torque. Figure 9.11 simultaneously plots the identification error and the joint torque estimation error for the third order model with linear change in EMG amplitude. Results from the linear and non-linear change in EMG amplitude are segregated into two charts. The results for standard and orthogonal polynomial basis functions are almost identical (at least to two significant figures). Figure 9.12 provides the A±SD of the SE's of the combined mutual errors between the true and estimated torque for individual (extensor, flexor) muscle groups. Results from the linear and non-linear change in EMG amplitude are segregated
into two charts. Again, the results for standard and orthogonal polynomial basis functions are identical to two significant figures. Lastly, Figure 9.13 lists the A±SD condition numbers for the standard and orthogonal polynomial basis fits. Results from linear and non-linear change in EMG amplitude have been combined in each entry.

9.4.4 Discussion of EMG to Torque Identification Performance

Although this study was brief, important trends appeared in the data. As the fit order was increased towards the true model order, errors in the estimation of the total joint torque as well as the individual (extensor, flexor) torque contributions decreased rapidly. (This result was not as distinct for the “third order” model since a pure cubic function, over the interval 0 to 0.8, can be well represented by a second order polynomial.) After the true model order was reached, progressively higher order fits provided limited improvement in the estimation of total joint torque. Clearly, any least squares problem would be expected to reduce fit error with an increase in fit order. But, in this case, improvement in total joint torque estimation with excessive model order (orders above the true model order) was identification of artifact in the particular input data set — likely noise from the EMG amplitude estimation. Further, Figure 9.12 shows that excessive fit order slightly degraded estimation of the individual torque contributions. In the experimental situation, therefore, a reduction in total joint torque estimation error with increased fit order might be misleading. The above trends suggested that appropriate fit order was reached when progressively higher order fits provided only limited improvement in the total joint torque fit error.

Although the condition numbers for standard versus orthogonal basis func-
Figure 9.8: Identification of a Quadratic Simulated Model

A quadratic standard polynomial relationship from extension and flexion EMG amplitude to each muscle group’s respective contribution to joint torque was assumed. Solid lines in the plots are the true EMG amplitude to torque relationships. Dashed lines are the estimated relationships. Extension and flexion results are shown for second, fourth and sixth order standard polynomial fits. EMG amplitude was ramped linearly.
Figure 9.9: Identification of a Cubic Simulated Model

A cubic standard polynomial relationship from extension and flexion EMG amplitude to each muscle group's respective contribution to joint torque was assumed. Solid lines in the plots are the true EMG amplitude to torque relationships. Dashed lines are the estimated relationships. Extension and flexion results are shown for second, fourth and sixth order standard polynomial fits. EMG amplitude was ramped linearly.
Linear Change in EMG Amplitude:

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<td>3</td>
<td>12.7 ± 1.5</td>
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<td>13.0 ± 1.2</td>
</tr>
<tr>
<td>5</td>
<td>11.4 ± 1.2</td>
<td>14.2 ± 3.1</td>
<td>12.7 ± 1.2</td>
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<td>6</td>
<td>10.9 ± 1.2</td>
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<td>12.4 ± 1.3</td>
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Non-Linear Change in EMG Amplitude:

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<td>7.0 ± 0.9</td>
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<td>5.7 ± 0.8</td>
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<tr>
<td>6</td>
<td>5.0 ± 0.5</td>
<td>5.9 ± 1.1</td>
<td>5.6 ± 0.8</td>
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**Figure 9.10: Standard Errors in Simulation Fits**

Average ± standard deviation errors are shown between the true and estimated total joint torque. Errors, expressed in units of simulated A/D units, are provided for both the linear and non-linear change in EMG amplitude trials. Results are for both standard and orthogonal polynomial basis functions.
**Figure 9.11: Simulation Standard Error Plot**

The mean and standard deviation standard error, averaged across 5 simulated EMG to torque relationships, are presented. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid circles denote standard errors with respect to the true EMG to torque relationship for each muscle group. Hollow circles denote standard errors with respect to joint torque. Results are for the third order model with linear change in EMG amplitude.
Linear Change in EMG Amplitude:

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<td>3</td>
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<td>16.3 ± 0.8</td>
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<td>15.0 ± 3.4</td>
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<td>16.5 ± 2.3</td>
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<td>16.5 ± 3.8</td>
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Non-Linear Change in EMG Amplitude:

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<td>26.6 ± 1.6</td>
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<td>40.3 ± 4.8</td>
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<td>26.4 ± 3.0</td>
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<td>5</td>
<td>30.2 ± 2.7</td>
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<td>45.6 ± 8.2</td>
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<td>6</td>
<td>31.3 ± 3.8</td>
<td>50.3 ± 17.2</td>
<td>50.7 ± 10.2</td>
</tr>
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</table>

Figure 9.12: Combined Mutual Standard Errors in Simulation Fits

Average ± standard deviation combined mutual errors are shown between the true and estimated torque for individual (extensor, flexor) muscle groups. Errors, expressed in units of simulated A/D units, are provided for both the linear and non-linear change in EMG amplitude trials. Results are for both standard and orthogonal polynomial basis functions.
### Standard Polynomial Fits:

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<th>Model Order = 2</th>
<th>Model Order = 3</th>
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<td>1.7 ± 0.0</td>
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<td>2</td>
<td>11.5 ± 0.8</td>
<td>11.4 ± 0.8</td>
<td>11.5 ± 0.8</td>
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<tr>
<td>3</td>
<td>78.5 ± 7.3</td>
<td>76.9 ± 7.9</td>
<td>79.7 ± 6.2</td>
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<td>4</td>
<td>550 ± 67.6</td>
<td>534 ± 52.5</td>
<td>565 ± 46.6</td>
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<td>5</td>
<td>3869 ± 500</td>
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<td>6</td>
<td>26832 ± 2834</td>
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### Orthogonal Polynomial Fits:

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<tbody>
<tr>
<td>1</td>
<td>1.7 ± 0.0</td>
<td>1.7 ± 0.0</td>
<td>1.7 ± 0.0</td>
</tr>
<tr>
<td>2</td>
<td>4.7 ± 0.3</td>
<td>4.5 ± 0.4</td>
<td>4.6 ± 0.4</td>
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<tr>
<td>3</td>
<td>13.7 ± 2.3</td>
<td>12.5 ± 2.1</td>
<td>13.6 ± 2.7</td>
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<tr>
<td>4</td>
<td>66.9 ± 19.3</td>
<td>61.6 ± 23.0</td>
<td>71.0 ± 27.1</td>
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<td>5</td>
<td>192 ± 61</td>
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<td>6</td>
<td>539 ± 211</td>
<td>512 ± 268</td>
<td>631 ± 307</td>
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### Figure 9.13: Condition Numbers in Simulation Fits

Average ± standard deviation condition numbers for the simulation fits are provided for both standard and orthogonal basis functions. Results from linear and non-linear change in EMG amplitude have been combined in each entry.
tions were clearly different, the SE's for the two different basis functions were essentially the same. Two comments resolve this apparent inconsistency. First, evaluated at each input vector, the two fit styles must be equivalent since the orthogonal basis was comprised of linear combinations of the standard basis (i.e. the minimum norm best fit for the two bases must be identical at all specified input vectors). The respective fit coefficients, however, would have different values. Second, although the two fit styles may interpolate/ extrapolate differently between/ from the specified input vectors, the overwhelming quantity of input data (compared to the model order) likely provided little flexibility between the styles. The maximum sixth order fits were fit at 10240 locations. Thus, no effective performance difference between the standard and orthogonal basis fits was found.

The results of the linear versus non-linear change in EMG amplitude differed in two aspects. First, estimation of the total joint torque from the non-linear change data was approximately twice as good as from the linear change data. Second, estimation of the individual torque contributions from the non-linear change data was approximately one half as good as from the linear change data. Thus, poorer identification was not always accurately reflected in the total joint torque fit error. Errors in identifying the individual torque contributions could cancel to improve the total joint torque fit for a particular input data set. Further, the performance of this identification algorithm seemed to be influenced by the time course of the EMG amplitude. In the experimental case, therefore, trial to trial repeatability of the estimated EMG to torque relationship was likely to be related to the trial to trial repeatability of the EMG amplitude.

Because a single white channel of the EMG waveform was simulated for
each muscle group, EMG amplitude estimation should have had an SNR performance of approximately 31.6 (based on a smoothing window length of 500 samples). This level of performance is certainly attainable experimentally if 6–8 channels per muscle group are utilized. The linear change in EMG amplitude, with co-contraction at a ratio of 4:1, produced estimates of individual torque contributions with SE’s of between 15 and 20 simulated A/D counts, for the appropriate fit order. Since the individual torque contributions had a range of approximately 500 A/D counts, the SE’s were approximately 3–4% of the range. The simple identification technique, then, may well be accurate enough for many practical uses of EMG to torque estimation.

9.5 Conclusion

Two aspects of the EMG to torque identification problem were briefly studied in simulation. A first study investigated appropriate input data for conditioning the identification problem. The results suggested that two contractions, one dominant in extensor EMG amplitude and the other dominant in flexor EMG amplitude, should be concatenated to form an input data set. Co-contraction should be minimized. The EMG amplitudes from these contractions should be normalized approximately over the range 0–1. The second study investigated the performance of the identification technique. The results suggested that proper fit order was achieved when progressively higher order fits provided only limited improvement in the total joint torque fit error. Overfitting the data by one or two orders of magnitude slightly degraded performance. The orthogonal polynomial basis functions performed no better than the standard polynomial basis functions, but the orthogonal basis was much better conditioned. Trial to trial
repeatability of the estimated EMG to torque relationship was influenced by the trial to trial repeatability of the EMG amplitude. The identification technique provided an accurate estimate of the individual simulated EMG to torque relationships.
Chapter 10

Experiment 3 —
Estimation of Joint Torque from the EMG Amplitude
10.1 Introduction

This chapter reports on an experimental study which estimated joint torque from the surface EMG waveform. The focus of the study was to compare the performance of single versus multiple channel and whitened versus unwhitened EMG amplitude estimators. The whitened amplitude estimators were designed based on the results of the Experiment 1. In addition to the above estimators, an adaptive whitening estimator was introduced. It was found that when contraction was lower than 10% MVC (a case not studied in the first two experiments), the whitened amplitude estimators did not perform as well as the unwhitened estimators. The adaptive whitening estimator attempted to incorporate the benefits of whitening for contractions above 10% MVC, while minimizing the cost for contractions below 10% MVC.

Multiple channels of the MSEMGA waveform were recorded from both muscle groups (elbow flexors and extensors) during isometric, quasi-isotonic (slowly varying) contraction about the elbow. As recommended by the previously described simulation, subjects produced contraction which alternated between extension dominance and flexion dominance, with minimal co-contraction. EMG amplitude estimates were computed and then fitted to the measured joint torque via standard and orthogonal polynomial basis functions. Prediction of joint torque from the polynomial fits was evaluated. As before, all amplitude estimators incorporated a fixed smoothing window of approximately 245ms.
10.2 Experimental Methods

Three subjects, two male and one female, ranging in age from 26 to 29 years, participated in three experiments. Subjects had no known neuromuscular deficits of the right shoulder, arm or hand. A subject was seated and secured into the instrumented chair. Five electrode-amplifiers were mounted over the elbow flexors, and five electrode-amplifiers were mounted over the elbow extensors, located as described in Chapter 5. Electrode-amplifiers zero through four were placed medial to lateral, respectively, across the elbow flexors. Electrode-amplifiers five through nine were placed medial to lateral, respectively, across the elbow extensors. During an experimental trial, the subject was instructed to lift his/ her right elbow above the pivot plate and to contact the instrumented beam only via the wrist cuff. The output voltage of the strain gauge circuit was presented on an oscilloscope to the subject. A target torque trajectory was also presented on the oscilloscope display. The subject attempted to maintain a consistent posture during and throughout all trials, so as to present repeatable contraction tasks. The subject tried to relax all muscles not directly involved in flexion/ extension about the elbow. By observing the oscilloscope display, the subject followed the target torque trajectory. For all trials, the oscilloscope gain was selected as the maximum possible gain. This gain selection maximized the available sensitivity of the oscilloscope display.

Initially, EMG data were recorded with the arm completely relaxed. (For this trial only, the subject relaxed his/ her right elbow on the pivot plate.) Then, isometric, isotonic contractions were recorded. These contractions calibrated the filters used in optimal amplitude estimation. Subjects produced one maximal
extension contraction, followed by one maximal flexion contraction. The maximal contraction trials provided a rough estimate of MVC, sufficient for a coarse gradation of contraction levels. All subsequent trials were calibrated as a percent of these MVC determinations. A 50% MVC extension target torque level was presented to the subject as the second display of a two channel oscilloscope. The subject was instructed to begin at rest, then, over a time period comfortable to the subject (typically 0.5–1s), gradually increase extension torque until the target torque level was achieved. By observing the oscilloscope display, the subject maintained the target torque level until a five second segment of data was recorded at a sampling rate of 2048 Hz. Two recordings were made corresponding to 50% MVC extension. Then, the target torque level was set for 50% MVC flexion, and two similar trials recorded. A rest period of three minutes after maximal contractions and two minutes after 50% MVC trials was provided. Between contractions, the subject was released from the wrist cuff to prevent impaired blood flow to/from the hand.

Next, a sequence of ten isometric, quasi-isotonic contractions was conducted. A thin straight segment of opaque tape was taped onto the oscilloscope display, marking a trajectory which began at 50% MVC extension and ended at 50% MVC flexion. The time base of the oscilloscope was set such that the strain gauge trace moved across the taped portion of the screen in approximately eight seconds. The second oscilloscope display channel was removed. The subject was instructed to begin at rest, then, over a time period comfortable to the subject (typically 0.5–1s), gradually increase extension torque to a level greater than 50% MVC. This contraction was timed such that after approximately one second of greater than 50% MVC extension, the oscilloscope time base was at
the beginning of the thin tape trajectory. As time progressed, the subject then maintained the strain gauge output at a level corresponding to the trajectory specified by the segment of tape. In this manner, the subject followed an eight second elbow joint torque trajectory which linearly ramped in time from greater than 50% MVC extension to 0% MVC extension/ flexion to greater than 50% MVC flexion. This torque trajectory provided a single task which included both extension and flexion dominant contraction. (The results of the EMG to torque simulation suggested that this torque trajectory could be considered quasi-isotonic. The experimental results of Lawrence and DeLuca, 1983, also support a quasi-isotonic assumption for this contraction method.) The ten EMG channels and the strain gauge output were recorded for ten seconds, centered about the eight second ramp task. All data were sampled at 2048 Hz. The trajectory tracking task was performed ten times. A rest period of four minutes after each tracking task was provided. Between contractions, the subject was released from the wrist cuff to prevent impaired blood flow to/ from the hand.

10.3 Methods of Analysis

Initially, four EMG amplitude estimators were constructed. 1) A single channel unwhitened amplitude estimator was formed as the simple MAV processor. 2) A single channel whitened amplitude estimator was formed by temporally whitening each data record, followed by MAV detection. A single five coefficient whitening filter was constructed for each electrode-amplifier for each subject from one of the five second isometric, isotonic contractions (at the 50% MVC level) per subject, and applied to all isometric, quasi-isotonic trials recorded by that electrode-amplifier. 3) A multiple channel unwhitened amplitude estimator
was formed by equalizing the variance of each channel (based on the 50% MVC isometric, isotonic calibration trial), then performing spatial-temporal MAV detection. 4) A multiple channel whitened amplitude estimator was formed by temporally whitening each channel, equalizing the variance of each channel, then performing spatial-temporal MAV detection. Spatial uncorrelation was not performed, since, based on Experiment 1, it provided little performance improvement. Whitening filters were the same as in the single channel case. These whitening filters inherently equalized the channel variances.

All of the EMG amplitude estimators used the MAV detector, since, in Experimental 1, it was found to perform better than the RMS detector. All EMG amplitude estimators incorporated a fixed smoothing window of approximately 245ms. As recommended by the results of the EMG to torque simulations, all EMG amplitude data were scaled to span the approximate range of [0,1]. In particular, all EMG amplitude estimators had a nominal value of one at 50% MVC.

Because each contraction trial for a particular subject produced a different range of total joint torques, an attempt was made to standardize the range of joint torques. The measured torque data from all ten trials were plotted. A negative maximum and positive maximum value achieved by all plots were determined visually. Individual torque plots were then reviewed and a beginning and end segment, corresponding to data less than the negative maximum torque and greater than the positive maximum torque, respectively, were noted. (Because the experimental contraction ramped from high extension to high flexion, the beginning data were less than the negative maximum torque and the end data were greater than the positive maximum torque.) After EMG amplitude estima-
tion, the beginning and end segments of data were discarded from the extensor EMG amplitude estimate, flexor EMG amplitude estimate and the measured total joint torque. The edited data were then ready for least squares fitting.

Least squares fitting of the EMG amplitudes (extensor and flexor) to standard and orthogonal polynomial basis functions was accomplished with the singular value decomposition method of Press et al. (1988, Section 14.3). As before, any linear combination of basis functions whose corresponding singular value was greater than $10^8$ was not allowed to contribute to the estimate of the fit parameters. Fit performance was measured as the SE between the measured total joint torque and that found by evaluating the fit at the estimated input extensor and flexor EMG amplitudes. EMG to torque prediction performance was also evaluated by calibrating an EMG to torque relationship from one experimental trial, and then applying that relationship to all experimental trials for that subject. Since each subject performed ten trials, a total of 100 SE's per subject could be computed in this manner for each EMG amplitude estimator style, each polynomial basis style, and each model order. Because the range of EMG amplitudes varied from trial to trial, an attempt was made to prevent an EMG to torque prediction from having to perform an extrapolation. Thus, calibration of EMG to torque fits were derived from the complete segment of data made available to the fit routines, but 500 additional samples (245ms) were deleted from the beginning and end of a data segment prior to a prediction trial. (The beginning and end segments of data corresponded to the large, and possibly out of range, EMG amplitudes.) Performance differences between results were evaluated for statistical significance by paired t-tests and unequal variance t-tests, as appropriate (Press et al., 1988, Section 13.4).
10.4 Surface EMG in Additive Gaussian Noise

10.4.1 The Problem of Additive Gaussian Noise

Preliminary EMG to torque results suggested that whitened EMG amplitude estimation performed more poorly than unwhitened EMG amplitude estimation for contractions lower than 10% MVC. This problem was not discovered during Experiments 1 and 2 because they did not study contractions less than 10% MVC. The problem was first noted when preliminary single channel EMG to torque performance results found a lower fit SE for unwhitened EMG amplitude estimators than for whitened EMG amplitude estimators. Consequently, EMG amplitude estimates were evaluated graphically. Figure 10.1 shows whitened and unwhitened EMG amplitude estimates both before and after MAV detection. This figure illustrates several points. First, when contraction was low, the whitened EMG waveform seemed to contain a bias, or offset, variance. This offset was reflected in the corresponding amplitude estimate. Second, the whitened amplitude estimate did not seem to differentiate low EMG amplitudes. The un-whitened (raw) EMG signal clearly had a larger amplitude at time=5.0s than at time=2.5s. This amplitude difference was well represented by the unwhitened amplitude estimate, but was barely noticeable in the whitened amplitude estimate. And third, when contraction was large, the whitened estimate seemed smoother than the unwhitened estimate. This third result was consistent with the results of Experiments 1 and 2.

10.4.2 A Surface EMG Model With Additive Noise

All of the observations illustrated in Figure 10.1 were consistent with a surface EMG waveform model which included additive noise. Consider the PSD’s drawn
Figure 10.1: Single Channel EMG Amplitude Estimators

The plots on the left are the raw EMG waveform (top) and its MAV detector amplitude estimate (bottom). The plots in the middle are the corresponding temporally whitened EMG waveform (top) and its MAV detector amplitude estimate (bottom). The plots on the right are the corresponding adaptively temporally whitened EMG waveform (top) and its MAV detector amplitude estimate (bottom). All data are single channel EMG from elbow extensors.
schematically in Figure 10.2. The solid line plots represent the PSD for three different EMG amplitudes. The plots are identical in shape, differing only in scale. The dashed line represents a white background noise. Assume the MSEM signal was a sum of a true EMG signal (solid line) and the background noise (dashed line). When the EMG amplitude was large (top solid line), the signal PSD was always greater than the noise PSD. (Since the sampling rate was 2048 Hz., the PSD was shown up to 1024 Hz.) But, when the EMG amplitude was small (bottom solid line), noise PSD exceeded signal PSD in the high frequencies.

This additive noise description can explain the points illustrated by Figure 10.1. First, a bias variance would exist due to the background noise. Second, note that temporal whitening selectively accentuates the high frequencies in the data. For low amplitude EMG, there is more noise than signal at high frequencies. Thus, the influence of noise dominates the output of the temporal whitening filter. Since the noise is constant, there is little differentiation between low EMG amplitudes. The relative contribution of the background noise to the EMG amplitude estimate progressively increases as the EMG amplitude decreases. Conversely, addressing the third point, as EMG amplitude increases, the relative effect of the background noise diminishes. In fact, the results of Experiment 1 suggested that temporal whitening over a 1024 Hz. bandwidth was appropriate for contractions at or above 10% MVC.

The source of the background noise was investigated by observing the measured EMG signal when an electrode-amplifier was secured to the skin above resting muscle (0% MVC) and when the two electrodes of the electrode-amplifier were shorted together. Approximately 40% of the RMS noise was attributed to all of the EMG electronics (electrode-amplifiers, gain amplifiers, A/D). The re-
Figure 10.2: Power Spectrum of Amplitude Modulated EMG

The solid line plots are identical in shape, differing only in amplitude. They represent the amplitude modulated EMG model. The dashed line represents a white background noise.
maining 60% of the RMS noise seemed to be associated with electronic noise on the skin surface and within the body (including any electrode-skin interaction, i.e. thermal noise due to the flow of currents into the electrode-amplifier). The total RMS noise in the acquired EMG signal was only approximately 20mV. Since the electronics (particularly the electrode-amplifiers) were state-of-the-art, it was assumed that little could be accomplished to reduce the background noise.

An EMG waveform model which included additive noise was proposed. The single channel model is shown in Figure 10.3. The model parallels the single channel model presented in Chapter 4, except an additive noise term is represented. The noise term is a zero mean, WSS, CE, jointly Gaussian noise process independent of the process $w_t$.

To investigate the new EMG waveform model, the effect of additive noise upon the PSD was simulated in Figure 10.4. Figure 10.4 shows an EMG PSD estimate with varying degrees of simulated additive white Gaussian noise. As the level of noise was increased, the PSD was altered in two fundamental manners. First, over the 1024 Hz. frequency band, the range of the signal power decreased sharply. Second, the high frequency end of the spectrum altered shape and became flat.

These two alterations were then compared to experimental data. Figure 10.5 shows EMG PSD estimates from isometric, isotonic contractions at 10, 25, 50 and 75% MVC. These data were from the Experiment 1. As the contraction level was decreased, the range of the signal decreased. The PSD at 75% MVC spans approximately five orders of magnitude, while the PSD at 10% MVC spans approximately three orders of magnitude. Also, there appear to be some differences in the shapes of the PSD’s, with a flattening of the high frequency
A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity $w_i$ is passed through the stable, causal, inversely stable, inversely causal, linear, time-invariant shaping filter $H_{time}(e^{jw})$, multiplied by the measured surface EMG amplitude $s$, and added to the zero mean, WSS, correlation-ergodic (CE), jointly Gaussian noise process $v_i$ to form the measured surface EMG waveform. The processes $w_i$ and $v_i$ are assumed to be independent and jointly Gaussian. The measured surface EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure 10.4: Simulated EMG Power Spectrum With Additive White Gaussian Noise

Top left plot shows an EMG PSD, estimated by a five coefficient autoregressive model. Remaining plots are the PSD estimate after adding varying degrees of simulated white Gaussian noise to the original EMG waveform. Noise is expressed in percent of the maximum value of the PSD of the original EMG waveform.
Figure 10.5: EMG Power Spectra at Different Contraction Levels

Discrete Fourier Transform power spectral density estimates from isometric, isotonic contractions at 10, 25, 50 and 75% MVC are shown.
region at the 10% MVC level. Thus, it was concluded that the experimental data were consistent with an EMG waveform model which included an additive noise term.

10.4.3 An Adaptive EMG Amplitude Estimator

Since the data from Experiment 1 were consistent with an additive noise model, it was desired to investigate solutions to the additive noise model. If the PSD of the EMG waveform and the PSD of the additive noise were identical, then an ML solution to the model of Figure 10.3 could be found. Appendix C shows that the MSEM signal, $m_i$, should first be whitened. Because both the true EMG waveform, $x_i$, and the additive noise, $v_i$, have the same PSD, both processes are whitened by the same fixed temporal whitening filter. If the output of the temporal whitening filter was denoted $y_i$, then optimum ML estimation of $s$ would be completed as

$$\hat{s}_i = \left[ \frac{1}{N} \sum_{j=0}^{N-1} y_{i-j}^2 - q \right]^2$$

where $q$ was the variance of the additive noise process after whitening. Note that if $\frac{1}{N} \sum_{j=0}^{N-1} y_{i-j}^2 < q$, then the optimal estimate was $\hat{s}_i = 0$.

Experimentally, however, it appeared that the EMG waveform and the additive noise did not have the same PSD — the EMG waveform PSD characteristically peaks near 100 Hz. and then decays, while the additive noise appeared to be white. Unfortunately, formal optimal solutions to the model of Figure 10.3 were not readily apparent in this case. Further, if an analytic solution exists, it likely contains some sort of adaptive filter. Formal development of such an adaptive filter was beyond the scope of this project.

Thus, an ad hoc adaptive filter was proposed. The goal of the adaptive
filter was to implement, in a simple strategy, an EMG amplitude estimator which would simultaneously preserve the benefits of temporal whitening for high contraction and the benefits of unwhitened estimation for low contraction. Such a filter would prove, in principle, the merits of a complete, formal solution to the additive noise model. Justification would then exist for the presumably more extensive effort required to develop a formal solution.

The argument provided thus far suggested that an adaptive filter should attempt not to disturb the previously developed temporal whitening procedures at high contractions, but should limit the high frequency gain of temporal whitening at low contractions. It did not make sense to equally weight the contributions from all frequencies when the ratio of EMG signal to additive noise was not the same at all frequencies. Thus, to perform adaptive whitening, the MSEMFG data were first whitened by a fixed linear filter, then adaptively low pass filtered. The adaptively whitened data were then MAV detected to form the EMG amplitude estimate. Figure 10.6 shows this processing scheme.

The adaptive low pass filter was designed to mimic a first order low pass filter. The bilinear transformation technique (Rabiner and Gold, 1975, Section 4.7) was used to design a digital filter. The desired analog filter response was

\[ H(j\omega) = \frac{\omega_c}{j\omega + \omega_c} \]

where \( \omega_c \) was the desired analog cutoff frequency in radians. The corresponding digital filter, via the bilinear transformation technique was

\[
H(e^{j\omega}) = H(j\omega)|_{j\omega=\frac{2}{T}\left(\frac{1-e^{-j\omega}}{1+e^{-j\omega}}\right)} = \frac{\left(\frac{-\omega_c T}{\omega_c T+2}\right) + \left(\frac{\omega_c T}{\omega_c T+2}\right) e^{-j\omega}}{1 - \left(\frac{2-\omega_c T}{2+\omega_c T}\right) e^{-j\omega}}
\]
Figure 10.6: Adaptive EMG Processor — Single Channel

To perform adaptive whitening, the data were first whitened by a fixed linear filter, then adaptively low pass filtered. The adaptively whitened data were then MAV detected to form the EMG amplitude estimate.

where $T$ was the sampling period. Denoting the output of the adaptive low pass filter as $y_i$ and the input as $u_i$, then the adaptive filter was implemented as the infinite impulse response filter

$$y_n = \left(\frac{\omega_c T}{\omega_c T + 2}\right)(u_n + u_{n-1}) + \left(\frac{2 - \omega_c T}{2 + \omega_c T}\right)y_{n-1}$$

Next, an adaptation scheme for the location of the low pass filter cutoff frequency, $f_c$, was developed. The cutoff frequency was allowed to change in a direct linear relation to the present EMG amplitude estimate, i.e.

$$f_c = \text{(slope)} \cdot \hat{s} + \text{(offset)}$$

The slope and offset of this linear relation were estimated experimentally as follows. A first order standard polynomial fit was made from one extension channel and one flexion channel using the adaptive processor. Initially, the
linear adaptation was set such that \( f_c = 25 \text{ Hz.} \) for \( \dot{s} = 0 \) and \( f_c = 5000 \text{ Hz.} \) for \( \dot{s} = 1 \). (Note that specifying two points was equivalent to specifying the slope and intercept.) The location of \( f_c \) for \( \dot{s} = 0 \) was then altered (in steps of approximately 100 Hz.) for the extension EMG channel until a local minimum in fit SE was achieved. Next, the location of \( f_c \) for \( \dot{s} = 1 \) was altered (in steps of 1000 Hz.) for the extension EMG channel until a local minimum in fit SE was achieved. Tuning then progressed to the flexion EMG channel. When the four successive minima had been located, the adaptive filter was specified. This entire process was performed four times, using data from two of the three subjects.

Although this minimization technique was ad hoc and may have been sensitive to the dynamics of \( \dot{s} \), two trends quickly developed in the results. First, the location of \( f_c \) for \( \dot{s} = 0 \) showed little variation and little influence on the fit SE. Thus, \( f_c = 25 \text{ Hz.} \) for \( \dot{s} = 0 \) was selected. Second, the location of \( f_c \) for \( \dot{s} = 1 \) showed a large variation from channel to channel. Closer evaluation of the data suggested that the variation was related to the intensity of additive noise in each EMG channel. Since experimental EMG data had been recorded when the subject was relaxed (0\% MVC), an estimate of additive noise intensity could be made for each EMG channel. The 0\% MVC data were, therefore, processed by the whitened EMG amplitude estimator, and the mean value of the output used as a measure of additive noise intensity. (The additive noise amplitude was estimated from the whitened amplitude estimator since the adaptive low pass filter was applied after fixed temporal whitening.) For the eight EMG channels studied, the additive noise amplitude was plotted versus the location of \( f_c \) for \( \dot{s} = 1 \). This plot is shown in Figure 10.7. The solid line in the plot shows the
Figure 10.7: Optimal Low Pass Cutoff Frequency at Unity EMG Amplitude

The frequency location of the optimal low pass cutoff frequency at unity EMG amplitude is shown versus the EMG noise amplitude for four pairs (agonist/antagonist EMG) of single channel EMG amplitude estimates. The optimal low pass cutoff frequencies were determined experimentally. Solid line shows the rule for assigning the unity EMG amplitude low pass cutoff frequency which was used by the adaptive EMG amplitude estimators.
relationship selected for choosing $f_c$ for $\delta = 1$. This relationship is

$$f_c = -25,000 \cdot (\text{EMG noise intensity}) + 12,500$$

for $\delta = 1$

The complete adaptation is shown in Figure 10.8.

Thus, for single channel adaptive EMG amplitude estimation, a fixed whitening filter was derived in the same manner as for the optimal single channel EMG amplitude estimator. Then, for each EMG channel for each subject, the additive noise intensity was estimated from a trial at 0% MVC. This intensity specified the slope of the adaptation for the data of that EMG channel. The output of the adaptive filter was MAV detected to form the EMG amplitude estimate.

For multiple channel adaptive EMG amplitude estimation, a complete single channel adaptive estimation was performed for each channel. The individual channel amplitude estimates were used only to program the adaptation for each respective adaptive filter. The multiple channel amplitude estimate was formed as a spatial-temporal MAV detection of the outputs of the adaptive filters.

### 10.5 Results

#### 10.5.1 Volume of Usable Data Collected

For each subject, data were collected from five flexion EMG channels, five extension EMG channels and the measured total joint torque. A sample set of data (only one extension/flexion EMG channel) is shown in Figure 10.9. Each data record from each subject was plotted and inspected. The usable range, in A/D units of measured torque data was (-600, 600) for subject AH, (-200, 250) for subject EB and (-400, 510) for subject GB, where negative torque denoted extension. As before, all data from any electrode-amplifier which saturated during
Figure 10.8: Adaptive Low Pass Filter Cutoff Frequency

Based on the EMG noise amplitude at 0% MVC, these lines show the location of the adaptive low pass filter cutoff frequency based on the present estimate of the EMG amplitude.
Figure 10.9: Data From Isometric, Quasi-Isotonic Contraction

A flexor EMG waveform (top left), and extensor EMG waveform (top right) and the corresponding total joint torque (bottom) are shown for an isometric, quasi-isotonic contraction.
Figure 10.10: Chart of Subjects for Experiment 3

The three subjects were denoted AH, EB and GB. Viable extension/ flexion electrode-amplifier numbers for each location are identified. The locations are: Lat = Lateral, Mid-Lat = Middle-Lateral, Mid = Middle, Mid-Med = Middle-Medial, and Med = Medial. Each pair corresponds to the extension electrode-amplifier number followed by the flexion electrode-amplifier number.

any portion of any experimental record were discarded from all further analysis. Viable electrode-amplifiers were grouped into extension/ flexion pairs as shown in Figure 10.10. In all, there were 13 viable electrode-amplifier pairs. For each single channel fit variation, 130 different fits could be performed. For multiple channel fits, all viable electrode-amplifiers were used to form an EMG amplitude estimate. Thus, for each multiple channel fit variation, 30 different fits could be performed.

10.5.2 Evaluation of EMG Crosstalk

Crosstalk was evaluated by computing the correlation coefficient between agonist/ antagonist EMG channels at various time lags. The channel located in the most lateral (middle, medial) position on the flexor muscle group was correlated with the channel located in the most lateral (middle, medial) position on the ex-
tensor muscle group. At each lag, the correlation coefficient was estimated from 4096 samples (2s), centered about the desired lag, with the time domain estimate method presented in Chapter 5. A time lag of zero samples corresponded to the 2560th sample (1.25th's) of the extension EMG channel. Negative lag referred to flexion EMG lagging extension EMG. Thus, the correlation coefficients were computed from the first 2.25s of an experimental contraction (extension dominant portion of an experimental contraction). Figure 10.11 shows plots of the correlation coefficient for various lags for a sample set of lateral, middle and medial electrode-amplifier pairs. Within the ±0.25s lag window, the average correlation coefficient, the average maximum correlation coefficient and the location of the average maximum correlation coefficient were computed. Averages across the 90 paired single channel records are given in Figure 10.12.

10.5.3 Fitting EMG to Torque

Figures 10.13 and 10.14 show extension and flexion, respectively, third order standard polynomial basis fits for the ten contractions from subject GB for the various EMG amplitude estimators. Figure 10.15 shows the second, fourth and sixth order standard polynomial basis fits, using the multiple channel adaptive EMG amplitude estimator, for the ten contractions from subject GB. Figures 10.16 and 10.17 show the SE's for the standard and orthogonal basis fits, respectively. Figure 10.18 simultaneously plots the unwhitened single channel and adaptive multiple channel fit errors for the standard polynomial basis. Figures 10.19 and 10.20 show the corresponding condition numbers for the fits. Because the standard and orthogonal basis fit results were almost identical, statistical tests were only performed on the results from the standard polynomial basis.
Figure 10.11: Correlation in Antagonist EMG Channel Pairs

The correlation coefficient at various lags is shown for pairs of extension/flexion EMG channels. Both electrode-amplifiers were placed on lateral, middle or medial regions of the muscles. Top plots show the correlation coefficients with up to a ±0.25s lag. Bottom plots show the correlation coefficients with up to a ±30ms lag.
| Electrode-Amplifier Locations | Average Maximum $|r|$ (Samples) | Location of Maximum $|r|$ (Samples) | Average $|r|$ |
|------------------------------|---------------------------------|-----------------------------------|----------------|
| Lateral                      | 0.38 ± 0.12                     | −0.6 ± 0.5                        | 0.03 ± 0.00    |
| Middle                       | 0.41 ± 0.15                     | 1.1 ± 3.7                         | 0.03 ± 0.00    |
| Medial                       | 0.40 ± 0.08                     | 2.3 ± 3.8                         | 0.04 ± 0.00    |

**Figure 10.12: Correlation Coefficients for Various Electrode-Amplifier Pair Locations**

The average maximum correlation coefficient, location of the average maximum correlation coefficient and the average correlation coefficient are shown when both (extension/ flexion) electrode-amplifiers were placed on lateral, middle or medial locations on the muscles. The correlation coefficient was calculated for lags of ±0.25s. The data were sampled at 2048 Hz. Each entry was the average ± standard deviation of 90 paired records.
Paired t-tests compared the results within the single/multiple channel categories. For single channel fit SE's, all differences in single channel performance results were significant ($p < 0.003$), except when comparing the first order MAV to adaptive estimators. For multiple channel fit SE's, all differences in multiple channel performance results were significant ($p < 0.004$) except when comparing the first order MAV to adaptive estimators, and when comparing the third through sixth order MAV to whitened estimators. For a given processor style (unwhitened = MAV, whitened, adaptive), single channel to multiple channel result differences were compared with unequal variance t-tests. All differences in second through sixth order results were significant ($p < 0.009$), except when comparing the second order adaptive estimators.
Figure 10.13: Extension EMG to Torque Fits

Extension third order standard polynomial basis fits for the ten contractions from subject GB are shown for the various EMG amplitude estimators. Single channel fits are for the middle electrode-amplifier pair.
Figure 10.14: Flexion EMG to Torque Fits

Flexion third order standard polynomial basis fits for the ten contractions from subject GB are shown for the various EMG amplitude estimators. Single channel fits are for the middle electrode-amplifier pair.
Figure 10.15: EMG to Torque Fits With Multiple Channel Adaptive EMG Amplitude Estimators

Second, fourth and sixth order standard polynomial basis fits for the ten contractions from subject GB are shown.
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**Figure 10.16**: Standard Errors in Standard Polynomial Basis Fits

Each single channel entry was the average ± standard deviation from 130 paired records. Each multiple channel entry was the average ± standard deviation from 30 records. Error was expressed in A/D units.
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**Figure 10.17: Standard Errors in Orthogonal Polynomial Basis Fits**

Each single channel entry was the average ± standard deviation from 130 paired records. Each multiple channel entry was the average ± standard deviation from 30 records. Error was expressed in A/D units.
Figure 10.18: Plot of Standard Errors in Standard Polynomial Basis Fits

Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid circles denote fit errors for unwhitened single channel EMG amplitude estimates (average of 130 fits). Hollow circles denote fit errors for adaptive multiple channel EMG amplitude estimates (average of 30 fits).
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<td>5</td>
<td>&gt; 10^5</td>
<td>&gt; 10^5</td>
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<tr>
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<td>&gt; 10^5</td>
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**Figure 10.19:** Condition Numbers in Single Channel Fits

Each single channel entry was the average ± standard deviation from 130 paired records. An entry of > 10^5 denoted that at least one of the records had a condition number greater than 10^5.
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<td>6</td>
<td>&gt; 10^5</td>
<td>2139 ± 2607</td>
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**Figure 10.20:** Condition Numbers in Multiple Channel Fits

Each multiple channel entry was the average ± standard deviation from 30 records. An entry of > 10^5 denoted that at least one of the records had a condition number greater than 10^5.
10.5.4 Prediction of Joint Torque

Figure 10.21 shows a sample result of measured (solid line) and predicted (dashed line) total joint torque. Joint torque was predicted from the EMG amplitude and a third order standard polynomial basis EMG to torque relationship. EMG amplitude was estimated (for EMG to torque calibration and for torque prediction) with the multiple channel adaptive processor. Figures 10.22 and 10.23 show the SE’s for the standard and orthogonal basis predictions, respectively. Because the standard and orthogonal basis prediction results were almost identical, statistical tests were only performed on the results from the standard polynomial basis. Paired t-tests compared the results within the single/multiple channel categories. For single channel prediction SE’s, all differences in first through fifth order single channel performance results were significant ($p < 0.006$), except when comparing the fifth order MAV and adaptive estimators. For multiple channel prediction SE’s, all differences in first through fifth order multiple channel performance results were significant ($p < 0.005$), except when comparing first and fifth order MAV to adaptive estimators, and when comparing the third through sixth order MAV to whitened estimators. For a given processor style (MAV, whitened, adaptive), single channel to multiple channel result differences were compared with unequal variance t-tests. All differences in results were significant ($p < 0.005$), except when comparing the sixth order whitened estimators.
Figure 10.21: Prediction of Joint Torque

Joint torque was predicted from the EMG amplitude and a third order standard polynomial basis EMG to torque relationship. Solid line is the measured torque. Dashed line is the predicted torque. EMG amplitude was estimated (for EMG to torque calibration and for torque prediction) with the multiple channel adaptive processor. The standard error is 27.3 A/D units. Data were from subject AH.
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<td>40.7 ± 27.3</td>
<td>48.5 ± 40.8</td>
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<td>5</td>
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<td>122 ± 577</td>
<td>111 ± 910</td>
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<td>30.8 ± 24.0</td>
<td>28.4 ± 23.5</td>
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<td>38.8 ± 59.9</td>
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<td>6</td>
<td>52.6 ± 121</td>
<td>50.1 ± 138</td>
<td>45.3 ± 125</td>
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Figure 10.22: Standard Errors in Standard Polynomial Basis Predictions

Each single channel entry was the average ± standard deviation from 1300 predictions. Each multiple channel entry was the average ± standard deviation from 300 predictions. Error was expressed in A/D units.
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<td>53.4 ± 27.2</td>
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<td>51.9 ± 30.5</td>
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<td>39.7 ± 20.0</td>
<td>46.8 ± 32.9</td>
<td>36.9 ± 26.0</td>
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<tr>
<td>3</td>
<td>40.7 ± 27.3</td>
<td>48.5 ± 40.8</td>
<td>36.6 ± 26.4</td>
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<tr>
<td>4</td>
<td>46.0 ± 57.1</td>
<td>52.2 ± 57.9</td>
<td>40.3 ± 50.7</td>
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<td>70.0 ± 259</td>
<td>82.5 ± 340</td>
<td>50.9 ± 121</td>
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<td>111.0 ± 541</td>
<td>136 ± 857</td>
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<td>41.0 ± 22.4</td>
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<td>2</td>
<td>29.6 ± 15.0</td>
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<td>27.9 ± 18.0</td>
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<td>3</td>
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<td>25.8 ± 16.0</td>
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<tr>
<td>4</td>
<td>31.7 ± 22.1</td>
<td>30.8 ± 24.0</td>
<td>28.4 ± 23.5</td>
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<td>5</td>
<td>32.7 ± 27.8</td>
<td>38.8 ± 59.9</td>
<td>33.1 ± 41.6</td>
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<td>6</td>
<td>60.5 ± 146</td>
<td>56.7 ± 187</td>
<td>45.3 ± 125</td>
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</table>

**Figure 10.23:** Standard Errors in Orthogonal Polynomial Basis Predictions

Each single channel entry was the average ± standard deviation from 1300 predictions. Each multiple channel entry was the average ± standard deviation from 300 predictions. Error was expressed in A/D units.
10.6 Discussion

10.6.1 Crosstalk

At a lag of approximately zero, a moderate correlation existed between extension and flexion EMG waveforms. The correlation was essentially the same regardless of the electrode-amplifier location. Since the lateral/medial electrode-amplifier pairs were more tightly spaced than the middle electrode-amplifiers, a difference in channel correlation, as a function of location, was expected if crosstalk existed. Perhaps co-contraction (not crosstalk) influenced the result at zero lag. Because of co-contraction, both extension and flexion EMG waveforms decreased in amplitude as contraction progressed from a high to a low level. (Recall that the correlation coefficient was computed during relaxation from 50% MVC extension.) Thus, a correlation in EMG amplitude would be reflected as a correlation in EMG waveforms. Unfortunately, with surface electrodes, no definitive test exists to differentiate between crosstalk and co-contraction.

It is not clear how sensitive the fit/prediction algorithms were to any possible crosstalk. Consider, however, a simple model for crosstalk in which the experimentally estimated EMG amplitude from each muscle group always contains a crosstalk component which is a fixed fraction of the EMG amplitude from the antagonist muscle group. That is, if $s_E$ and $s_F$ are the true extensor/flexor EMG amplitudes and $\hat{s}_E$ and $\hat{s}_F$ are their experimental estimates, then

$$s_E = \alpha_E \hat{s}_E + \beta_F \hat{s}_F$$
$$s_F = \beta_E \hat{s}_E + \alpha_F \hat{s}_F$$

where $\alpha_E$, $\alpha_F$, $\beta_E$ and $\beta_F$ are constants. Next, recall the algebraic relation
between joint torque and flexor/extensor contraction as

\[ T_{\text{ext}} = T_F - T_E \]

Then, for first order standard polynomial fitting to the true EMG amplitudes,

\[ T_{\text{ext}} = f_{F,1} s_F - f_{E,1} s_E \]

\[ = (f_{F,1} \alpha_F - f_{E,1} \beta_F) \hat{s}_F - (f_{E,1} \alpha_E - f_{F,1} \beta_E) \hat{s}_E \]

where the \( f_{i,j} \) are the standard polynomial fit parameters. The true and experimentally estimated EMG amplitudes are both related to the joint torque by a first order relation, but the parameters of the relation differ. Thus, the experimentally determined relation predicts total joint torque with the same performance as data with no crosstalk, but the identification of individual (extensor/flexor) contributions to total joint torque is biased by crosstalk. The above equation shows that the bias grows with the degree of crosstalk. For second order standard polynomial fitting to the true EMG amplitudes,

\[ T_{\text{ext}} = f_{F,1} s_F + f_{F,2} s_F^2 - f_{E,1} s_E - f_{E,2} s_E^2 \]

\[ = (f_{F,1} \alpha_F - f_{E,1} \beta_F) \hat{s}_F + (f_{F,2} \alpha_F^2 - f_{E,2} \beta_F^2) \hat{s}_F^2 \]

\[ - (f_{E,1} \alpha_E - f_{F,1} \beta_E) \hat{s}_E - (f_{E,2} \alpha_E^2 - f_{F,2} \beta_E^2) \hat{s}_E^2 \]

\[ + 2(f_{F,2} \alpha_F \beta_E - f_{E,2} \alpha_E \beta_F) \hat{s}_E \hat{s}_F \]

In this case, the true and experimentally estimated EMG amplitudes do not share a common relation to total joint torque. In fact, for all relations higher than first order, cross terms exist. Thus, identification of both total joint torque
and individual joint torque contributions differ. As in the first order case, errors seem to grow with the degree of crosstalk, but the situation is more complex.

For the investigations reported in this study, if there were some EMG crosstalk, it is likely that the results would not have altered. This study was primarily concerned with a comparison of several EMG amplitude estimators. If crosstalk existed, then that crosstalk would have been presented to each estimator. Thus, result differences would likely still have reflected performance differences in the EMG amplitude estimators.

10.6.2 EMG to Torque Fitting

The results of Figures 10.16 and 10.17 provide three clear conclusions. First, the adaptive processors (single/multiple) had a lower fit SE than the MAV or whitened processors. (Although the MAV and adaptive estimators performed equally for first order fits, they both performed poorly. Thus, comparison of fit results at the first order was not appropriate.) For third order standard polynomial basis fits, the average SE of the single (multiple) channel adaptive estimators was only 65–78% that of the MAV or whitened single (multiple) channel estimators. Second, each multiple channel estimator performed better than its respective single channel estimator. For third order standard polynomial basis fits, the average SE of the multiple channel estimators was 62–71% that of each respective single channel estimator. Also, for third order standard polynomial basis fits, the multiple channel adaptive estimator ("best estimator") had approximately half the average SE of the single channel MAV estimator ("standard estimator"). And third, there was little difference in results between the standard and orthogonal bases. The small differences that existed were not sta-
tistically significant and reflected roundoff error in the least squares algorithm. (Chapter 9 discussed why the fit SE’s from the two bases should be identical.)

Comparison of the fit SE’s with the results of the EMG to torque simulations suggested that the appropriate fit order was in the range of 2–4. In the simulations, an appropriate order was found when progressively higher order fits provided only limited improvement in the total joint torque fit error. Experimentally, the fit SE did decline rapidly with the first increase in fit order. Thereafter, the decay in fit SE slowed. The SE decreased 36–54% between fit orders 1–2, 15–29% between fit orders 2–3, 7–11% between fit orders 3–4, and 5–11% between fit orders 4–5. The shape of the EMG to torque relationship was nearly linear, a result similar to that of other investigators (see the review in Chapter 3).

The condition numbers for all of the fits were larger than those reported in the simulations. However, Figures 10.19 and 10.20 suggested that conditioning of the problem was influenced by the performance of EMG amplitude estimation. For example, condition numbers from the multiple channel estimator fits were uniformly lower than each respective single channel estimator. Recall that the simulation EMG amplitude estimates had an SNR performance of approximately 31.6. With at most five electrode-amplifiers per muscle group, even the “best” (multiple channel adaptive) EMG amplitude estimator of the experimental data would not have performed as well. Thus, the higher condition numbers in this experimental trial likely reflected the performance of EMG amplitude estimation.
10.6.3 EMG to Torque Prediction

Although efforts were taken to prevent extrapolation in the EMG to torque fits, the results indicated that extrapolation occurred, and that extrapolation caused a small number of very large errors. Apparently, outside the range over which identification was performed, the basis functions were poorly behaved. Figure 10.24 shows a sixth order, standard polynomial basis EMG to torque prediction. EMG amplitude was estimated (for EMG to torque calibration and for torque prediction) with the single channel adaptive processor. Because of extrapolation error at the beginning and end of the data record (corresponding to the high extension and flexion portions of the experimental contraction, respectively), the SE for this prediction was 7937 A/D units! The integrity of the sixth order fit was not maintained outside of the range of EMG amplitude provided to the fit. Recall that the range of total joint torques was uniform across all trials and that the predictions excluded a 500 sample (245ms) segment of data from the beginning and end of each record prior to prediction. It appeared that these efforts were not sufficient. In the future, the range of EMG amplitude (not total joint torque) provided to the fit and prediction should be made uniform.

For the purpose of this project, torque prediction without extrapolation was accomplished by performing a second prediction study, this study excluding a 2048 sample (1 second) segment of data from the beginning and end of each record prior to prediction. Only the standard polynomial basis was studied. Figure 10.25 shows the results. Paired t-tests compared the results within the single/multiple channel categories. For single channel prediction SE’s, all differences in performance were significant ($p < 0.000001$), except when comparing the first order MAV and adaptive estimators. For multiple channel prediction
Figure 10.24: Prediction of Joint Torque With Extrapolation Error

Joint torque was predicted from the EMG amplitude and a sixth order standard polynomial basis EMG to torque relationship. Solid line is the measured torque. Dashed line is the predicted torque. EMG amplitude was estimated (for EMG to torque calibration and for torque prediction) with the single channel adaptive processor. Because of the extrapolation error, the standard error was 7937 A/D units. Data were from subject EB.
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<td>26.6 ± 12.8</td>
<td>26.6 ± 15.5</td>
<td>23.9 ± 14.8</td>
</tr>
</tbody>
</table>

**Figure 10.25:** Standard Errors in Standard Polynomial Basis Predictions — Reduced Prediction Range

Each single channel entry was the average ± standard deviation from 1053 predictions. Each multiple channel entry was the average ± standard deviation from 243 predictions. Error was expressed in A/D units.
SE's, all differences in performance were significant \( (p < 0.01) \), except when comparing third through sixth order MAV to whitened estimators. For a given processor style (MAV, whitened, adaptive), single channel to multiple channel result differences were compared with unequal variance t-tests. All differences were significant \( (p < 0.000001) \).

These reduced prediction range results followed the same trends as the fit results. Each multiple channel predictor performed better than its respective single channel predictor. The adaptive processors (single/multiple) had a lower prediction SE than the MAV or whitened processors. The adaptive multiple channel predictor had an SE approximately 90% of the unwhitened (MAV) multiple channel predictor. Although this improvement was small, it demonstrated the potential of adaptive whitening. Considering the large performance improvements in EMG amplitude estimation demonstrated by Experiments 1 and 2 (for contractions above 10% MVC), more advanced adaptive whiteners (i.e. formal solutions to the EMG model with additive noise) seem well worth pursuing.

Figure 10.26 shows the reduced range prediction SE and the fit SE for the multiple channel adaptive predictor. As the model order is increased, the prediction SE attains a minimum at third order, then remains near this value out to sixth order. These results follow the same trends as the simulation results. The best prediction performance was provided by the third order multiple channel adaptive processor. For each respective subject, this predictor had an average SE approximately 3% of the maximum torque range for that subject.
Figure 10.26: Multiple Adaptive Standard Polynomial Errors

EMG to torque results for the multiple channel adaptive EMG amplitude estimator with standard polynomial basis functions. Mean values are graphed as small circles. Standard deviations are graphed as error bars about the mean. Solid circles denote prediction errors (average of 243 predictions). Hollow circles denote fit errors (average of 30 fits). Prediction results are for the reduced prediction range.
10.7 Conclusion

This chapter reported an experimental study which estimated joint torque from the surface EMG waveform. As a consequence of the poor low amplitude performance of whitened EMG amplitude estimators, a new model for the EMG waveform was introduced. This model included an additive noise term. Based upon the new model, an ad hoc adaptive EMG amplitude estimator was designed. The adaptive estimator showed that the benefits of whitening for high EMG amplitude and the benefits of standard MAV detection for low EMG amplitude could simultaneously be attained. The new EMG waveform model should be investigated further. This study also demonstrated that an improvement in EMG to torque prediction was realized with improved EMG amplitude estimation. All multiple channel predictors performed better than their respective single channel version. Additionally, the adaptive predictors performed better than the MAV or whitened predictors. The results suggested that a third order model of the EMG to torque relationship was appropriate, for both EMG to torque fit and prediction, with these EMG amplitude estimators.
Chapter 11

Discussion and Prospectus for Future Research
11.1 Overview of Research Context

This research project considered two fundamental, and related, aspects of electromyography. First, theoretic models, followed by physiologic and simulation studies, investigated estimators of the EMG amplitude. EMG amplitude estimates are presently used as command inputs to upper-extremity elbow, wrist and hand prostheses. Such prostheses utilize EMG amplitude estimates which typically have poor SNR performance. Previous investigators have introduced two separate techniques for improving performance; 1) temporal whitening of a single EMG channel and 2) the combination of multiple EMG channels. This project developed a stochastic model of the EMG waveform. Optimal estimation of EMG amplitude, based on the stochastic model, simultaneously encompassed both of the previously introduced techniques. Experimental studies investigated the performance and sensitivity of the optimal estimation, and showed that the improvements realized by whitening and multiple channel combination were additive.

The second aspect of electromyography considered by this project was the relationship between EMG amplitude and joint torque. Certainly, knowledge of such a relationship is of value to prosthetics. Additionally, the study of human movement and biomechanics would benefit from a non-invasive assessment of the torques created by muscular activation. The indeterminacy (in the presence of muscular co-contraction) of the EMG to torque relationship was studied through a simulation. The simulation suggested that appropriate experimental design and interpretation could resolve the indeterminate problem. Experimentally, whitened and unwhitened, single and multiple channel EMG to torque estimators
were compared. Results suggested that the improvements in EMG amplitude estimation lead to an improvement in estimating joint torque from the EMG waveform.

11.2 Models of the EMG Waveform and the EMG to Torque Relationship

A survey of the field of EMG signal processing demonstrates that the EMG waveform models presented in this project represent the first complete solution to multiple electrode-amplifier estimation of EMG amplitude. Triolo and Moskowitz (1985) performed a short preliminary simulation study, but few details are described. Hogan and Mann (1980a) developed a multiple channel model, but did not include a description of the temporal correlation in EMG data. Their model seems to be the most developed model of constant force EMG amplitude estimators available in the literature. They presented both single and multiple channel models. The single channel models of this thesis contain a few components not found in previous models. The models of this thesis assume joint Gaussianity, correlation-ergodicity, and a temporal correlation filter which is stable, causal and whose inverse exists and is stable and causal. (All of the above assumptions are probably implicit, but not stated, in the model of Hogan and Mann.) The assumption of an invertible temporal correlation filter greatly reduced the complexity of developing the theoretical justification for whitening. In fact, no theoretic justification was needed. Since the whitening filter was invertible, no information was lost by whitening. Thus, whitening could be considered separately from the formal ML development. Additionally, regardless of the underlying PDF for the EMG waveform, whitening could still
simplify the optimization. In particular, the models showed that if the EMG data were distributed as a Laplacian random variable, then whitening followed by MAMAV detection was optimal in the ML sense (assuming that whitening produced independent samples).

The EMG to torque model provided a manner to relate the EMG waveform to torque. The relationship was not, however, direct. The model actually related the EMG amplitude to torque. This technique allowed the EMG to torque problem to be separated into two subproblems — estimating EMG amplitude from the EMG waveform, followed by estimating torque from the EMG amplitude estimate. This approach decreased the complexity of the problem. For isometric, isotonic contraction, EMG amplitude estimator performance could be considered from deviations about the mean value for the estimate regardless of the actual mean value. No direct knowledge of muscle force was required, so long as muscle force was essentially constant. When considering the relationship between EMG and torque, only minor attention had to be given to the details of EMG amplitude estimation (except, of course, for the need to include the adaptive estimators). As the isometric and isotonic assumptions are relaxed, the two subproblem approach may no longer be viable.

The EMG to torque model also separately identified the contributions to the total joint torque due to contraction of each muscle group. This technique would not be appropriate if significant EMG crosstalk were present. If crosstalk is a problem, new electrode-amplifier configurations, for example the double differencing electrode-amplifiers of Broman et al. (1985) and DeLuca and Merletti (1988) or the spatial filtering technique of Reucher et al. (1987a, 1987b), may be appropriate.
11.3 Single Channel Optimization

The results of Experiment 1 suggested that a single temporal whitening filter per electrode-amplifier could be used for contractions in the range of 10–75% MVC. This filter was relatively easy to determine, and is some sense was “forgiving”. That is, many filters provided a modest improvement in SNR performance, and the same whitening filter was appropriate for several joint angles.

The next major issue in single channel optimization is that of robustness. This project demonstrated two problems in robustness — non-characteristic EMG data with high frequency noise and additive background noise for contractions less than 10% MVC. Non-characteristic EMG data might be detected via a multiple channel error-checking monitor. In particular, individual optimal EMG amplitude estimates could be made from multiple channels of EMG and the various estimates monitored. If all channels are functioning normally, all individual amplitude estimates would rise and fall in unison. A channel which does not follow the others would indicate an error, and corrective action could be applied. Thus, multiple channels of EMG provide certain as yet undeveloped modes of self-test.

The additive background noise problem was addressed with the adaptive temporal whitening technique. This technique demonstrated the effectiveness of adaptive whitening. Future research should be directed towards more formal solutions to the additive noise model of the EMG waveform. A formal, general solution may, however, be quite difficult. Two compromise solution ideas may be effective. First, a formal solution might be found if more assumptions are made in the model. In particular, assuming both the multiplicative and additive
Figure 11.1: Wiener Filter Adaptive EMG Amplitude Estimator

The discrete time EMG waveform $m_i$ includes additive noise $v_i$, multiplicative noise $n_i$ and the EMG amplitude $s$ as $m_i = r_i + v_i$, where $r_i = s n_i$.

Noise PSD's to be autoregressive moving average (ARMA) would allow them to be written in closed form. An analytic least squares estimator might then be formulated. The second compromise solution, shown in Figure 11.1, treats the estimation problem in two parts. Consider writing the EMG waveform with additive noise model as $m_i = r_i + v_i$, where $r_i = s n_i$, $n_i$ is the multiplicative noise, $v_i$ is the additive noise, and $s$ is the EMG amplitude. Form the linear least squares estimate of $r_i$ based on $m_i$. This is the Wiener filter. Then, estimate $s$ from $\hat{r}_i$. The Wiener filter provides the adaptation since its frequency response $H_W(e^{j\omega})$ is (Press et al., 1988),

$$H_W(e^{j\omega}) = \frac{s^2 |H_{nn}(e^{j\omega})|^2}{s^2 |H_{nn}(e^{j\omega})|^2 + |H_{vv}(e^{j\omega})|^2}$$

where $|H_{xx}(e^{j\omega})|^2$ represents the magnitude response squared of a linear shaping filter whose output PSD is $S_{xx}(e^{j\omega})$ when its input is white noise. (The symbol $s$
is the EMG amplitude, not the Laplace Transform frequency variable). Note that
the filter gain at each frequency is inversely related to the additive noise intensity
at that frequency. Although the Wiener filter approach may not be optimal
in the ML sense, it provides a standard estimation methodology nonetheless.
Finally, although an additive noise model was consistent with the data, it does
not uniquely describe the observed phenomenon. Perhaps the EMG PSD does
change with the EMG amplitude. Although the results of Experiment 1 showed
that such a change need not be considered for contractions in the range of 10–
75% MVC, the low contraction case may need to be considered more carefully.

11.4 Multiple Channel Combination

For up to eight channels with modest correlation, multiple channel combina-
tion was quite robust. Optimal or equal variance spatial uncorrelation filters
performed equably and could be calibrated from a short segment of data. The
only problem experienced with multiple channel combination was robustness
problems of temporal whitening. As mentioned previously, however, multiple
channels might help to detect robustness problems within individual channels.

The implementation of a real-time multiple channel EMG amplitude esti-
mator should be evaluated. For such a device, integer mathematics should be
considered. In particular, real-time smoothing could be performed very rapidly
with the use of an integer algorithm. Rather than summing $N \cdot L$ elements each
estimate, only the newest $L$ samples need be added to the sum and the oldest $L$
samples subtracted from the sum. Floating point math can not use this method
since roundoff error could cause a bias to accumulate. Integer mathematics might
also be used for temporal whitening and spatial uncorrelation.
11.5 Relating EMG to Torque

Although the EMG to torque simulation study was limited in scope, it did suggest that simultaneous identification of flexor and extensor muscle group contributions to the total joint torque could be accomplished with a small, easily acquired set of muscle contractions. Few previous studies have even considered co-contraction, yet the experiments of this project found co-contraction during all moderate (approximately 25% MVC) or greater agonist muscular contraction. Co-contraction may always exist, but at low levels during low levels of contraction.

The simulation study also suggested that some of the trial to trial variation in the EMG to torque relationship was due to differences in the “contour” of contraction. That is, trial to trial variations in the time course of EMG amplitude contributed to trial to trial differences in the identified EMG to torque relationship. Because the volume of data used in the fit calculations was so large, perhaps editing of the EMG amplitudes could provide an appropriate standardized subset of inputs to the fit calculations. This subset should still span the complete range of EMG amplitudes, but could be chosen to always follow one particular contour. This technique could first be investigated in simulation. (With sparser data sets, orthogonal polynomial basis functions may prove helpful.)

Finally, this project did not address directly whether the EMG to torque relationship was linear or non-linear. This question seems secondary to determining how accurately the EMG to torque prediction can be made — regardless of the basis functions. Prediction accuracies of approximately 2-6% (of 50%MVC) for up to 40-50% MVC were achieved. Future studies should concentrate on
prediction accuracy rather than the linearity of the basis.

11.6 Future EMG to Torque Investigation

The immediate future EMG to torque projects will likely continue to estimate EMG amplitude, then predict joint torque based on the EMG amplitude estimate. For the isometric, isotonic case, robust temporal whitening schemes should be considered for EMG amplitude estimation. Also, a more complete understanding of the additive background noise model, and its solutions, is needed. With better additive noise model solutions, the work of Experiment 3 should be repeated. A next experimental trial should investigate the influence of angle on the EMG to torque relationship.

Then, the non-stationary case could be evaluated. For amplitude estimation, several investigators have suggested adaptive smoothing window length estimators (D’Alessio, 1984, 1985; Filligoi and Mandarini, 1984; Fullmer et al., 1984; Jacobsen et al., 1984; Xiong and Shwedyk, 1987). A long smoothing window is used when the EMG amplitude is slowly varying, and a short smoothing window is used when the EMG amplitude is quickly varying. The EMG amplitude to torque identification problem will increase in complexity when muscle dynamics and non-isometric contractions are considered. Finally, the future will also bring consideration of muscle fatigue as another variant in the EMG to torque problem.
Chapter 12

Project Summary
This thesis presented theoretic models, physiologic experiments and computer simulations which investigated the surface EMG amplitude and its relationship to joint torque. A functional stochastic model of multiple channel EMG was presented. Analytic solutions to the EMG amplitude estimation problem for several variants of the model were derived. In particular, a solution to the complete multiple channel EMG amplitude estimation problem — including both spatially and temporally correlated data — was determined.

Experiment 1 examined the sensitivities, design considerations and performance of EMG amplitude estimates at one joint angle. Surface EMG waveforms from elbow flexors/extensors were recorded for non-fatiguing, isometric, isotonic contractions at one joint angle. An MARMS estimator provided an A±SD SNR of 10.7 ± 3.3 for single channel estimators. Temporal whitening with one five-coefficient whitening filter per electrode per subject improved the A±SD SNR to 17.6 ± 6.0. Whitening filters performed best when they were fourth order or higher and were calibrated from three or more seconds of data from a high contraction trial. Calibration from as little as 62.5 ms of data dropped SNR performance by less than 5%. Optimal multiple channel combination with four channels improved the A±SD SNR to 24.6 ± 10.4. On one subject eight optimally combined channels were achieved, providing an A±SD SNR of 35.0 ± 13.4. Spatial uncorrelation filters could be derived from as little as 15.625 ms of data. The equal variance combiner performed as well, or almost as well as the optimal combiner. Additional sensitivities, design considerations and performances were investigated both experimentally and with simulation studies.

Experiment 2 investigated the influence of joint angle on the calibration and performance of optimal amplitude estimators. Surface EMG waveforms from
elbow flexors/extensors were recorded for non-fatiguing, isometric, isotonic contractions from joint angles over a span of 90°. Results indicated little or no loss in SNR performance when optimized estimators were calibrated from one joint angle and applied to the EMG data from all joint angles. The actual performance at all angles was approximately the same. Sensitivities, design considerations and performance of the single/multiple channel EMG amplitude estimation algorithms, as a function of joint angle, were investigated experimentally.

An isometric, isotonic model of the elbow which included simultaneous flexion and extension contraction (co-contraction) was developed. The model parameterized a relationship between EMG amplitude and joint torque for each of the flexor and extensor muscle groups. Methods for identifying the model were investigated in a simulation study. The simulation explicitly included muscular co-contraction. The simulation determined experimental data required to perform EMG to torque identification and evaluated the performance of the identification technique.

Experiment 3 investigated the relationship between EMG amplitude and joint torque. Surface EMG waveforms from elbow flexors and extensors, and joint torque were simultaneously recorded for non-fatiguing, isometric, quasi-isotonic contractions. These experiments established the need for, and the character of, a more detailed functional model of the surface EMG waveform which included an additive background noise. An adaptive EMG amplitude estimator was implemented to test the new additive noise model. Single/multiple channel unwhitened/whitened/adaptive EMG amplitude estimates were fit to joint torque via the parameterized model. Each multiple channel estimator had a fit error approximately 65% of its respective single channel estimator. The mul-
multiple channel adaptive estimator had a fit error approximately half that of the unwhitened single channel estimator. An initial attempt at joint torque prediction from the EMG waveform was evaluated. Each multiple channel predictor had an SE approximately 70% of its respective single channel predictor. The multiple channel adaptive predictor had an SE approximately 90% of the unwhitened multiple channel predictor.
Appendix A

Random Variables, Vectors and Processes

\[^{1}\text{This discussion of random variables, vectors and processes is adapted from Drake (1967), Papoulis (1984) and Shapiro and Willsky (1988).}\]
A.1 Random Variables

A probability space for an experiment consists of a sample space and a probability measure. A sample space, denoted $\Omega$, is a mutually exclusive list of every possible distinguishable outcome of an experiment. An event is a subset of the sample space $\Omega$. A probability measure, denoted $Pr(A)$, where $A$ is an event, assigns probabilities to events. A probability measure must obey the following three axioms;

1) For any event $A_1$, $Pr(A_1) \geq 0$,

2) $Pr(\Omega) = 1$,

3) For any two disjoint events $A_1$ and $A_2$,

$$Pr(A_1 \cup A_2) = Pr(A_1) + Pr(A_2)$$

For two events $A_1$ and $A_2$ within a sample space, the conditional probability of $A_1$ given $A_2$ (or, the probability of the event $A_1$ conditioned on the event $A_2$) is the probability of event $A_1$ given that event $A_2$ has occurred. This conditional probability is denoted $Pr(A_1|A_2)$ and is formally defined as

$$Pr(A_1|A_2) = \frac{Pr(A_1 \cap A_2)}{Pr(A_2)} \text{ for } Pr(A_2) \neq 0$$

Two events, $A_1$ and $A_2$, within a sample space are independent if

$$Pr(A_1 \cap A_2) = Pr(A_1)Pr(A_2)$$

or, equivalently if

$$Pr(A_1|A_2) = Pr(A_1)$$

A random variable is defined by a real-valued function which assigns a value of the random variable to each element of the sample space. A probability density
function (PDF) for a random variable is a rule for assigning a probability measure
to every possible event within the sample space. A PDF for a random variable \( x \)
is denoted \( p_x(x) \) (read as "the probability that the experimental value of random
variable \( x \) obtained on a performance of the experiment will take on the value
\( X \)). The expected value of a function \( g(x) \) of a random variable \( x \) is defined as

\[
\mathcal{E}_x[g(x)] = \int_{X=-\infty}^{\infty} g(x) p_x(X) \, dX
\]

The subscript \( x \) on the expectation operator \( \mathcal{E} \) denotes the form of the probability
density within the expectation integral. In general, the form of the probability
density is unambiguous and, therefore, the expectation will be written more
compactly without the subscript, i.e.

\[
\mathcal{E}_x[g(x)] \equiv \mathcal{E}[g(x)]
\]

However, when the form of the probability density is unclear, the more robust
subscript notation will be utilized. The \( n^{th} \) moment of a random variable \( x \) is
defined as

\[
\text{\( n^{th} \) moment of \( x = \mathcal{E}[x^n] \)}
\]

and the \( n^{th} \) central moment of a random variable \( x \) is defined as

\[
\text{\( n^{th} \) central moment of \( x = \mathcal{E}[(x - \mathcal{E}[x])^n] \)}
\]

Three frequently encountered moments are;

1) Mean Value (First Moment): \( \mathcal{E}[x] \equiv \mu_x \)

2) Mean-Square Value (Second Moment): \( \mathcal{E}[x^2] \)

3) Variance (Second Central Moment): \( \mathcal{E}[(x - \mu_x)^2] \equiv \sigma_x^2 \equiv k_{xx} \)
The above three moments are related as

\[ \sigma_x^2 = \mathbb{E}[x^2] - \mu_x^2 \]

or

\[ \mathbb{E}[x^2] = \sigma_x^2 + \mu_x^2 \]

The standard deviation \( \sigma_x \) of a random variable \( x \) is the square root of its variance: \( \sigma_x = \sqrt{\sigma_x^2} \).

A set of random variables \( x_1, x_2, x_3, \ldots, x_N \) has a joint density which is denoted as

\[ p_{x_1, x_2, x_3, \ldots, x_N}(X_1, X_2, X_3, \ldots, X_N) \]

The marginal PDF for any particular random variable within the set can be determined from the joint PDF by integration of the joint PDF over the complete domain of all other random variables, i.e.

\[ p_{x_i}(X_i) = \int_{X_1=-\infty}^{\infty} \int_{X_2=-\infty}^{\infty} \int_{X_3=-\infty}^{\infty} \cdots \int_{X_{i-1}=-\infty}^{\infty} \int_{X_{i+1}=-\infty}^{\infty} \cdots \int_{X_N=-\infty}^{\infty} \]

\[ \cdot p_{x_1, x_2, x_3, \ldots, x_N}(X_1, X_2, X_3, \ldots, X_N) \cdot \]

\[ \cdot dX_1 \; dX_2 \; dX_3 \cdots dX_{i-1} \; dX_{i+1} \cdots dX_N \]

Note that there is no general method for determining the joint density from all of the marginal densities. The expected value of a function \( g(x_1, x_2, x_3, \ldots, x_N) \) of a set of random variables is defined as

\[ \mathbb{E}[g(x_1, x_2, x_3, \ldots, x_N)] = \int_{X_1=-\infty}^{\infty} \int_{X_2=-\infty}^{\infty} \int_{X_3=-\infty}^{\infty} \cdots \int_{X_N=-\infty}^{\infty} \]

\[ \cdot g(x_1, x_2, x_3, \ldots, x_N) \cdot \]

\[ \cdot p_{x_1, x_2, x_3, \ldots, x_N}(X_1, X_2, X_3, \ldots, X_N) \cdot \]

\[ \cdot dX_1 \; dX_2 \; dX_3 \cdots dX_N \]

Some important expectations are;
1) Linearity Property: For $a_i$ and $a_j$ scalars,
\[ \mathcal{E}[a_i x_i + a_j x_j] = a_i \mathcal{E}[x_i] + a_j \mathcal{E}[x_j] \]

2) Correlation:
\[ c_{x_i,x_j} = \mathcal{E}[x_i x_j] \]

3) Covariance:
\[ k_{x_i,x_j} = \mathcal{E}[(x_i - \mu_{x_i})(x_j - \mu_{x_j})] = c_{x_i,x_j} - \mu_{x_i} \mu_{x_j} \]

The correlation coefficient is defined as
\[ r_{x_i,x_j} = \frac{k_{x_i,x_j}}{\sigma_{x_i} \sigma_{x_j}} \]

Two random variables $x_i$ and $x_j$ are uncorrelated if $k_{x_i,x_j} = 0$. This condition is equivalent to $\mathcal{E}[x_i x_j] = \mathcal{E}[x_i] \mathcal{E}[x_j]$ or $r_{x_i,x_j} = 0$. A set of random variables is completely pair-wise uncorrelated if $k_{x_i,x_j} = 0$ for all $i \neq j$. A pair of random variables $x_i$ and $x_j$ is statistically independent (SI) if
\[ p_{x_i,x_j}(X_i, X_j) = p_{x_i}(X_i) p_{x_j}(X_j) \]

A set of random variables is completely pair-wise SI if
\[ p_{x_i,x_j}(X_i, X_j) = p_{x_i}(X_i) p_{x_j}(X_j) \quad \text{for all } i \neq j \]

Complete pair-wise statistical independence implies complete pair-wise uncorrelation, but the converse is not true in general.

The (joint) conditional PDF for a set of random variables conditioned on the event $A$ is defined as
\[ p_{x_1,x_2,x_3,\ldots,x_N|A}(X_1, X_2, X_3, \ldots, X_N|A) = \]
\[ = \frac{p_{x_1,x_2,x_3,\ldots,x_N,A}(X_1, X_2, X_3, \ldots, X_N,A)}{Pr(A)} \quad \text{for } Pr(A) \neq 0 \]

This conditional PDF is the probability that the experimental values of random variables $x_1, x_2, x_3, \ldots, x_N$ obtained on a performance of the experiment will take on the values $X_1, X_2, X_3, \ldots, X_N$, respectively, given that event $A$ has occurred.

371
The expected value of a function \( g(x_1, x_2, x_3, \ldots, x_N) \) of a set of random variables conditioned on the event \( A \) is defined as

\[
\mathbb{E}_{x_1, x_2, x_3, \ldots, x_N | A}[g(x_1, x_2, x_3, \ldots, x_N)] = \int_{x_1 = -\infty}^{\infty} \int_{x_2 = -\infty}^{\infty} \int_{x_3 = -\infty}^{\infty} \cdots \int_{x_N = -\infty}^{\infty} \\
\cdot g(x_1, x_2, x_3, \ldots, x_N) \cdot \\
\cdot p_{x_1, x_2, x_3, \ldots, x_N | A}(X_1, X_2, X_3, \ldots, X_N | A) \cdot \\
\cdot dX_1 \, dX_2 \, dX_3 \cdots dX_N
\]

A function of a random variable defines a new random variable. Linear transformation of the random variable \( x \) by the scalars \( a \) and \( b \) can form the random variable \( y \) as

\[ y = ax + b \]

In this case, \( y \) has a mean value of

\[ \mu_y = a \mu_x + b \]

and a variance of

\[ \sigma_y^2 = a^2 \sigma_x^2 \]

Similarly, a function of a pair of random variables defines a random variable. If \( a_i, a_j \) and \( b \) are scalars, then \( y \) can be formed from a linear combination of the random variables \( x_i \) and \( x_j \) as

\[ y = a_i x_i + a_j x_j + b \]

Here, \( y \) has a mean value of

\[ \mu_y = a_i \mu_{x_i} + a_j \mu_{x_j} + b \]

and a variance of

\[ \sigma_y^2 = a_i^2 \sigma_{x_i}^2 + 2a_i a_j \rho_{x_i x_j} + a_j^2 \sigma_{x_j}^2 \]
Note that if $x_i$ and $x_j$ are uncorrelated, then

$$
\sigma_y^2 = \sigma_{x_i}^2 a_i^2 + \sigma_{x_j}^2 a_j^2
$$

### A.2 Random Vectors

A set of random variables is more conveniently and compactly referred to as a random vector. A random vector $\mathbf{x}$ can be constructed from the random variables $x_1, x_2, x_3, \ldots, x_N$ as

$$
\mathbf{x} = \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    \vdots \\
    x_N
\end{bmatrix}_{N \times 1}
$$

A random vector $\mathbf{x}$ has a joint PDF denoted as

$$p_{\mathbf{x}}(X)$$

The expected value of a function $g(\mathbf{x})$ of a random vector $\mathbf{x}$ is defined as

$$
\mathbb{E}[g(\mathbf{x})] = \int_{X_1=-\infty}^{\infty} \int_{X_2=-\infty}^{\infty} \int_{X_3=-\infty}^{\infty} \cdots \int_{X_N=-\infty}^{\infty} \\
\cdot g(\mathbf{x}) \cdot p_{\mathbf{x}}(X) \cdot \\
\cdot dX_1 \cdot dX_2 \cdot dX_3 \cdots dX_N
$$

and denoted as

$$
\mathbb{E}[g(\mathbf{x})] = \int_{X=-\infty}^{\infty} g(\mathbf{x}) \cdot p_{\mathbf{x}}(X) \cdot dX
$$

Some important expectations are;

1) Mean Vector: $\mu_{\mathbf{x}} \equiv \mathbb{E}[\mathbf{x}] = \begin{bmatrix}
    \mu_{x_1} \\
    \mu_{x_2} \\
    \mu_{x_3} \\
    \vdots \\
    \mu_{x_N}
\end{bmatrix}_{N \times 1}$
2) Correlation Matrix: \( C_{xx} \equiv E[xx^T] \)
\[
\begin{bmatrix}
c_{x_1x_1} & c_{x_1x_2} & c_{x_1x_3} & \cdots & c_{x_1x_N} \\
c_{x_2x_1} & c_{x_2x_2} & c_{x_2x_3} & \cdots & c_{x_2x_N} \\
c_{x_3x_1} & c_{x_3x_2} & c_{x_3x_3} & \cdots & c_{x_3x_N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
c_{x_Nx_1} & c_{x_Nx_2} & c_{x_Nx_3} & \cdots & c_{x_Nx_N}
\end{bmatrix}_{N \times N}
\]

3) Covariance Matrix: \( K_{xx} \equiv E[(\bar{x} - \mu_x)(\bar{x} - \mu_x)^T] = C_{xx} - \mu_x\mu_x^T \)
\[
\begin{bmatrix}
k_{x_1x_1} & c_{x_1x_2} & c_{x_1x_3} & \cdots & k_{x_1x_N} \\
k_{x_2x_1} & k_{x_2x_2} & c_{x_2x_3} & \cdots & k_{x_2x_N} \\
k_{x_3x_1} & k_{x_3x_2} & k_{x_3x_3} & \cdots & k_{x_3x_N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
k_{x_Nx_1} & k_{x_Nx_2} & k_{x_Nx_3} & \cdots & k_{x_Nx_N}
\end{bmatrix}_{N \times N}
\]

4) Matrix of Correlation Coefficients:
\[
R_{xx} =
\begin{bmatrix}
n_{x_1x_1} & r_{x_1x_2} & r_{x_1x_3} & \cdots & n_{x_1x_N} \\
n_{x_2x_1} & n_{x_2x_2} & r_{x_2x_3} & \cdots & n_{x_2x_N} \\
n_{x_3x_1} & n_{x_3x_2} & n_{x_3x_3} & \cdots & n_{x_3x_N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
n_{x_Nx_1} & n_{x_Nx_2} & n_{x_Nx_3} & \cdots & n_{x_Nx_N}
\end{bmatrix}_{N \times N}
\]

Note that since \( c_{x_i x_j} = E[x_i x_j] = E[x_j x_i] = c_{x_j x_i} \), the correlation matrix is symmetric. Similarly, the covariance matrix and matrix of correlation coefficients are symmetric.

A function of a random vector defines a new random vector. Linear transformation of the random vector \( \underline{x} \) by the matrix \( D \) can form the random vector \( y \) as
\[
y = D\underline{x}
\]

In this case, \( y \) has a mean vector equal to
\[
\mu_y = D\mu_x
\]
and a covariance matrix equal to

\[ K_{yy} = DK_{xx}D^T \]

The symmetry of the covariance matrix presents a unique application to this transformation. In particular, a real symmetric matrix has a complete set of real linearly independent eigenvectors. As such, these eigenvectors can be chosen to be orthonormal. Denoting \( N \) orthonormal eigenvectors of \( K_{xx} \) as \( \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_N \), with corresponding eigenvalues (which also must be real) \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N \), each eigenvector satisfies the eigenvalue equation

\[ K_{xx}\mathbf{u}_i = \lambda_i \mathbf{u}_i \quad \text{for} \quad 1 \leq i \leq N \]

If the matrix \( D^T \) is formed from the \( N \) eigenvectors as the partitioned matrix

\[ D^T = [\mathbf{u}_1 | \mathbf{u}_2 | \mathbf{u}_3 | \ldots | \mathbf{u}_N] \]

then the \( N \) eigenvalue equations can be written in one equation as

\[ K_{xx}D^T = D^T \text{ diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N) \]

Since \( D^T \) was constructed such that its columns are orthonormal, \( D^{-1} = D^T \), and thus

\[ DK_{xx}D^T = \text{ diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N) \]

Thus the new random variables \( y_1, y_2, y_3, \ldots, y_N \) which comprise the new random vector \( \mathbf{y} = D\mathbf{u} \) are uncorrelated. Further,

\[ K_{yy} = \text{ diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N) \]
A set of random vectors \( \bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L \) can be constructed from the random variables \( x_{11}, \ldots, x_{1N_1}, x_{21}, \ldots, x_{2N_2}, x_{31}, \ldots, x_{3N_3}, \ldots, x_{L1}, \ldots, x_{LN_L} \) as

\[
\bar{x}_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1N_1} \end{bmatrix}_{N_1 \times 1} \quad \bar{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2N_2} \end{bmatrix}_{N_2 \times 1} \quad \bar{x}_3 = \begin{bmatrix} x_{31} \\ x_{32} \\ \vdots \\ x_{3N_3} \end{bmatrix}_{N_3 \times 1} \quad \ldots
\]

\[
\bar{x}_L = \begin{bmatrix} x_{L1} \\ x_{L2} \\ \vdots \\ x_{LN_L} \end{bmatrix}_{N_L \times 1}
\]

(Notice that distinct random vectors can have distinct lengths.) Such a set of random vectors has a joint PDF which is denoted as

\[
p_{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L}(X_1, X_2, X_3, \ldots, X_L)
\]

The expected value of a function \( g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L) \) of a set of random vectors is defined as

\[
\mathcal{E}[g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \cdot g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L) \cdot 
\]

\[
\cdot p_{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_L}(X_1, X_2, X_3, \ldots, X_L) \cdot 
\]

\[
\cdot dX_1 \cdot dX_2 \cdot dX_3 \ldots dX_L
\]

Some important expectations are;

1) Cross-Correlation Matrix: \( C_{\bar{x}_i \bar{x}_j} \equiv \mathcal{E}[\bar{x}_i \bar{x}_j^T] \)

\[
= \begin{bmatrix}
    c_{x_{11}x_{1j}} & c_{x_{11}x_{2j}} & \cdots & c_{x_{11}x_{Nj}} \\
    c_{x_{12}x_{1j}} & c_{x_{12}x_{2j}} & \cdots & c_{x_{12}x_{Nj}} \\
    c_{x_{13}x_{1j}} & c_{x_{13}x_{2j}} & \cdots & c_{x_{13}x_{Nj}} \\
    \vdots & \vdots & \vdots & \vdots \\
    c_{x_{1N_1}x_{1j}} & c_{x_{1N_1}x_{2j}} & \cdots & c_{x_{1N_1}x_{Nj}} \\
\end{bmatrix}_{N_1 \times N_j}
\]
2) Cross-Covariance Matrix: $K_{\xi_i, \xi_j} \equiv E[(\xi_i - \mu_{\xi_i})(\xi_j - \mu_{\xi_j})^T] = C_{\xi_i, \xi_j} - \mu_{\xi_i} \mu_{\xi_j}^T$

\[
\begin{bmatrix}
 k_{\xi_{11}\xi_{j1}} & k_{\xi_{11}\xi_{j2}} & k_{\xi_{11}\xi_{j3}} & \cdots & k_{\xi_{11}\xi_{jN_j}} \\
 k_{\xi_{12}\xi_{j1}} & k_{\xi_{12}\xi_{j2}} & k_{\xi_{12}\xi_{j3}} & \cdots & k_{\xi_{12}\xi_{jN_j}} \\
 k_{\xi_{13}\xi_{j1}} & k_{\xi_{13}\xi_{j2}} & k_{\xi_{13}\xi_{j3}} & \cdots & k_{\xi_{13}\xi_{jN_j}} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 k_{\xi_{iN_i}\xi_{j1}} & k_{\xi_{iN_i}\xi_{j2}} & k_{\xi_{iN_i}\xi_{j3}} & \cdots & k_{\xi_{iN_i}\xi_{jN_j}} \\
\end{bmatrix}_{N_i \times N_j}
\]

3) Matrix of Cross-Correlation Coefficients:

$R_{\xi_i, \xi_j} \equiv \begin{bmatrix}
 r_{\xi_{11}\xi_{j1}} & r_{\xi_{11}\xi_{j2}} & r_{\xi_{11}\xi_{j3}} & \cdots & r_{\xi_{11}\xi_{jN_j}} \\
 r_{\xi_{12}\xi_{j1}} & r_{\xi_{12}\xi_{j2}} & r_{\xi_{12}\xi_{j3}} & \cdots & r_{\xi_{12}\xi_{jN_j}} \\
 r_{\xi_{13}\xi_{j1}} & r_{\xi_{13}\xi_{j2}} & r_{\xi_{13}\xi_{j3}} & \cdots & r_{\xi_{13}\xi_{jN_j}} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 r_{\xi_{iN_i}\xi_{j1}} & r_{\xi_{iN_i}\xi_{j2}} & r_{\xi_{iN_i}\xi_{j3}} & \cdots & r_{\xi_{iN_i}\xi_{jN_j}} \\
\end{bmatrix}_{N_i \times N_j}$

Note that the cross-correlation matrix, cross-covariance matrix and matrix of cross-correlation coefficients are symmetric if $N_i$ equals $N_j$. Two random vectors $\xi_i$ and $\xi_j$ are uncorrelated if $K_{\xi_i, \xi_j} = 0$. This condition is equivalent to $E[\xi_i \xi_j^T] = E[\xi_i]E[\xi_j]^T$ or $R_{\xi_i, \xi_j} = 0$. A set of random vectors is completely pair-wise uncorrelated if $K_{\xi_i, \xi_j} = 0$ for all $i \neq j$. A pair of random vectors is SI if

$p_{\xi_i, \xi_j}(X_i, X_j) = p_{\xi_i}(X_i)p_{\xi_j}(X_j)$

A set of random vectors is completely pair-wise SI if

$p_{\xi_i, \xi_j}(X_i, X_j) = p_{\xi_i}(X_i)p_{\xi_j}(X_j)$ \hspace{1cm} \text{for all } i \neq j

Complete pair-wise statistical independence implies complete pair-wise uncorrelation, but the converse is not true in general.
The (joint) conditional PDF for a set of random vectors conditioned on the event $A$ is defined as

$$p_{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L | A}(X_1, X_2, X_3, \ldots, X_L | A) =$$

$$= \frac{p_{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L, A}(X_1, X_2, X_3, \ldots, X_L | A)}{P_r(A)}$$

for $P_r(A) \neq 0$

This conditional PDF is the probability that the experimental values of random vectors $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L$ obtained on a performance of the experiment will take on the values $X_1, X_2, X_3, \ldots, X_L$, respectively, given that event $A$ has occurred.

The expected value of a function $g(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L)$ of a set of random vectors conditioned on the event $A$ is defined as

$$E_{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L | A}[g(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L) \cdot p_{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L | A}(X_1, X_2, X_3, \ldots, X_L | A) \cdot dX_1 \cdot dX_2 \cdot dX_3 \cdots dX_L$$

A function of a pair of random vectors defines a new random vector. If $A_i$ and $A_j$ are matrices and $\mathbf{b}$ is a vector, then $\mathbf{y}$ can be formed from a linear combination of the random vectors $\mathbf{z}_i$ and $\mathbf{z}_j$ as

$$\mathbf{y} = A_i \mathbf{z}_i + A_j \mathbf{z}_j + \mathbf{b}$$

Here, $\mathbf{y}$ has a mean vector equal to

$$\mu_\mathbf{y} = A_i \mu_{\mathbf{z}_i} + A_j \mu_{\mathbf{z}_j} + \mathbf{b}$$

and a covariance matrix equal to

$$K_{\mathbf{y}\mathbf{y}} = A_i K_{\mathbf{z}_i \mathbf{z}_i} A_i^T + A_i K_{\mathbf{z}_i \mathbf{z}_j} A_j^T + A_j K_{\mathbf{z}_j \mathbf{z}_i} A_i^T + A_j K_{\mathbf{z}_j \mathbf{z}_j} A_j^T$$

Note that if $\mathbf{z}_i$ and $\mathbf{z}_j$ are uncorrelated, then

$$K_{\mathbf{y}\mathbf{y}} = A_i K_{\mathbf{z}_i \mathbf{z}_i} A_i^T + A_j K_{\mathbf{z}_j \mathbf{z}_j} A_j^T$$
A.3 Random Processes

A random process $x(t)$ is a rule by which at each instant of time a random variable is assigned to $x$. Thus, if time is held fixed at some value $t_1$, then $x(t_1)$ is a random variable. The first order PDF for $x(t)$, denoted as $p_{x(t_1)}(X(t_1))$, characterizes the random variable $x(t_1)$ for all possible values of $t_1$. The $N^{th}$ order PDF for $x(t)$, denoted

$$p_{x(t_1),x(t_2),x(t_3),...,x(t_N)}(X(t_1), X(t_2), X(t_3),...,X(t_N))$$

caracterizes the set of random variables $x(t_1), x(t_2), x(t_3),...,x(t_N)$ for all possible combinations of the times $t_1, t_2, t_3,\ldots,t_N$. In general, complete characterization of the stochastic process $x(t)$ requires the $N^{th}$ order PDF for every $N$ and every choice of $t_1, t_2, t_3,\ldots,t_N$. The first-order characterization of a random process is its mean function $\mu_x(t)$. At a fixed time $t_1$, $\mu_x(t_1)$ is the mean value of the random variable $x(t_1)$. Second-order characterization consists of the mean function combined with either the correlation function or the covariance function. The correlation function is defined as

$$C_{xx}(t_1,t_2) \equiv \mathcal{E}[x(t_1)x(t_2)]$$

and is a function of the times $t_1$ and $t_2$. If time is fixed, then $C_{xx}(t_1,t_2)$ is the correlation of the two random variables $x(t_1)$ and $x(t_2)$. The covariance function is defined as

$$K_{xx}(t_1,t_2) \equiv \mathcal{E}[\{x(t_1)-\mu_x(t_1)\}{x(t_2)-\mu_x(t_2)}]$$

$$= C_{xx}(t_1,t_2) - \mu_x(t_1)\mu_x(t_2)$$

The correlation coefficient function can be defined as

$$R_{xx}(t_1,t_2) = \frac{K_{xx}(t_1,t_2)}{\sigma_x(t_1)\sigma_x(t_2)}$$
If time is fixed, then $K_{xx}(t_1, t_2)$ and $R_{xx}(t_1, t_2)$ are the covariance and correlation coefficient, respectively, of the two random variables $x(t_1)$ and $x(t_2)$.

A random process $x(t)$ is wide-sense stationary (WSS) if its mean is constant and its correlation function depends only on the time difference $\tau = t_1 - t_2$. Thus,

$$C_{xx}(t_1, t_2) = C_{xx}(t_1 - t_2, 0) = C_{xx}(\tau, 0)$$

and is denoted

$$C_{xx}(\tau)$$

Note also that $\mu_x(t)$ can be written as $\mu_x$. For a WSS continuous time process, the power spectral density (PSD) $S_{xx}(j\omega)$ is defined as the Fourier Transform of the correlation function;

$$S_{xx}(j\omega) = \int_{-\infty}^{\infty} C_{xx}(\tau) e^{-j\omega \tau} d\tau$$

In discrete time, the PSD of a WSS process is defined as the Discrete Fourier Transform of the correlation function;

$$S_{xx}(e^{j\omega}) = \sum_{\tau=-\infty}^{\infty} C_{xx}(\tau) e^{-j\omega \tau}$$

A process is called a white process of intensity $q$ if

$$C_{xx}(\tau) = q \delta(\tau)$$

where

$$\delta(\tau) = \begin{cases} 1, & \tau = 0 \\ 0, & \text{otherwise} \end{cases}$$

Such a process has a PSD of constant value $q$.

A random process is ergodic if its ensemble averages equal appropriate time averages. In particular, a random process $x(t)$ with constant mean $\mu_x(t) = \mu_x$
is mean-ergodic if the time average value of the process approaches $\mu_x$ as the averaging time grows toward infinity. A WSS random process $x(t)$ is mean-ergodic if the area under the absolute value of its covariance function is bounded or if the random variables $x(t)$ and $x(t + \tau)$ are uncorrelated for large $\tau$. A WSS random process $x(t)$ is correlation-ergodic if the time average value of the product $x(t)x(t + \tau)$, for all $t$ and $\tau$, approaches the correlation function $C_{xx}(\tau)$ as the averaging time grows toward infinity.

A set of random processes $x_1(t), x_2(t), x_3(t), \ldots, x_L(t)$ has a first order joint PDF which is denoted as

$$p_{x_1(t), x_2(t), x_3(t), \ldots, x_L(t)}(X_1(t), X_2(t), X_3(t), \ldots, X_L(t))$$

Important second order characterizations are the cross-correlation function,

$$C_{x_i x_j}(t_1, t_2) \equiv \mathbb{E}[x_i(t_1)x_j(t_2)]$$

the cross-covariance function,

$$K_{x_i x_j}(t_1, t_2) \equiv \mathbb{E}[(x_i(t_1) - \mu_{x_i}(t_1))(x_j(t_2) - \mu_{x_j}(t_2))]$$

$$= C_{x_i x_j}(t_1, t_2) - \mu_{x_i}(t_1)\mu_{x_j}(t_2)$$

and the cross-correlation coefficient function,

$$R_{x_i x_j}(t_1, t_2) = \frac{K_{x_i x_j}(t_1, t_2)}{\sigma_{x_i(t_1)}\sigma_{x_j(t_2)}}$$

Two random processes $x_i(t)$ and $x_j(t)$ are uncorrelated if $K_{x_i x_j}(t_1, t_2) = 0$ for all $t_1$ and $t_2$. This condition is equivalent to $\mathbb{E}[x_i(t_1)x_j(t_2)] = \mathbb{E}[x_i(t_1)]\mathbb{E}[x_j(t_2)]$ for all $t_1$ and $t_2$, or $R_{x_i x_j}(t_1, t_2) = 0$ for all $t_1$ and $t_2$. A set of random processes is completely pair-wise uncorrelated if $K_{x_i x_j}(t_1, t_2) = 0$ for all $i \neq j$ and for all $t_1$ and $t_2$. A pair of random processes is SI if

$$p_{x_i(t), x_j(t)}(X_i(t), X_j(t)) = p_{x_i(t)}(X_i(t))p_{x_j(t)}(X_j(t))$$
A set of random processes is completely pair-wise SI if

\[ p_{x_i(t), x_j(t)}(X_i(t), X_j(t)) = p_{x_i(t)}(X_i(t)) p_{x_j(t)}(X_j(t)) \text{ for all } i \neq j \]

Complete pair-wise statistical independence implies complete pair-wise uncorrelation, but the converse is not true in general.

A pair of random processes \( x_i(t) \) and \( x_j(t) \) are jointly wide sense stationary (JWSS) if they are individually WSS and their cross-correlation function depends only on the time difference \( \tau = t_1 - t_2 \). Thus,

\[ C_{x_i x_j}(t_1, t_2) = C_{x_i x_j}(t_1 - t_2, 0) = C_{x_i x_j}(\tau, 0) \]

and is denoted

\[ C_{x_i x_j}(\tau) \]

For a JWSS continuous time process, the cross power spectral density (CPSD) \( S_{x_i x_j}(j\omega) \) is defined as the Fourier Transform of the cross-correlation function. In discrete time, the CPSD of a JWSS process is defined as the Discrete Fourier Transform of the cross-correlation function. Note that two uncorrelated JWSS processes have a null-valued CPSD.

If a real-valued continuous time WSS random process \( x(t) \) is the input to a linear time-invariant system with frequency response \( H(j\omega) \), then the output of the linear system \( y(t) \) is also a real-valued continuous time WSS random process. The output has a mean value of

\[ \mu_y = \mu_x \int_{-\infty}^{\infty} h(\tau) \, d\tau = \mu_x H(0) \]

where \( h(t) \) is the impulse response of the linear system. The PSD of the output is

\[ S_{yy}(j\omega) = H(j\omega) H(-j\omega) S_{xx}(j\omega) = |H(j\omega)|^2 S_{xx}(j\omega) \]
The CPSD of the output to the input is

\[ S_{yx}(j\omega) = H(j\omega) S_{xx}(j\omega) \]

If the above system is realized in discrete time, the real-valued WSS output has a mean value of

\[ \mu_y = \mu_x \sum_{\tau=-\infty}^{\infty} h(\tau) = \mu_x H(1) \]

a PSD of

\[ S_{yy}(e^{j\omega}) = H(e^{j\omega}) H(e^{-j\omega}) S_{xx}(e^{j\omega}) = |H(e^{j\omega})|^2 S_{xx}(e^{j\omega}) \]

and a CPSD of

\[ S_{yx}(e^{j\omega}) = H(e^{j\omega}) S_{xx}(e^{j\omega}) \]

If time is discrete and the random process is observed over a finite duration, then the observed segment of the random process can be described more easily as a random vector. Let a discrete-time random process \( x(t) \) be observed at the times \( t_1, t_2, t_3, \ldots, t_N \). Let the random variables \( x_1, x_2, x_3, \ldots, x_N \) be the random variables assigned to \( x(t) \) at each of the designated times \( t_1, t_2, t_3, \ldots, t_N \), respectively. Then, the random vector

\[
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}_{N \times 1}
\]

can be formed. Complete characterization of the random vector \( \mathbf{x} \) is equivalent to complete characterization of the discrete time random process \( x(t) \) over the observed segment.
A.4 Maximum Likelihood Estimation of a Single Parameter

In a single parameter estimation problem a probabilistic model for a system is developed, this model being parameterized by a single unknown parameter. The role of estimation is to observe the system and then choose the most desirable value for the unknown parameter. In maximum likelihood estimation (MLE) of a single parameter, the term "most desirable" is interpreted to mean that value of the parameter which, based on the system model, provides the largest probability of occurrence of the observed data.

If the parameter to be estimated is denoted \( a \), then its estimate will be denoted \( \hat{a} \). Let the system be observed at discrete times, the observation data being denoted by the vector \( \mathbf{x} \). The probabilistic model for the system is then denoted \( p_{x|a}(X|A) \) (read as "the probability that the experimental value of random vector \( \mathbf{x} \) obtained on a performance of the experiment will take on the value \( X \), given that parameter \( a \) has the value \( A \)). Here, the MLE problem can be written mathematically as

\[
\hat{a} = \arg \max_A p_{x|a}(X|A)
\]

Thus, the maximization takes place over all possible values of \( A \). In general, \( p_{x|a}(X|A) \) is maximized with respect to \( A \) by use of elementary calculus.

Performance evaluation of the maximum likelihood estimate is based on the estimation error, \( \text{error} \), defined as

\[
\text{error} = A - \hat{a}
\]

The bias \( \mu_{\text{error}} \) is the conditional expected value of the estimation error;

\[
\mu_{\text{error}} = \mathbb{E}_{\mathbf{x}|a}[\text{error}]
\]
The error variance $\sigma_{\text{error}}^2$ is the conditional variance of the estimation error;

$$\sigma_{\text{error}}^2 = \mathbb{E}_{\mathbb{E}}[(\text{error} - \mu_{\text{error}})^2]$$

The mean-square error $MSE$ is the conditional second moment of the estimation error;

$$MSE_{\text{a}} = \mathbb{E}_{\mathbb{E}}[\text{error}^2] = \sigma_{\text{error}}^2 + \mu_{\text{error}}^2$$

Note that all of these performance measures can be a function of the unknown parameter $A$. 
Appendix B

Gaussian Random Variables, Vectors and Processes

1This discussion of Gaussian random variables, vectors and processes is adapted from Drake (1967), Papoulis (1984) and Shapiro and Willsky (1988).
\section*{B.1 The Gaussian Distribution}

A random variable \( x \) is a Gaussian (or normal) random variable if and only if it has a PDF of the form

\[
p_x(X) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2s^2}} \quad -\infty \leq X \leq \infty
\]

for some parameters \( m \) and \( s^2 \geq 0 \). Such a Gaussian PDF is denoted as

\[p_x(X) = \mathcal{G}(X; m, s^2)\]

This random variable has a mean value \( \mu_x = m \) and a variance \( \sigma_x^2 = s^2 \). Figure B.1 shows the Gaussian PDF. Note that a Gaussian random variable is completely specified by its mean value and variance. For \( p_x(X) = \mathcal{G}(X; m, s^2) \) and two constants \( a_1 \) and \( a_2 \), the random variable formed as \( y = a_1 x + a_2 \) is also Gaussian with PDF

\[
p_y(Y) = \mathcal{G}(Y; a_1 \mu_x + a_2, a_1^2 \sigma_x^2)
\]

\[
= \mathcal{G}(Y; a_1 m + a_2, a_1^2 s^2)
\]

A set of \( N \) Gaussian random variables \( x_1, x_2, x_3, \ldots, x_N \) is said to be jointly Gaussian if and only if for every set of non-trivial constants \( a_1, a_2, a_3, \ldots, a_N \) the random variable formed as

\[y = a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots + a_N x_N\]

is Gaussian with PDF

\[p_y(Y) = \mathcal{G}(Y; \mu_y, \sigma_y^2)\]

where

\[
\mu_y = a_1 \mu_{x_1} + a_2 \mu_{x_2} + a_3 \mu_{x_3} + \ldots + a_N \mu_{x_N}
\]
Figure B.1: The Gaussian Probability Distribution

The probability density function for the Gaussian random variable $x$ with mean value $m$ and variance $s^2$ is

$$p_x(X) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{(X-m)^2}{2s^2}} \quad -\infty \leq X \leq \infty$$
\begin{align*}
\sigma_y^2 &= \sum_{i=1}^{N} \sum_{j=1}^{N} a_i k_{x_i x_j} a_j \\
&= a_1 m_{x_1} + a_2 m_{x_2} + a_3 m_{x_3} + \ldots + a_N m_{x_N}
\end{align*}

Note that if the set of random variables \(x_1, x_2, x_3, \ldots, x_N\) is completely pair-wise uncorrelated, then

\begin{align*}
\sigma_y^2 &= a_1^2 \sigma_{x_1}^2 + a_2^2 \sigma_{x_2}^2 + a_3^2 \sigma_{x_3}^2 + \ldots + a_N^2 \sigma_{x_N}^2 \\
&= a_1^2 \sigma_{x_1}^2 + a_2^2 \sigma_{x_2}^2 + a_3^2 \sigma_{x_3}^2 + \ldots + a_N^2 \sigma_{x_N}^2
\end{align*}

A set of jointly Gaussian random variables is completely specified by the marginal mean values and all covariances. A pair of jointly Gaussian random variables has the special property that the two variables are independent if and only if they are uncorrelated. Accordingly, a set of jointly Gaussian random variables is completely pair-wise SI if and only if it is completely pair-wise uncorrelated.

A jointly Gaussian random vector \(\mathbf{x}\) can be constructed from the jointly Gaussian random variables \(x_1, x_2, x_3, \ldots, x_N\) as

\[
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}_{N \times 1}
\]

This random vector has the joint PDF

\[
p_{\mathbf{x}}(\mathbf{X}) = \frac{e^{\frac{-(\mathbf{X}-\mu_\mathbf{x})^T K_{\mathbf{xx}}^{-1} (\mathbf{X}-\mu_\mathbf{x})}{2}}}{(2\pi)^{N/2} |K_{\mathbf{xx}}|^{1/2}} \quad -\infty \leq \mathbf{X} \leq \infty
\]

denoted as

\[
p_{\mathbf{x}}(\mathbf{X}) = \mathcal{N}(\mathbf{X}; \mu_\mathbf{x}, K_{\mathbf{xx}})
\]

where \(\mu_\mathbf{x}\) and \(K_{\mathbf{xx}}\) are the mean vector and covariance matrix, respectively, of \(\mathbf{x}\).

Note that a jointly Gaussian random vector is completely specified by its mean.
vector and covariance matrix. In vector notation, joint Gaussianity implies that for every set of non-trivial constants

\[ a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix} \]

the random variable formed as

\[ y = a^T x = a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots + a_N x_N \]

is Gaussian with PDF

\[ p_y(Y) = G(Y; a^T \mu_x, a^T K_{xx} a) \]

Linear transformation of the jointly Gaussian random vector \( x \) by the matrix \( D \) forms the jointly Gaussian random vector \( y \) as

\[ y = D x \]

As shown in Appendix A, if the columns of \( D^T \) are constructed from \( N \) orthonormal eigenvectors of \( K_{xx} \), then \( y \) has a mean vector equal to

\[ \mu_y = D \mu_x \]

and a covariance matrix equal to

\[ K_{yy} = D K_{xx} D^T = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N) \]

where \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N \) are the eigenvalues of \( K_{xx} \). Such a transformation is important in two aspects. First, since the new random vector is jointly Gaussian, it is completely specified by its mean vector and covariance matrix. As shown
above, both of these quantities can be determined analytically from knowledge of the original random vector (namely $\mu_x$ and $K_{xx}$) and the transformation matrix $D$. Second, recall that for jointly Gaussian random variables uncorrelation implies statistical independence. Thus, not only are the entries of the new random vector uncorrelated, they are SI.

A joint set of jointly Gaussian random vectors $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \ldots, \mathbf{z}_L$ is completely specified by the marginal mean vectors, marginal covariance matrices and all cross-covariance matrices. A joint pair of jointly Gaussian random vectors is SI if and only if the two vectors are uncorrelated. Accordingly, a joint set of jointly Gaussian random vectors is completely pair-wise SI if and only if it is completely pair-wise uncorrelated.

A jointly Gaussian random process $z(t)$ is a random process for which at each instant of time one of a set of jointly Gaussian random variables is assigned to $z$. A jointly Gaussian random process is completely specified by its mean function and its covariance function. A WSS jointly Gaussian random vector is both mean-ergodic and correlation-ergodic if its correlation function $C_{xx}(\tau)$ approaches zero as $|\tau|$ approaches infinity. A joint set of jointly Gaussian random processes $x_1(t), x_2(t), x_3(t), \ldots, x_L(t)$ is completely specified by the marginal mean functions, marginal covariance functions and all cross-covariance functions. A joint pair of jointly Gaussian random processes is independent if and only if the two processes are uncorrelated. Accordingly, a joint set of jointly Gaussian random processes is completely pair-wise SI if and only if it is completely pair-wise uncorrelated. If a WSS Gaussian random process is the input to a linear time-invariant system, then the output of the linear system is a WSS Gaussian random process. Thus, the output is completely specified via its mean function.
and covariance function.

B.2 The Chi Distribution

The Chi distribution is an important distribution which is derived from the Gaussian distribution. A random variable $\chi$ is a Chi random variable if it is formed from a joint set of $N$ independent, identically distributed Gaussian random variables $x_1, x_2, x_3, \ldots, x_N$ of zero mean and variance $s^2$ as

$$
\chi = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_N^2}
$$

The Chi random variable has a PDF of the form

$$
p_{\chi}(\chi) = \begin{cases} 
\frac{2^{\frac{N-1}{2}} e^{-\frac{\chi^2}{2s^2}}}{(s \sqrt{2})^N \Gamma(N/2)} & \chi > 0 \\
0 & \text{otherwise}
\end{cases}
$$

where $\Gamma(\alpha)$ is the gamma function, defined for $\alpha$ positive as

$$
\Gamma(\alpha) = \int_{y=0}^{\infty} y^{\alpha-1} e^{-y} \, dy \quad \alpha > 0
$$

The first and second order characteristics of the Chi distribution are (Hogan and Mann, 1980a)²

$$
\mu_{\chi} = s \sqrt{2} \frac{\Gamma(N+1)}{\Gamma(N/2)}
$$

$$
\mathcal{E}(\chi^2) = N s^2
$$

$$
\sigma^2_{\chi} = s^2 \left[ N - 2 \frac{\Gamma^2(N+1)}{\Gamma^2(N/2)} \right]
$$

²Hogan and Mann (1980a) actually determine the moments of the related random variable

$$\chi^* = \left[ \frac{x_1^2 + x_2^2 + x_3^2 + \cdots + x_N^2}{N} \right]^{1/2a}
$$

where $a$ is a constant. By assigning the value one to the constant $a$ and multiplying by $\sqrt{N}$, the relevant moments have been calculated.
For $N$ an integer, all of the arguments to the gamma functions in the above expressions will be either an integer or an integer plus one half. Evaluation of the gamma function in these expressions can be simplified by use of the relations;

$$
\Gamma(\alpha + 1) = \alpha \Gamma(\alpha)
$$

$$
\Gamma(1) = 1
$$

$$
\Gamma(1/2) = \sqrt{\pi}
$$

In particular, it can be shown that

$$
\Gamma(\alpha/2) = \sqrt{\pi} \frac{(1)(3)(5)\ldots(\alpha-2)}{2^{\frac{\alpha-1}{2}}} \quad \text{for } \alpha \text{ a positive odd integer}
$$

$$
\Gamma(\alpha/2) = \frac{(2)(4)(6)\ldots(\alpha-2)}{2^{\frac{\alpha-1}{2}}} = \left(\frac{\alpha}{2} - 1\right)! \quad \text{for } \alpha \text{ a positive even integer}
$$

For example, if $N$ is even, then

$$
\mu_x = s \sqrt{\frac{\pi}{2}} \frac{(1)(3)(5)\ldots(N-1)}{(2)(4)(6)\ldots(N-2)}
$$
Appendix C

Processing the Constant-Force Contraction Multichannel Electromyogram \(^1\)

\(^1\)Much of this discussion of optimal EMG processing is adapted from Hogan and Mann (1980a, 1980b).
C.1 A Functional Model of Measured Surface EMG

The measured surface electromyogram (MSEMG) waveform is a complex spatial-temporal interference pattern of the electrical activity of the various underlying muscle tissues. As shown in Figure C.1, this interference pattern is zero mean (a characteristic imposed by the AC coupled EMG electrodes), and excursions about the mean increase with increased muscle activation. If the MSEMG waveform is viewed as a random signal, a given sample of the MSEMG waveform closely resembles a Gaussian distributed random variable. The standard deviation of the MSEMG signal increases with an increase in muscle force. If the MSEMG signal is observed over a window of time, the observed signal resembles a band-limited Gaussian random process. Thus, several investigators have modeled the MSEMG signal as being formed from the multiplication of a unit intensity, zero mean, WSS, band-limited, CE, jointly Gaussian process and a control signal. Figure C.2 depicts this model. The control signal represents the EMG amplitude of the muscle. Such a model describes the electrical phenomenon, but in no way accounts for details of the underlying physiology.

The band-limited Gaussian process can be modeled as being formed from a zero mean, WSS, white, CE, jointly Gaussian process of unit intensity passed through an LTI shaping filter which is stable, causal, and whose inverse exists and is stable and causal. (Recall from Appendix A, that since the input to the shaping filter is WSS, the filter output is also WSS.) It will be assumed that the shaping filter characterizes the filtering effects of EMG propagation through the body tissues, as well as the recording properties of the electrodes. The shaping filter accounts for all of the time dependence in the MSEMG signal. The present
Figure C.1: Measured Surface EMG Waveform and Corresponding Joint Torque

A) Measured surface EMG waveform recorded from the triceps muscle with a bipolar electrode during an isometric non-fatiguing contraction. EMG is normalized to its maximum value in this trial.
B) Torque generated about the elbow during elbow extension for the same trial shown in (A) above. Torque is normalized to its maximum value in this trial.
Figure C.2: Multiplicative Model of Measured Surface EMG Waveform

A) Discrete-time case. The measured surface EMG waveform $m_i$ is formed from the multiplication of a unit intensity, zero mean, wide-sense stationary band-limited, correlation-ergodic, jointly Gaussian process $n_i$ and the EMG amplitude $s_i$.

B) Continuous-time case. The measured surface EMG waveform $m(t)$ is formed from the multiplication of a unit intensity, zero mean, wide-sense stationary band-limited, correlation-ergodic, jointly Gaussian process $n(t)$ and the EMG amplitude $s(t)$. 

MSEMG waveform analysis will be limited to the case of constant force, non-fatiguing contractions. Thus, the EMG amplitude simplifies to the constant $s$. This single channel model is shown in Figure C.3. It is assumed that the level of constant force contraction is directly related to the EMG amplitude $s$ (the standard deviation of the MSEMG signal). Thus, the single channel MSEMG waveform processing problem is formulated as performing a standard deviation estimate on a zero mean, WSS, CE, jointly Gaussian process.

When several electrodes are placed over a muscle, several, possibly correlated, MSEMG signals are recorded. Figure C.4 extends the single channel MSEMG waveform model to a multiple channel model. $L$ independent, zero mean, JWSS, white, CE, jointly Gaussian processes of unit intensity are passed through an $L$-input, $L$-output, LTI shaping filter, which is stable, causal, and whose inverse exists and is stable and causal. This multi-dimensional shaping filter is restricted to account only for the spatial dependence between channels, including differences in signal strength. Such a restriction requires that outputs of the multi-dimensional filter can only be based on knowledge of the present inputs. Any use of past inputs would imply a contribution to the temporal correlation in the MSEMG signal. Hence, the multi-dimensional filter has no dynamics and can be represented as a linear transformation. The $L$ outputs from the multi-dimensional shaping filter are passed through a bank of LTI shaping filters to form $L$ dependent, zero mean, JWSS, non-white, CE, jointly Gaussian processes. The bank of shaping filters account for all of the time dependence in the MSEMG signal. These shaping filters are stable, causal, and have an inverse which is stable and causal. Herein, the multiple channel MSEMG waveform processing problem is formulated as estimating the common standard deviation
Zero Mean, WSS, CE, Jointly Gaussian, White Process of Unit Intensity

Filtering Effects of Muscle Tissue, Bone, Skin and Electrodes

Measured Surface EMG

Figure C.3A: Discrete-Time Functional Model of a Single Channel of EMG

A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity is passed through the stable, causal, inversely stable, inversely causal, linear, time-invariant shaping filter $H_{time}(e^{j\omega})$ and multiplied by the EMG amplitude $s$ to form the measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure C.3B: Continuous-Time Functional Model of a Single Channel of EMG

A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity is passed through the stable, causal, inversely stable, inversely causal, linear, time-invariant shaping filter $H_{\text{time}}(j\omega)$ and multiplied by the EMG amplitude $s$ to form the measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing. [Adapted from Hogan and Mann, 1980a]
Figure C.4A: Discrete-Time Functional Model of Multiple Channels of EMG

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{space}$ which accounts only for the spatial dependence between channels. These filter outputs are each passed through a shaping filter $H_{time,j}(e^{j\omega})$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure C.4B: Continuous-Time Functional Model of Multiple Channels of EMG

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{space}$ which accounts only for the spatial dependence between channels. These filter outputs are each passed through a shaping filter $H_{time,j}(j\omega)$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
(EMG amplitude) of $L$ zero mean, JWSS, CE, jointly Gaussian processes.

C.2 Optimization of Several EMG Configurations

In practice, the complete single and multiple channel models depicted in Figures C.3 and C.4, respectively, may neither be necessary nor desired. If the MSEM signal is sampled slowly, the temporal correlation of samples within a channel can be ignored. For many muscles, certain electrode configurations would preclude the need to consider the spatial correlation between channels. Particular applications will determine the appropriate MSEM waveform model. Accordingly, MSEM waveform processing will be discussed separately for the following six cases:

I) Single Channel: Uncorrelated Samples
II) Single Channel: Correlated Samples
III) Multiple Channels: Uncorrelated Samples, Uncorrelated Channels
IV) Multiple Channels: Uncorrelated Samples, Correlated Channels
V) Multiple Channels: Correlated Samples, Uncorrelated Channels
VI) Multiple Channels: Correlated Samples, Correlated Channels

For each case, an optimal estimate of the EMG amplitude will be derived. In all of the optimization cases, "optimal" will be taken to mean optimal estimation of the EMG amplitude $s$ in the maximum likelihood sense. Each formulation will consider both the discrete-time case and the continuous-time case. Only causal estimators will be considered since they are appropriate for eventual implementation in real-time systems. Finally, all of the analysis will assume that the estimate is made by observing the MSEM signal over a finite time duration.
C.2.1 Case I — Single Channel: Uncorrelated Samples

To achieve uncorrelated samples in the single channel model, the shaping filter becomes an all-pass filter, yielding the model shown in Figure C.5. In the discrete-time case, if a single channel of continuous MSEM activity \( m(t) \) is sampled periodically over a finite time duration \( T \), the \( N \) samples of MSEM activity can be denoted as the \( N \) random variables \( m_1, m_2, m_3, \ldots, m_N \). By assumption, these random variables are uncorrelated and each random variable has the Gaussian PDF with mean value zero and standard deviation \( \sigma \). To perform MLE, it will be assumed that \( \sigma \) has the known value \( \hat{\sigma} \). This event gives the conditional density

\[
p_{m_i \mid \hat{\sigma}}(M_i \mid \hat{\sigma}) = G(M_i; 0, \hat{\sigma}^2) = \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{(M_i)^2}{2\hat{\sigma}^2}} \quad -\infty \leq M_i \leq \infty
\]

for all \( i \). Since the random variables are both jointly Gaussian and uncorrelated, they are independent. Hence, the conditional joint PDF for the \( N \) random variables is just the product of the individual conditional PDF's;

\[
p_{m_1, m_2, m_3, \ldots, m_N \mid \hat{\sigma}}(M_1, M_2, M_3, \ldots, M_N \mid \hat{\sigma}) =
\]

\[
= \prod_{i=1}^{N} p_{m_i \mid \hat{\sigma}}(M_i \mid \hat{\sigma})
\]

\[
= \prod_{i=1}^{N} \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{-\frac{M_i^2}{2\hat{\sigma}^2}} \quad -\infty \leq M_i \leq \infty
\]

\[
= \frac{1}{(2\pi\hat{\sigma}^2)^{N/2}} e^{-\frac{\sum_{i=1}^{N} M_i^2}{2\hat{\sigma}^2}} \quad -\infty \leq M_i \leq \infty
\]

The maximum likelihood estimate of the standard deviation is the value of \( \hat{\sigma} \) which maximizes the above density. A monotonic transformation of the density does not alter the location of the maximum. Herein, it will prove advantageous to maximize the natural logarithm of the density. Taking the natural logarithm
A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity is multiplied by the EMG amplitude $s$ to form the measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.

A) Discrete-time case.
B) Continuous-time case.
of the density yields
\[
\ln p_{m_1, m_2, m_3, \ldots, m_N | \hat{s}}(M_1, M_2, M_3, \ldots, M_N | \hat{s}) = -\frac{N}{2} \ln 2\pi - N \ln \hat{s} - \frac{1}{2\hat{s}^2} \sum_{i=1}^{N} M_i^2
\]
Differentiating the above with respect to \( \hat{s} \) gives
\[
\frac{\partial}{\partial \hat{s}} \ln p_{m_1, m_2, m_3, \ldots, m_N | \hat{s}}(M_1, M_2, M_3, \ldots, M_N | \hat{s}) = -\frac{N}{\hat{s}} + \frac{1}{\hat{s}^3} \sum_{i=1}^{N} M_i^2
\]
Setting this derivative to zero and solving for \( \hat{s} \) gives the desired estimate;
\[
\hat{s} = \left[ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right]^{1/2}
\]
For completeness, it will be shown that \( \hat{s} \) above is, in fact, a maximum (and not, for example, a local minimum). To do so, the second derivative of the maximizing function is taken;
\[
\frac{\partial^2}{\partial \hat{s}^2} \ln p_{m_1, m_2, m_3, \ldots, m_N | \hat{s}}(M_1, M_2, M_3, \ldots, M_N | \hat{s}) = \frac{N}{\hat{s}^2} - \frac{3}{\hat{s}^4} \sum_{i=1}^{N} M_i^2
\]
Now, substitution of the estimate \( \hat{s} \) into the second derivative gives
\[
\frac{\partial^2}{\partial \hat{s}^2} \ln p_{m_1, m_2, m_3, \ldots, m_N | \hat{s}}(M_1, M_2, M_3, \ldots, M_N | \hat{s}) = \frac{N^2}{\sum_{i=1}^{N} M_i^2} - \frac{3N^2 \sum_{i=1}^{N} M_i^2}{[\sum_{i=1}^{N} M_i^2]^2}
\]
\[
= \frac{-2N^2}{\sum_{i=1}^{N} M_i^2} \leq 0
\]
Since this second derivative is always less than or equal to zero, \( \hat{s} \) is a maximum. (Note that, in general, maximization requires evaluating the function at all local maxima and all boundaries. For this problem, \( \hat{s} \) is bounded at zero, which does not alter the result.)

Alternatively in the discrete time case, the \( N \) MSEM samples could have been formulated as the random vector \( \mathbf{m} \), where
\[
\mathbf{m} = \begin{bmatrix}
m_1 \\
m_2 \\
m_3 \\
\vdots \\
m_N
\end{bmatrix}
\]
The conditional PDF for this random vector, given that each element has the standard deviation \( \hat{s} \), is
\[
p_{m|m}(M|\hat{s}) = \frac{e^{-\frac{M^T K_{mm}^{-1} M}{\hat{s}^2}}}{(2\pi)^{N/2} |K_{mm}|^{1/2}} \quad -\infty \leq M \leq \infty
\]

Since the random vector \( m \) is merely a convenient manner in which to refer to the set of random variables \( m_1, m_2, m_3, \ldots, m_N \), the above density must be equivalent to that of the previous paragraph. Equivalence can be shown by knowledge of the covariance matrix. Since the random variables are uncorrelated and zero mean,
\[
\mathcal{E}[m_i m_j] = \begin{cases} 0 & i \neq j \\ \hat{s}^2 & i = j \end{cases}
\]

The covariance matrix is then
\[
K_{mm} = \begin{bmatrix}
\hat{s}^2 & 0 & 0 & \cdots & 0 \\
0 & \hat{s}^2 & 0 & \cdots & 0 \\
0 & 0 & \hat{s}^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & 0 \\
0 & 0 & \cdots & 0 & \hat{s}^2 \\
\end{bmatrix}_{N \times N} = \hat{s}^2 I
\]

Thus,
\[
K_{mm}^{-1} = \frac{1}{\hat{s}^2} I
\]

and
\[
|K_{mm}|^{1/2} = \hat{s}^N
\]

The conditional PDF for the random vector can now be written as
\[
p_{m|m}(M|\hat{s}) =
\]
\[
= \frac{e^{-\frac{\sum_{i=1}^{N} M_i^2}{2\hat{s}^2}}}{(2\pi)^{N/2} \hat{s}^N} \quad -\infty \leq M \leq \infty
\]
\[
= \frac{e^{-\frac{\sum_{i=1}^{N} M_i^2}{2\hat{s}^2}}}{(2\pi\hat{s}^2)^{N/2}} \quad -\infty \leq M_i \leq \infty
\]
This conditional density for the random vector is now clearly identical to the previous formulation. The vector formulation, however, has provided insight into the covariance between MSEMG samples. Such insight will prove useful in subsequent analysis cases.

To evaluate the performance of this discrete-time maximum likelihood estimator, note that \( \hat{s} \) is just a scaled version of the Chi random variable (i.e. scaled by the factor \( 1/\sqrt{N} \)). Applying the results of Appendix B, the first two moments and the variance of the estimate are:

\[
E[\hat{s}] = s \sqrt{\frac{2}{N}} \frac{\Gamma\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)}
\]

\[
E[\hat{s}^2] = s^2 \left[ 1 - \frac{2}{N} \frac{\Gamma^2\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} \right]
\]

Thus, the estimate bias is found as

\[
\mu_{\text{error}} = E_{\text{ml}}[s - \hat{s}] = s \left[ 1 - \sqrt{\frac{2}{N}} \frac{\Gamma\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} \right]
\]

In practice this bias is quite small. For example, with \( N = 30 \), the bias is already less than one percent of the true value of the parameter. The estimate \( MSE \) is

\[
MSE_{\hat{s}} = E_{\text{ml}}[(s - \hat{s})^2] = 2s^2 \left[ 1 - \sqrt{\frac{2}{N}} \frac{\Gamma\left(\frac{N+1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} \right]
\]

The estimate error variance is

\[
\sigma^2_{\text{error}} = s^2 \left[ 1 - \frac{2}{N} \frac{\Gamma^2\left(\frac{N+1}{2}\right)}{\Gamma^2\left(\frac{N}{2}\right)} \right]
\]

Finally, it is of interest to define a signal to noise ratio (SNR) for this problem. At constant force the true EMG amplitude \( s \) is modeled as constant. For the purposes of a SNR calculation, the true value of \( s \) will be taken to be the expected
value of the estimate \( \hat{s} \). A SNR can then be defined as

\[
SNR \equiv \left[ \frac{\mathcal{E}[\hat{s}]}{\mathcal{E}[\hat{s} - \mu_s]^2} \right]^{1/2}
\]

For the present problem,

\[
SNR_3 = \left[ \frac{2 \Gamma^2(N+1)}{N \Gamma^2(N/2) - 2 \Gamma^2(N+1)} \right]^{1/2}
\]

For computational purposes, the following form for the SNR may be best;

\[
SNR_3 = \left[ \frac{N}{2} \left( \frac{\Gamma(N/2)}{\Gamma(N+1/2)} \right)^2 - 1 \right]^{-1/2}
\]

Hogan and Mann (1980a)\(^2\) give an approximation to this SNR when \( N \) is large as

\[
SNR_3 \approx \sqrt{2N} \quad \text{for } N \text{ large}
\]

A solution to the continuous-time problem for this case can be found from the discrete-time solution. Namely, let the number of MSEM samples \( N \) over the finite time duration \( T \) grow to infinity. Formally, note that if the sampling period is \( \Delta \tau \), then \( N = T/\Delta \tau \) samples occur in the time \( T \). These discrete samples, denoted \( m_i \), correspond to the value of the continuous-time MSEM waveform \( m(\tau_i) \) at the times \( \tau_i \). Now, letting \( N \) grow to infinity is equivalent to letting \( \Delta \tau \) shrink to zero. Thus, the continuous-time estimate \( \hat{s}(t) \) of the EMG amplitude is found as

\[
\hat{s}(t) = \lim_{\Delta \tau \to 0} \left[ \hat{s}_{N=\frac{t}{\Delta \tau}, M_i=m(\tau_i)} \right] = \lim_{\Delta \tau \to 0} \left[ \frac{\Delta \tau}{T} \sum_{i=1}^{T/\Delta \tau} m^2(\tau_i) \right]^{1/2}
\]

\(^2\)Hogan and Mann (1980a) actually provide the SNR approximation for the related estimate

\[
\hat{s} = \left[ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right]^{1/2a}
\]

where \( a \) is a constant. By assigning the value one to the constant \( a \), the relevant SNR approximation has been calculated.
Moving the limit inside the square root and $\Delta \tau$ inside the sum yields an integral equation as
\[
\hat{s}(t) = \left[ \frac{1}{T} \lim_{\Delta \tau \to 0} \sum_{i=1}^{T/\Delta \tau} m^2(\tau_i) \Delta \tau \right]^{1/2} = \left[ \frac{1}{T} \int_{t-T}^{t} m^2(\tau) \, d\tau \right]^{1/2}
\]
Since $N$ has grown to infinity, the performance of this continuous-time estimate is ideal. That is, $\hat{s}(t)$ is unbiased, has an MSE of zero and an infinite SNR. Physically, however, a continuous-time MSEM signal waveform with a white PSD is unrealizable. Thus, the present analysis case is never realized in continuous time.

In practice, the EMG amplitude $s$ is not constant and, viewed as a stochastic process, the discrete/ continuous MSEM signal is not WSS. However, the bandwidth of muscle force and, by assumption, the EMG amplitude is from zero to approximately five Hz, while the MSEM signal contains little information below approximately twenty Hz. Thus, the MSEM signal can be considered quasi-stationary. That is, if viewed via short time windows, the EMG amplitude is constant within a particular window, but can vary from one window to the next. Therefore, if a channel of continuous MSEM activity is sampled periodically for an indefinite period of time, the discrete-time non-linear filter depicted in Figure C.6A can be utilized to provide sequential optimal, causal estimates of the EMG amplitude. Similarly, Figure C.6B shows the continuous time non-linear filter which provides a continuous, optimal, causal estimate of the EMG amplitude based on the continuous MSEM signal.

### C.2.2 Case II — Single Channel: Correlated Samples

When successive samples of MSEM signal activity are correlated, the general single channel model of Figure C.3 models the generation of the MSEM signal.
Figure C.6: Optimal EMG Processor — Single Channel: Uncorrelated Samples

A) Discrete-time case. Samples of measured surface myoelectric activity are denoted $m_i$, $\hat{s}_i$ are the optimal causal estimates of the EMG amplitude, and $N$ is the window length.

B) Continuous-time case. Continuous measured surface myoelectric activity is denoted $m(t)$, $\hat{s}(t)$ is the optimal causal estimate of the EMG amplitude, and $T$ is the window time duration.
The shaping filter has a non-constant value. Again in discrete-time, denote $N$ periodically sampled values of the continuous MSEM activity as the random vector $\mathbf{m}$. Given that each element of $\mathbf{m}$ has the standard deviation $\hat{s}$,

$$p_{\text{ml}}(\mathbf{M}|\hat{s}) = \frac{e^{-\frac{\mathbf{M}^T K^{-1}_{\text{mm}} \mathbf{M}}{2}}}{(2\pi)^{N/2}|K_{\text{mm}}|^{1/2}} \quad -\infty \leq \mathbf{M} \leq \infty$$

This formulation is identical to the vector formulation of Case I, except now there is no simple form for the inverse of the covariance matrix $K_{\text{mm}}$. From Figure C.3, it is clear that

$$K_{\text{mm}} = s^2 K_{nn}$$

Assume that the inverse of $K_{nn}$ exists, denoted as

$$K_{nn}^{-1} = \begin{bmatrix} k'_{n_1n_1} & k'_{n_1n_2} & \cdots & k'_{n_1n_N} \\ k'_{n_2n_1} & k'_{n_2n_2} & \cdots & k'_{n_2n_N} \\ \vdots & \vdots & \ddots & \vdots \\ k'_{n_Nn_1} & k'_{n_Nn_2} & \cdots & k'_{n_Nn_N} \end{bmatrix}_{N \times N}$$

The conditional density for the MSEM samples can now be written as

$$p_{\text{ml}}(\mathbf{M}|\hat{s}) = \frac{e^{-\frac{1}{2\hat{s}^2} \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{ni, nj} M_i M_j}}{(2\pi \hat{s}^2)^{N/2}|K_{nn}|^{1/2}} \quad -\infty \leq \mathbf{M} \leq \infty$$

The maximum likelihood estimate of the standard deviation is the value of $\hat{s}$ which maximizes the above density. A monotonic transformation of the density does not alter the location of the maximum. Herein, it will prove advantageous to maximize the natural logarithm of the density. Taking the natural logarithm of the density yields

$$\ln p_{\text{ml}}(\mathbf{M}|\hat{s}) = -\frac{N}{2} \ln 2\pi - N \ln \hat{s} - \frac{1}{2} \ln |K_{nn}| - \frac{1}{2\hat{s}^2} \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{ni, nj} M_i M_j$$

Differentiating the above with respect to $\hat{s}$ gives

$$\frac{\partial}{\partial \hat{s}} \ln p_{\text{ml}}(\mathbf{M}|\hat{s}) = -\frac{N}{\hat{s}} + \frac{1}{\hat{s}^3} \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{ni, nj} M_i M_j$$
Setting this derivative to zero and solving for \( \hat{s} \) gives the desired estimate:

\[
\hat{s} = \left[ \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j \right]^{1/2}
\]

For completeness, it will be shown that \( \hat{s} \) above is, in fact, a maximum (and not, for example, a local minimum). To do so, the second derivative of the maximizing function is taken:

\[
\frac{\partial^2 \ln p_{m|s}(M|\hat{s})}{\partial \hat{s}^2} = \frac{N}{\hat{s}^2} - \frac{3N}{\hat{s}^4} \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j
\]

Now, substitution of the estimate \( \hat{s} \) into the second derivative gives:

\[
\frac{\partial^2 \ln p_{m|s}(M|\hat{s})}{\partial \hat{s}^2} = \frac{N^2}{\sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j} - \frac{3N^2 \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j}{\left[ \sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j \right]^2}
\]

\[
= \frac{-2N^2}{\sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j}
\]

This second derivative is less than or equal to zero according to:

\[
\frac{\partial^2 \ln p_{m|s}(M|\hat{s})}{\partial \hat{s}^2} \leq 0
\]

\[ \iff \]

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} k'_{n,n_j} M_i M_j \geq 0
\]

If this second derivative constraint is not satisfied, then maximization with respect to \( \hat{s} \) of the probability density will occur at the boundary constraint where \( \hat{s} = 0 \).

There are several problems inherent in this solution. First, although the covariance matrix \( K_{nn} \) is symmetric (in fact, it is a symmetric Toeplitz matrix), it need not be invertible. Second, even if an inverse exists, computation of the inverse will be quite difficult (consider that the matrix dimension \( N \) may be several hundred). Third, the estimate is computationally burdensome (an
$N^2$ procedure). And fourth, this algorithm does not suggest a solution to the continuous-time problem.

Fortunately, and alternative solution to this discrete-time estimation problem can be based on the filtering concept discussed in Case I. In particular, Case I showed that a filter could be utilized to produce successive estimates of the EMG amplitude from successive samples of the MSEM signal. For the present case, successive estimates of the EMG amplitude are also desired. Thus, begin the solution by assigning a filter $G_{\text{optimal}}(e^{j\omega})$ which, given an indefinite length sequence of periodically sampled values of the continuous MSEM signal, produces sequential causal optimal estimates of $s$. Figure C.7A depicts this filter. Since an optimal estimate exists for any, and thus every, sequence of $N$ sequential MSEM samples, the filter $G_{\text{optimal}}(e^{j\omega})$ must exist.

Next, consider the output $v_i$ of a stable, causal filter $G_{\text{white}}(e^{j\omega})$, whose inverse exists and is stable and causal, formed from the input $m_i$. Since the filter $G_{\text{white}}(e^{j\omega})$ is invertible, no information is lost due to the filter. Thus, optimal estimation of $s$ from the sequence $v_i$ is equivalent to optimal estimation of $s$ from the samples $m_i$. This equivalence must be so, since optimal estimation of $s$ from the sequence $v_i$ could always be accomplished by cascade of the filters $G_{\text{white}}^{-1}(e^{j\omega})$ and $G_{\text{optimal}}(e^{j\omega})$. Figure C.8A illustrates this argument.

Filtering of the sequence $m_i$ to the sequence $v_i$ can be exploited if $G_{\text{white}}(e^{j\omega})$ is (1) linear and (2) selected such that the sequence $v_i$ is white of intensity $s$. Since the $m_i$ are zero mean, WSS, CE, jointly Gaussian distributed, a linear filter $G_{\text{white}}(e^{j\omega})$ assures that the $v_i$ are also zero mean, WSS, CE, jointly Gaussian distributed. If additionally the sequence $v_i$ is white of intensity $s$, then, given
Figure C.7: Generic Optimal Filter — Single Channel: Correlated Samples

A) Discrete-time case. The filter $G_{\text{optimal}}(e^{j\omega})$ produces sequential causal optimal estimates of $s$ from an indefinite length sequence of periodically sampled values of the continuous measured surface myoelectric signal.

B) Continuous-time case. The filter $G_{\text{optimal}}(j\omega)$ produces a continuous causal optimal estimate of $s$ from the continuous measured surface myoelectric signal.
Figure C.8: Alternate Generic Optimal Filter — Single Channel: Correlated Samples

A) Discrete-time case. Cascade of the two filters $G_{\text{white}}(e^{j \omega})$ and $G_{\text{white}}^{-1}(e^{j \omega})$ forms an all-pass network which, combined with the filter $G_{\text{optimal}}(e^{j \omega})$, produces the optimal estimates $\hat{s}_i$.

B) Continuous-time case. Cascade of the two filters $G_{\text{white}}(j \omega)$ and $G_{\text{white}}^{-1}(j \omega)$ forms an all-pass network which, combined with the filter $G_{\text{optimal}}(j \omega)$, produces the optimal estimate $\hat{s}(t)$. 
that $s$ has the known value $\delta$, the random vector

$$u = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_N \end{bmatrix}_{N \times 1}$$

formed from $N$ successive members of the sequence $u_i$ must have the Gaussian joint PDF

$$p_{\mathbf{u}|\mathbf{y}}(\mathbf{y}|\delta) = \mathcal{G}(\mathbf{y}; \mathbf{0}, \delta^2 I)$$

$$= \frac{-\sum_{i=1}^{N} v_i^2}{(2\pi\delta^2)^{N/2}} \quad -\infty \leq V_i \leq \infty$$

From the previous analysis case, the optimal estimate of $s$ based on such a vector $y$ would be

$$\hat{s} = \left[ \frac{1}{N} \sum_{i=1}^{N} V_i \right]^{1/2}$$

What remains is a means by which to select the requisite $G_{white}(e^{j\omega})$, if it exists. The model for the MSEM waveform presented in Figure C.3 shows that the selection $G_{white}(e^{j\omega}) = H_{time}^{-1}(e^{j\omega})$ is the only possible filter which can produce a white sequence as its output. Further, since $H_{time}(e^{j\omega})$ was constrained to be linear, time-invariant, stable, causal, invertibly stable and invertibly causal, its inverse must exist and be stable, causal and linear time-invariant. Thus, $G_{white}(e^{j\omega})$ does exist and is found as $G_{white}(e^{j\omega}) = H_{time}^{-1}(e^{j\omega})$. The resultant discrete-time optimal causal filter for a single MSEM waveform channel with correlated samples is shown in Figure C.9A. The filter $G_{white}(e^{j\omega})$ is known as a whitening filter.

An argument identical to that above can be utilized to develop the optimal continuous time processor. Begin by assigning the filter $G_{optimal}(j\omega)$, shown in Figure C.7B, as the filter which produces the optimal causal estimate of $s$.
Figure C.9: Optimal EMG Processor — Single Channel: Correlated Samples

The estimate is formed as the square root of the smoothed, whitened measured surface myoelectric signal.
A) Discrete-time case.
B) Continuous-time case.
based on $m(t)$. Next, insert the all-pass network comprised of the stable, causal, causally invertible filter $G_{\text{white}}(j\omega)$ cascaded with its inverse $G_{\text{white}}^{-1}(j\omega)$, as in Figure C.8B. Since the filter $G_{\text{white}}(j\omega)$ is invertible, optimal estimation of $s$ from the filter output $v(t)$ is equivalent to optimal estimation of $s$ from $m(t)$. If $G_{\text{white}}(j\omega)$ is selected to be $H_{\text{time}}^{-1}(j\omega)$, then $v(t)$ is a WSS jointly Gaussian white process of intensity $s$. From the previous analysis case, optimal causal estimation of $s(t)$ from $v(t)$ is then

$$\hat{s}(t) = \left[ \frac{1}{T} \int_{t-T}^{t} v^{2}(\tau) \, d\tau \right]^{1/2}$$

Thus, the continuous time optimal causal filter for a single correlated MSEM signal waveform channel is shown in Figure C.9B.

Performance for both the discrete-time and the continuous-time maximum likelihood estimators can utilize the results of the previous analysis case. For the previous case of uncorrelated samples, $N$ denoted the sample size. More generally, the previous performance formulae are based on the number of degrees of freedom in the MSEM signal. For uncorrelated data, there are precisely $N$ degrees of freedom. In practice, however, perfect whitening can not be achieved. For correlated data, Hogan and Mann(1980b) suggest the use of an effective number of degrees of freedom $N_{\text{effective}}$ as

$$N_{\text{effective},II} = 2B_s T \quad \text{for Case II}$$

where $B_s$ is the statistical bandwidth of the MSEM signal. (Recall that for discrete data $T = N \triangle \tau$.) For a continuous waveform, $B_s$ is given as

$$B_s = \frac{\int_{-\infty}^{\infty} S_{mm}(j\omega) \, d\omega}{\int_{-\infty}^{\infty} S_{mm}^2(j\omega) \, d\omega}$$

for a continuous waveform.
For a discrete waveform, \( B_s \) will be estimated as
\[
B_s = \frac{\int_0^\pi S_{mm}(e^{i\omega}) \, d\omega}{\int_0^\pi S_{mm}^2(e^{i\omega}) \, d\omega}
\]
for a discrete waveform

### C.2.3 Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

To achieve uncorrelated samples and uncorrelated channels, the multiple channel MSEM waveform model reduces to that of Figure C.10. The channel gains \( g_j \), where \( g_j > 0 \), reflect the relative signal level of the different channels. In the discrete-time case, denote \( N \) periodically sampled values of the \( j^{th} \) channel of continuous MSEM activity \( m_j(t) \) as the random vector \( m_j \), where
\[
m_j = \begin{bmatrix} m_{j,1} \\ m_{j,2} \\ m_{j,3} \\ \vdots \\ m_{j,N} \end{bmatrix}_{N \times 1}
\]

Given that each element of \( m_j \) has the standard deviation \( g_j \hat{s} \),
\[
p_{m_j|M_j}(M_j|\hat{s}) = \frac{e^{-\frac{M_j^2 \alpha_{m_j}^{-1} M_j}{2}}}{(2\pi)^{N/2} |K_{m_j,m_j}|^{1/2}}
\]
- \( -\infty \leq M_j \leq \infty \)

By assumption, the random variables \( m_{j,1}, m_{j,2}, m_{j,3}, \ldots, m_{j,N} \) which comprise the random vector \( m_j \) are uncorrelated, thus \( m_j \) has the covariance matrix
\[
K_{m_j,m_j} = \begin{bmatrix}
g_j^2 \hat{s}^2 & 0 & 0 & \cdots & 0 \\
0 & g_j^2 \hat{s}^2 & 0 & \cdots & 0 \\
0 & 0 & g_j^2 \hat{s}^2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 \\
0 & 0 & \cdots & 0 & g_j^2 \hat{s}^2
\end{bmatrix}_{N \times N} = g_j^2 \hat{s}^2 I
\]

For \( L \) channels, the \( L \) random vectors \( m_1, m_2, m_3, \ldots, m_L \) are defined. Since the channels are uncorrelated,
\[
K_{m_j,m_k} = 0 \quad \text{for } j \neq k
\]

421
Figure C.10A: Discrete-Time EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are multiplied first by the gain factors $g_j$ ($g_j > 0$) and second by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure C.10B: Continuous-Time EMG Model Case III — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are multiplied first by the gain factors $g_j$ ($g_j > 0$) and second by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
There are, therefore, a total of $L \cdot N$ distinct random variables. These $L \cdot N$ random variables can be arranged as the single composite random vector $m_{j,i}$ defined as the partitioned matrix

$$m_{j,i} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_L \end{bmatrix}_{N \cdot L \times 1}$$

Since the channels are jointly Gaussian, this vector is jointly Gaussian with PDF as

$$p_{m_{j,i}|s}(M_{j,i}|s) = \frac{e^{-\frac{M_{j,i}^T K_{m_{j,i,m_{j,i}}}^{-1} M_{j,i}}{2}}}{(2\pi)^{NL/2} \sqrt{|K_{m_{j,i,m_{j,i}}}|^{1/2}}} - \infty \leq M_{j,i} \leq \infty$$

Further, the covariance matrix for $m_{j,i}$ can be related to the covariance matrices $K_{m_j, m_k}$ as the partitioned matrix

$$K_{m_{j,i}, m_{j,i}} = \begin{bmatrix} K_{m_1, m_1} & K_{m_1, m_2} & K_{m_1, m_3} & \cdots & K_{m_1, m_L} \\ K_{m_2, m_1} & K_{m_2, m_2} & K_{m_2, m_3} & \cdots & K_{m_2, m_L} \\ K_{m_3, m_1} & K_{m_3, m_2} & K_{m_3, m_3} & \cdots & K_{m_3, m_L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{m_L, m_1} & K_{m_L, m_2} & K_{m_L, m_3} & \cdots & K_{m_L, m_L} \end{bmatrix}_{N \cdot L \times N \cdot L}$$

For the present case of uncorrelated samples and uncorrelated channels, $K_{m_{j,i}, m_{j,i}}$ becomes the $N \cdot L$ by $N \cdot L$ diagonal matrix

$$K_{m_{j,i}, m_{j,i}} = \text{diag}(g_1^2 s^2, \ldots, g_1^2 s^2, g_2^2 s^2, \ldots, g_2^2 s^2, \ldots, g_L^2 s^2, \ldots, g_L^2 s^2)$$

Both the inverse and determinant of this matrix are easily computed as

$$K_{m_{j,i}, m_{j,i}}^{-1} = \text{diag} \left( \frac{1}{g_1^2 s^2}, \ldots, \frac{1}{g_1^2 s^2}, \frac{1}{g_2^2 s^2}, \ldots, \frac{1}{g_2^2 s^2}, \ldots, \frac{1}{g_L^2 s^2}, \ldots, \frac{1}{g_L^2 s^2} \right)$$

and

$$|K_{m_{j,i}, m_{j,i}}| = s^{2NL} g_1^{2N} g_2^{2N} g_3^{2N} \cdots g_L^{2N} = s^{2NL} \prod_{j=1}^{L} g_j^{2N}$$

424
Thus, the conditional PDF for the composite random vector can be written as

$$p_{m_j,i,s}(M_{j,i}|s) = \frac{e^{-\sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{2\sigma_j^2}}}{(2\pi)^{NL/2} \hat{s}^{NL} \prod_{j=1}^{L} g_j^N} \quad -\infty \leq M_{j,i} \leq \infty$$

MLE of the parameter $s$ is facilitated by taking the natural logarithm of the above PDF:

$$\ln p_{m_j,i,s}(M_{j,i}|s) = -\frac{NL}{2} \ln 2\pi - NL \ln \hat{s} - \frac{1}{2\hat{s}^2} \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j}$$

Differentiating the above with respect to $\hat{s}$ gives

$$\frac{\partial \ln p_{m_j,i,s}(M_{j,i}|s)}{\partial \hat{s}} = -\frac{NL}{\hat{s}} + \frac{1}{\hat{s}^3} \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j}$$

Setting this derivative to zero and solving for $\hat{s}$ gives the desired estimate as

$$\hat{s} = \left[ \frac{1}{NL} \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j} \right]^{1/2}$$

Figure C.11A shows the discrete-time non-linear filter which provides sequential, optimal, causal estimates of the EMG amplitude based on periodic samples of the continuous MSEM signal. For completeness, it will be shown that $\hat{s}$ above is, in fact, a maximum (and not, for example, a local minimum). To do so, the second derivative of the maximizing function is taken;

$$\frac{\partial^2 \ln p_{m_j,i,s}(M_{j,i}|s)}{\partial \hat{s}^2} = \frac{NL}{\hat{s}^2} - \frac{3}{\hat{s}^4} \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j}$$

Now, substitution of the estimate $\hat{s}$ into the second derivative gives

$$\frac{\partial^2 \ln p_{m_j,i,s}(M_{j,i}|s)}{\partial \hat{s}^2} = \frac{(NL)^2}{\sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j}} - \frac{3(NL)^2}{\left[ \sum_{j=1}^{L} \sum_{i=1}^{N} \frac{M_{j,i}^2}{g_j} \right]^2} \leq 0$$
Figure C.11A: Discrete-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

The optimal estimate is formed as the square root of the average of the $L$ channels of smoothed, variance adjusted measured surface EMG waveform.
Figure C.11B: Continuous-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Uncorrelated Channels

The optimal estimate is formed as the square root of the average of the $L$ channels of smoothed, variance adjusted measured surface EMG waveform.
Since this second derivative is always less than or equal to zero (recall that \( g_j > 0 \)), \( \hat{s} \) is a maximum. (Note that, in general, maximization requires evaluating the function at all local maxima and all boundary points. For this problem, \( \hat{s} \) is bounded at zero, which does not alter the result.)

Performance for this discrete-time maximum likelihood estimator can be based on the performance formulae of Case I. These formulae are a function of the number of degrees of freedom in the data (denoted \( N \) in Case I). For the present case, all \( N \cdot L \) samples are uncorrelated. Since these samples are jointly Gaussian, they are also independent. Thus, the number of degrees of freedom in the data is \( N \cdot L \).

A solution to the continuous-time problem for this case can be found from the discrete-time solution. Namely, let the number of MSEM samples \( N \) over the finite time duration \( T \) grow to infinity. Formally, note that if the sampling period is \( \Delta \tau \), then \( N = T/\Delta \tau \) samples per channel occur in the time \( T \). These discrete samples, denoted \( m_{ji} \), correspond to the value of the continuous time MSEM waveform \( m_j(\tau_i) \) of channel \( j \) at the times \( \tau_i \). Now, letting \( N \) grow to infinity is equivalent to letting \( \Delta \tau \) shrink to zero. Thus, the continuous time estimate \( \hat{s}(t) \) of the EMG amplitude is found as

\[
\hat{s}(t) = \lim_{\Delta \tau \to 0} \left[ \hat{s}_{N \cdot L, m_{ji} = m_j(\tau_i)} \right] \\
= \lim_{\Delta \tau \to 0} \left[ \frac{\Delta \tau}{T \cdot L} \sum_{j=1}^{L} \sum_{i=1}^{T/\Delta \tau} \frac{m_j^2(\tau_i)}{g_j} \right]^{1/2} \\
= \left[ \frac{1}{T \cdot L} \sum_{j=1}^{L} \lim_{\Delta \tau \to 0} \sum_{i=1}^{T/\Delta \tau} \frac{m_j^2(\tau_i)}{g_j} \Delta \tau \right]^{1/2} \\
= \left[ \frac{1}{T \cdot L} \sum_{j=1}^{L} \int_{\tau=t-T}^{t} \frac{m_j^2(\tau)}{g_j} d\tau \right]^{1/2}
\]

Figure C.11B shows the continuous-time non-linear filter which provides a continuous, optimal, causal estimate of the EMG amplitude for this analysis case.

Since \( N \cdot L \) grows to infinity, the performance of this continuous time estimate
is ideal. That is, $\hat{s}(t)$ is unbiased, has an MSE of zero and an infinite SNR. Physically, however, a continuous-time MSEM signal with a white PSD is unrealizable. Thus, the present analysis case is never realized in continuous time.

C.2.4 Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels

To achieve uncorrelated samples and correlated channels, the multiple channel MSEMG waveform model becomes that shown in Figure C.12. Again, in discrete-time, denote $N$ periodically sampled values of $j$ channels of continuous MSEMG activity as the composite $N \cdot L$ by 1 random vector $m_{ji}$. Given that each element of $m_{ji}$ has the standard deviation $\hat{s}$,

$$p_{m_{ji}|s}(M_{ji}|s) = \frac{1}{(2\pi)^{NL/2}} \left| K_{m_{ji}m_{ji}} \right|^{1/2}$$

$-\infty \leq M_{ji} \leq \infty$

Since samples within a given channel are uncorrelated,

$$K_{m_{ji}m_{ji}} = \begin{bmatrix}
g_j \hat{s}^2 & 0 & 0 & \cdots & 0 \\
0 & g_j \hat{s}^2 & 0 & \cdots & 0 \\
0 & 0 & g_j \hat{s}^2 & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 \\
0 & 0 & \cdots & 0 & g_j \hat{s}^2
\end{bmatrix}_{N \times N}$$

where the channel gains $g_j (g_j > 0)$ reflect the relative signal levels of the different channels. Also, since separate channels are correlated,

$$K_{m_{ji}m_{ki}} \neq 0 \quad \text{for } j \neq k$$

The PDF for $m_{ji}$ is identical in form to the formulation in Case III, except now there is no simple form for the inverse of the covariance matrix $K_{m_{ji}m_{ji}}$. Thus, direct solution by use of elementary calculus will not be employed.
Figure C.12A: Discrete-Time EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{\text{space}}$, which accounts only for the spatial dependence between channels. The outputs of $H_{\text{space}}$ are each multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
Figure C.12B: Continuous-Time EMG Model Case IV — Multiple Channels: Uncorrelated Samples, Correlated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{\text{space}}$, which accounts only for the spatial dependence between channels. The outputs of $H_{\text{space}}$ are each multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
A solution to this discrete-time estimation problem can be based upon the filtering technique discussed in Case II. In particular, Case II showed that optimal causal estimation of $s$ from the MSEM signal is equivalent to optimal estimation of $s$ from a filtered version of the MSEM signal, if the filter was stable, causal and had a stable causal inverse. Further, if a linear filter is selected, the filter output is Gaussian distributed. Consider, then, filtering the $L$ MSEM signals via an $L$-input, $L$-output, linear, stable, causal filter whose inverse exists and is stable and causal. Denote a channel of output from this filter as $v_j$. This filtering operation can be exploited if the output channels $v_j$ have uncorrelated samples, uncorrelated channels and are of equal variance $s^2$. Namely, the optimum processor is specified by the results of Case III.

From Figure C.12, it is clear that if an appropriate filter exists, it must be the filter $H_{\text{space}}^{-1}$. Because $H_{\text{space}}$ is stable, causal, stably invertible, causally invertible and LTI, its inverse must exist and be stable, causal and LTI. Thus, $H_{\text{space}}^{-1}$ exists. The discrete-time optimal causal filter for multiple MSEM signal channels with uncorrelated samples and correlated channels is given in Figure C.13A.

Identification of a general multi-dimensional filter is a difficult task. Restrictions placed on the multi-dimensional shaping filter $H_{\text{space}}$ combined with the assumption that individual channels have uncorrelated samples allow for a simple determination of the filter $H_{\text{space}}^{-1}$. Specifically, for this analysis case, the correlation between channels of MSEM activity is limited to samples observed during the same instant in time. The MSEM activity from separate time frames is
Figure C.13A: Discrete-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels

The filter $H_{\text{space}}^{-1}$ produces $L$ channels of spatially uncorrelated data from $L$ channels of spatially correlated data. The uncorrelated data are then smoothed, averaged and square rooted to form the optimal estimate.
Figure C.13B: Continuous-Time Optimal EMG Processor — Multiple Channels: Uncorrelated Samples, Correlated Channels

The filter $H_{\text{space}}^{-1}$ produces $L$ channels of spatially uncorrelated data from $L$ channels of spatially correlated data. The uncorrelated data are then smoothed, averaged and square rooted to form the optimal estimate.
uncorrelated. Thus, the cross-covariance matrix $K_{m_j m_k}$ is diagonal in form:

$$K_{m_j m_k} = \begin{bmatrix}
  k_{m_j,1 m_k,1} & 0 & 0 & \cdots & 0 \\
  0 & k_{m_j,2 m_k,2} & 0 & \cdots & 0 \\
  0 & 0 & k_{m_j,3 m_k,3} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \ddots & 0 \\
  0 & 0 & \cdots & 0 & k_{m_j,N m_k,N} \\
\end{bmatrix}_{N \times N}$$

The above implies that the random variables $m_{1,i}$, $m_{2,i}$, $m_{3,i}$, $\ldots$, $m_{L,i}$ observed from separate channels during the same instant in time are correlated with each other, but uncorrelated with any other $m_{k,i}$ observed during another instant in time. Thus, the filter $H^{-1}_{\text{space}}$ need only uncorrelate the $L$ samples from each time frame, while not introducing any temporal correlation.

To find such a filter, consider the ensemble random vector $m_{\ast,i}$ formed from the $L$ samples of MSEM activity corresponding to time $i$ as

$$m_{\ast,i} = \begin{bmatrix} m_{1,i} \\ m_{2,i} \\ m_{3,i} \\ \vdots \\ m_{L,i} \end{bmatrix}_{L \times 1}$$

Since the channels of MSEM activity are jointly Gaussian, $m_{\ast,i}$ is jointly Gaussian. Linear transformation of the vector $m_{\ast,i}$ by the matrix $D$ forms the jointly Gaussian vector $v_i'$ as

$$v_i' = D m_{\ast,i}$$

As discussed in Appendix B, if the columns of $D^T$ are constructed from $L$ orthonormal eigenvectors of $K_{m_{\ast,i} m_{\ast,i}} = \frac{1}{N} K_{m_{\ast,i} m_{\ast,i}}$, where

$$n_{\ast,i} = \begin{bmatrix} n_{1,i} \\ n_{2,i} \\ n_{3,i} \\ \vdots \\ n_{L,i} \end{bmatrix}_{L \times 1}$$

435
then $\mathbf{u}'_i$ is zero mean and has a covariance matrix equal to

$$K_{\mathbf{u}'_i,\mathbf{u}'_i} = D K_{m_*,i} D^T = \text{diag}(s^2 \lambda_1, s^2 \lambda_2, s^2 \lambda_3, \ldots, s^2 \lambda_L)$$

where $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_L$ are the eigenvalues of $K_{m_*,i}$. Recall from Appendix A that these eigenvalues and eigenvectors are real. Without loss of generality, let all of the eigenvalues be non-negative (eigenvalues and corresponding eigenvectors can be scaled by an arbitrary constant). Define the matrix $\Lambda^{-\frac{1}{2}}$ as

$$\Lambda^{-\frac{1}{2}} = \text{diag}\left(\frac{1}{\sqrt{\lambda_1}}, \frac{1}{\sqrt{\lambda_2}}, \frac{1}{\sqrt{\lambda_3}}, \ldots, \frac{1}{\sqrt{\lambda_L}}\right)$$

Linear transformation of the vector $\mathbf{u}'_i$ by the matrix $\Lambda^{-\frac{1}{2}}$ forms the jointly Gaussian vector $\mathbf{u}_i$ as

$$\mathbf{u}_i = \Lambda^{-\frac{1}{2}} \mathbf{u}'_i = \Lambda^{-\frac{1}{2}} D m_*,i$$

where $\mathbf{u}_i$ is zero mean and

$$K_{\mathbf{u}_i,\mathbf{u}_i} = s^2 I$$

The above transformation can now be applied to the MSEM activity corresponding to times $i + 1, i + 2, i + 3, \ldots$, yielding the random vectors $\mathbf{u}_{i+1}, \mathbf{u}_{i+2}, \mathbf{u}_{i+3}, \ldots$. Since each $\mathbf{u}_i$ was formed from the data in non-overlapping time intervals, no temporal correlation has been introduced to the $\mathbf{u}_i$. The transformed data are uncorrelated in both space and time. Thus, this transformation defines the filter $H_{\text{space}}^{-1}$. In practice, the eigenvalues and eigenvectors of $K_{m_*,i}$ are found by determining the eigenvalues and eigenvectors of $K_{n_*,i}$ for some reference value of $s$.

The optimum continuous time processor can also be derived based upon the filtering technique discussed in Case II. Consider filtering the $L$ continuous MSEM signals via the $L$-input, $L$-output, linear, stable, causal filter $H_{\text{space}}^{-1}$.
as shown in Figure C.13B. Because $H_{\text{space}}^{-1}$ is stable, causal, invertibly stable, invertibly causal and LTI, the filter $H_{\text{space}}^{-1}$ exists. The output channels $v_j(t)$ of $H_{\text{space}}^{-1}$ are uncorrelated in time, uncorrelated in space are are of equal variance $s^2$. Thus, the $v_j(t)$ signals can be optimally processed by the result derived in Case III, where all gains $g_j$ are equal to unity. Figure C.13B shows this optimal causal filter for multiple continuous MSEMGE waveform channels which are uncorrelated in time and correlated in space.

Restrictions placed on the multi-dimensional shaping filter $H_{\text{space}}$ combined with the assumption that individual channels are uncorrelated in time allow for a simple determination of the filter $H_{\text{space}}^{-1}$. In particular, the filter $H_{\text{space}}^{-1}$ need only uncorrelate the data from each single instant in time of the $L$ MSEMGE signals, while not introducing any temporal correlation. To form this filter, denote the MSEMGE signal at time $t$ from $L$ channels as $m_1(t), m_2(t), m_3(t), \ldots, m_L(t)$. At a given instant of time $t$, construct the ensemble random vector $\mathbf{m}_{*,(t)}$ as

$$\mathbf{m}_{*,(t)} = \begin{bmatrix} m_1(t) \\ m_2(t) \\ m_3(t) \\ \vdots \\ m_L(t) \end{bmatrix}_{L \times 1}$$

Since channels are discrete, $\mathbf{m}_{*,(t)}$ is a discrete random vector which is jointly Gaussian distributed with zero mean and covariance $K_{m_{*,(t)}m_{*,(t)}}$. Form the $L$-dimensional vector $\mathbf{u}_{(t)}$ as

$$\mathbf{u}_{(t)} = \Lambda_{(t)}^{-\frac{1}{2}} D_{(t)} \mathbf{m}_{*,(t)}$$

where the columns of $D_{(t)}^T$ are constructed from $L$ orthonormal eigenvectors of $K_{m_{*,(t)}m_{*,(t)}}$, and $\Lambda_{(t)}^{-\frac{1}{2}}$ is constructed from the $L$ (non-negative) eigenvalues $\lambda_1,(t),$
\( \lambda_2(t), \lambda_3(t), \ldots, \lambda_L(t) \) of \( K_{m_*(t) m_*(t)} \) as

\[
\Lambda_{t}^{-1} = \text{diag} \left( \frac{1}{\sqrt{\lambda_1(t)}}, \frac{1}{\sqrt{\lambda_2(t)}}, \frac{1}{\sqrt{\lambda_3(t)}}, \ldots, \frac{1}{\sqrt{\lambda_L(t)}} \right)
\]

As before, the elements of \( u(t) \) are uncorrelated. The above transformation can be applied at all times. Since each \( u(t) \) is applied to the data from a single instant, no temporal correlation has been introduced. The transformed data are uncorrelated in both space and time, thus the transformation defines the filter \( H_{\text{space}}^{-1} \).

Performance for the discrete-time maximum likelihood estimator can be based on the performance formulae of Case I. These formulae are a function of the number of degrees of freedom in the data (denoted \( N \) in Case I). If perfect spatial uncorrelation is achieved, then the number of degrees of freedom in the data is \( N \cdot L \). In practice, however, perfect spatial uncorrelation can not be achieved. Consider, as before, use of an effective number of degrees of freedom. Since each time frame is independent for this case, the effective number of degrees of freedom in the data is equal to the sum of the effective number of degrees of freedom in each time frame;

\[
N_{\text{effective}, IV} = \sum_{i=1}^{N} N_{\text{effective}, i} \quad \text{for Case IV}
\]

where \( N_{\text{effective}, i} \) is the effective number of degrees of freedom in time frame \( i \). The effective number of degrees of freedom in time frame \( i \) is equal to the effective number of degrees of freedom in the ensemble random vector \( m_{*,i} \). Since \( m_{*,i} \) is stationary, \( N_{\text{effective}, i} \) has the same value \( N_{\text{effective, ensemble}} \) for all \( i \). Thus,

\[
N_{\text{effective}, IV} = N \cdot N_{\text{effective, ensemble}} \quad \text{for Case IV}
\]
From the discussion in Case II,

\[ N_{\text{effective, ensemble}} = 2 B_{m_{s,i}}, T \]

where \( B_{m_{s,i}} \) is the statistical bandwidth of the discrete random vector \( m_{s,i} \).

Thus,

\[ N_{\text{effective, IV}} = 2 N T B_{m_{s,i}} \quad \text{for Case IV} \]

A problem exists in applying the above formula. For the ensemble vector \( m_{s,i} \), \( T \) is ill-defined. \( T \) is meant to be the time duration for observing a single signal.

For this ensemble formulation, an intuitive empirical solution is to take \( T \) as the product \( L \Delta \tau_{\text{equiv}} \), where \( \tau_{\text{equiv}} \) is the sampling period of a single MSEM signal which will yield a covariance matrix similar to the covariance matrix of the ensemble random vector \( m_{s,i} \). Akin to the case of a discrete-time signal where \( T = N \Delta \tau \), the term \( L \) represents the number of discrete entries and the term \( \Delta \tau_{\text{equiv}} \) represents the equivalent time spacing.

In the continuous-time case, temporally uncorrelated data means that there are an infinite number of degrees of freedom in the data. Thus, \( \hat{s}(t) \) is unbiased, has an MSE of zero and has an infinite SNR. Physically, however, a continuous-time MSEM signal waveform with a white PSD is unrealizable. Thus, the present analysis case is never realized in continuous time.

**C.2.5 Case V — Multiple Channels:**

**Correlated Samples, Uncorrelated Channels**

When multiple uncorrelated channels of MSEM signal waveforms have correlated samples, the MSEM signal model of Figure C.14 results. The discrete-
Figure C.14A: Discrete-Time EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are each passed through a shaping filter $H_{time,j}(e^{j\omega})$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguining.
$w_1(t) \rightarrow H_{\text{time},1}(j\omega) \rightarrow n_1(t) \rightarrow m_1(t)$

$w_2(t) \rightarrow H_{\text{time},2}(j\omega) \rightarrow n_2(t) \rightarrow m_2(t)$

$w_3(t) \rightarrow H_{\text{time},3}(j\omega) \rightarrow n_3(t) \rightarrow m_3(t)$

$\vdots$  $\vdots$  $\vdots$  $\vdots$  $\vdots$

$w_L(t) \rightarrow H_{\text{time},L}(j\omega) \rightarrow n_L(t) \rightarrow m_L(t)$

Independent Zero Mean, JWSS, CE, Jointly Gaussian, White Processes of Unit Intensity
Filtering Effects of Muscle Tissue, Bone, Skin and Electrodes

**Figure C.14B:** Continuous-Time EMG Model Case V — Multiple Channels: Correlated Samples, Uncorrelated Channels

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are each passed through a shaping filter $H_{\text{time},i}(j\omega)$ and multiplied by the EMG amplitude $s$ to form the $L$ channels of measured surface EMG waveform. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
time random vector \( m_{j,i} \) has the conditional PDF

\[
p_{m_{j,i}}(M_{j,i} | \delta) = \frac{e^{-\frac{M_{j,i}^T K_{m_{j,i}}^{-1} M_{j,i}}{2}}}{(2\pi)^{NL/2} |K_{m_{j,i}}|^{1/2}} \quad -\infty \leq M_{j,i} \leq \infty
\]

Because separate channels are uncorrelated

\[
K_{m_j m_k} = 0 \quad \text{for } j \neq k
\]

Because samples within a channel are correlated, the matrix \( K_{m_j m_j} \) will contain off-diagonal terms. Thus, the PDF for \( m_{j,i} \) is identical in form to the formulation in Case III, except no simple form exists for the inverse of \( K_{m_j, m_j} \). Direct solution by use of elementary calculus will not be employed.

A solution to this discrete-time estimation problem can be based on the filtering technique discussed in Case II. In particular, Case II established that optimal causal estimation of \( s \) from a channel of the MSEM signal is equivalent to optimal estimation of \( s \) from a filtered version of the channel of MSEM activity, if the filter was stable, causal and had a stable causal inverse. Consider, then, the solution shown in Figure C.15A. Each channel of MSEM activity is filtered by the stable, causal, invertibly stable and causal, LTI filter \( H_{time,j}^{-1}(e^{j\omega}) \). The output of each filter is a zero mean, JWS, CE, jointly Gaussian, white process of intensity \( s^2 \). Optimal estimation of \( s \) based on \( L \) such outputs is given as the result of Case III, where all gains \( g_j \) are equal to unity. The result of Case III is cascaded with the \( L \) whitening filters \( H_{time,j}^{-1}(e^{j\omega}) \) to form the estimator shown in Figure C.15A.

The continuous-time optimal causal estimator is shown in Figure C.15B. From the discussion in Case II, each MSEM signal channel can be whitened with the filter \( H_{time,j}^{-1}(j\omega) \) to form a zero mean, JWSS, CE, jointly Gaussian,
Figure C.15A: Discrete-Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels

The estimate is formed as the square root of the average of $L$ smoothed, whitened channels of measured surface EMG waveform.
Figure C.15B: Continuous-Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Uncorrelated Channels

The estimate is formed as the square root of the average of $L$ smoothed, whitened channels of measured surface EMG waveform.
white process of intensity $s^2$. Optimal estimation of $s$ based upon the outputs of the $L$ whitening filters is given as the result of Case III, where all gains are equal to unity.

Performance for both the discrete-time and continuous-time maximum likelihood estimators can be based on the performance formulae of Case I. These formulae are a function of the number of degrees of freedom in the data (denoted $N$ in Case I). If perfect temporal whitening is achieved, then the number of degrees of freedom in the data is $N \cdot L$. In practice, however, perfect temporal whitening can not be achieved. Consider, as before, use of an effective number of degrees of freedom. Since the $L$ channels are uncorrelated, and therefore independent, the effective number of degrees of freedom in the data $N_{\text{effective, } V}$ must be the sum of the effective number of degrees of freedom in each channel:

$$N_{\text{effective, } V} = \sum_{j=1}^{L} N_{\text{effective, } j} \quad \text{for Case V}$$

where $N_{\text{effective, } j}$ is the effective number of degrees of freedom in the data of channel $j$. Case II gives the effective number of degrees of freedom in a particular channel. Thus,

$$N_{\text{effective, } V} = 2 T \sum_{j=1}^{L} B_{s,j} \quad \text{for Case V}$$

where $B_{s,j}$ is the statistical bandwidth of the MSEM signal of channel $j$ as defined in Case II.

**C.2.6 Case VI — Multiple Channels: Correlated Samples, Correlated Channels**

When multiple channels of MSEM signal are correlated both in time and space, the general multiple channel model of Figure C.4 is applicable. The
discrete-time random vector \( \tilde{m}_{ji} \) is jointly Gaussian, conditioned on the knowledge that \( s = \tilde{s} \). Because correlation exists in both time and space,

\[
K_{\tilde{m}_{ji}, \tilde{m}_{ki}} \neq 0 \quad \text{for all } j, k
\]

Thus, the PDF for \( \tilde{m}_{ji} \) is identical in form to the formulation in Case III, except no simple form exists for the inverse of \( K_{\tilde{m}_{ji}, \tilde{m}_{ji}} \). Direct solution by use of elementary calculus will not be employed.

Again, the filtering technique discussed in Case II can be utilized to find the desired estimator. Prewhiten each MSEM signal by passing the data through the filters \( H_{\text{time}, j}^{-1}(e^{j\omega}) \) as shown in Figure C.16A. Since both \( H_{\text{time}}(e^{j\omega}) \) and \( H_{\text{space}} \) are stable, causal, invertibly stable and causal, and LTI, \( H_{\text{time}, j}^{-1}(e^{j\omega}) \) exist. The \( L \) output channels \( v_j \) are now uncorrelated in time, but still correlated in space. Estimation of \( s \) from the channels \( v_j \) is now exactly the problem solved in Case IV. Thus, the remainder of the solution is as given in Figure C.16A. Note that the filter \( H_{\text{space}}^{-1} \) must perform a transformation based on the covariance structure of the intermediate random variable \( v_j \) and not the original MSEM signal. Also, since the bank of whitening filters and \( H_{\text{space}}^{-1} \) are linear, their position in the solution figure can be switched.

The continuous-time optimal causal estimator for this case is shown in Figure C.16B. Each channel can be whitened by the filters \( H_{\text{time}, j}^{-1}(j\omega) \). The \( L \) output channels of the bank of whitening filters are uncorrelated in time and correlated in space. Estimation of \( s \) from these whitened channels is given by the processor in Case IV. Again, note that the filter \( H_{\text{space}}^{-1} \) must perform a transformation based on the covariance structure of the whitened data and not the original MSEM signal.

Performance for both the discrete-time and continuous-time maximum like-
The bank of filters $H_{\text{time},j}^{-1}(e^{j\omega})$ uncorrelate each channel temporally. The filter $H_{\text{space}}^{-1}$ uncorrelates the data spatially. The resultant signals are smoothed, averaged and square rooted to form the optimal estimate.
Figure C.16B: Continuous-Time Optimal EMG Processor — Multiple Channels: Correlated Samples, Correlated Channels

The bank of filters $H_{time,j}^{-1}(j\omega)$ uncorrelate each channel temporally. The filter $H_{space}^{-1}$ uncorrelates the data spatially. The resultant signals are smoothed, averaged and square rooted to form the optimal estimate.
lihood estimators can be based on the performance formulae of Case I. These formulae are a function of the number of degrees of freedom in the data (denoted $N$ in Case I). If perfect temporal whitening and spatial uncorrelation is achieved, then the number of degrees of freedom in the data is $N \cdot L$. In practice, however, perfect temporal whitening or spatial uncorrelation can not be achieved. Consider, as before, use of an effective number of degrees of freedom. For the present case, $N_{\text{effective, VI}}$ must account for both the number of effectively uncorrelated time frames and the number of effectively uncorrelated channels within a time frame. Since each channel can have a different statistical bandwidth, each channel can have a different number of effectively uncorrelated time frames. Thus, use of the following ad hoc value of $N_{\text{effective, VI}}$ is suggested:

$$N_{\text{effective, VI}} = N_{\text{channel, average}} \cdot N_{\text{effective, ensemble}} \quad \text{for Case VI}$$

$N_{\text{channel, average}}$ is the average over $L$ channels of the number of effectively uncorrelated time frames. For a single channel, the number of effectively uncorrelated time frames is given in Case II. $N_{\text{effective, ensemble}}$ is the number of effectively uncorrelated channels, as given in Case IV.

### C.3 Some Alternative EMG Processors

#### C.3.1 The Mean Absolute Value Processor

A common, ad hoc, single channel MSEM algorithm is the mean absolute value (MAV) processor. If the white Gaussian model of the MSEM algorithm, described previously, is assumed, then the statistics of the MAV processor can be derived. In the discrete-time case, let a single channel of continuous MSEM activity $m(t)$ be sampled periodically over a finite time duration $T$, the
$N$ samples of MSEM activity being denoted the $N$ random variables $m_1, m_2, m_3, \ldots, m_N$. As before, given that the EMG amplitude $s$ has the known value $s$, the conditional joint PDF for the MSEM samples is, in random vector form,

$$p_{m|x}(M|s) = \mathcal{G}(M; \Omega, s^2 I)$$

Instead of forming the optimal ML estimate, form the following ad hoc estimate: Divide the total interval of $N$ samples into $Q$ successive non-overlapping intervals $T_1, T_2, T_3, \ldots, T_Q$, each of length $J$, such that $N = QJ$. For each interval $T_q$, compute the optimal ML estimate $\hat{s}_q$ for $s$ From prior analysis it is clear that

$$\hat{s}_q = \left[\frac{1}{J} \sum_{i=qJ}^{(q+1)J-1} M_i^2\right]^{1/2}$$

Further,

$$\mathbb{E}[\hat{s}_q] = s \sqrt{\frac{2}{J} \frac{\Gamma(\frac{J+1}{2})}{\Gamma(\frac{J}{2})}} = \mathbb{E}[\hat{s} | J \text{ Degrees of Freedom}]$$

$$\mathbb{E}[\hat{s}_q^2] = s^2 = \mathbb{E}[s^2]$$

$$\sigma_{\hat{s}_q}^2 = s^2 \left[1 - \frac{2}{J} \frac{\Gamma^2(\frac{J+1}{2})}{\Gamma^2(\frac{J}{2})}\right] = \sigma_{\hat{s}}^2 | J \text{ Degrees of Freedom}$$

and

$$SNR_{\hat{s}_q} = \left[\frac{2 \Gamma^2(\frac{J+1}{2})}{J \Gamma^2(\frac{J}{2}) - 2 \Gamma^2(\frac{J+1}{2})}\right]^{1/2} = SNR_{\hat{s}} | J \text{ Degrees of Freedom}$$

Hence, the performance of the estimate formed from each interval is independent of the interval.

Next, form the ad hoc estimator $\hat{s}_{ad \text{ hoc}}$ as the average of the above estimates, i.e.

$$\hat{s}_{ad \text{ hoc}} = \frac{1}{Q} \sum_{q=1}^{Q} \hat{s}_q$$

The expected value of this estimate is

$$\mathbb{E}[\hat{s}_{ad \text{ hoc}}] = \mathbb{E} \left[\frac{1}{Q} \sum_{q=1}^{Q} \hat{s}_q\right]$$
\[
\begin{align*}
E[\hat{s}_q] &= \frac{1}{Q} \sum_{q=1}^{Q} E[\hat{s}_q] \\
E[\hat{s}_{ad\, hoc}] &= \frac{1}{Q} Q E[\hat{s} | J \text{ Degrees of Freedom}] \\
E[\hat{s}_{ad\, hoc}] &= E[\hat{s} | J \text{ Degrees of Freedom}]
\end{align*}
\]

The second moment of the ad hoc estimate is
\[
E[\hat{s}_{ad\, hoc}] = E\left[ \frac{1}{Q} \sum_{q=1}^{Q} \hat{s}_q \frac{1}{Q} \sum_{i=1}^{Q} \hat{s}_i \right] = \frac{1}{Q^2} \sum_{q=1}^{Q} \sum_{i=1}^{Q} E[\hat{s}_q \hat{s}_i]
\]

When \( q = i \) (\( Q \) occurrences), then
\[
E[\hat{s}_q \hat{s}_i] = E[\hat{s}_q^2] = E[\hat{s}_i^2], \quad q = i
\]

When \( q \neq i \) (\( Q^2 - Q \) occurrences), the correlation \( E[\hat{s}_q \hat{s}_i] \) involves MSEM signals from non-overlapping time intervals. Since the MSEM signals are uncorrelated, so must be \( \hat{s}_q \) and \( \hat{s}_i \). Therefore,
\[
E[\hat{s}_q \hat{s}_i] = E[\hat{s}_q] E[\hat{s}_i] = E^2[\hat{s} | J \text{ Degrees of Freedom}], \quad q \neq i
\]

Thus,
\[
E[\hat{s}_q \hat{s}_i] = \begin{cases} 
E[\hat{s}^2], & q = i, \ (Q \text{ occurrences}) \\
E^2[\hat{s} | J \text{ Degrees of Freedom}], & q \neq i, \ (Q^2 - Q \text{ occurrences})
\end{cases}
\]

and
\[
E[\hat{s}_{ad\, hoc}^2] = \frac{1}{Q^2} \left[ Q E[\hat{s}^2] + (Q^2 - Q) E^2[\hat{s} | J \text{ Degrees of Freedom}] \right] = \frac{1}{Q} \left[ E[\hat{s}^2] + (Q - 1) E^2[\hat{s} | J \text{ Degrees of Freedom}] \right]
\]

The variance of the ad hoc estimate is
\[
\sigma^2_{ad\, hoc} = E[\hat{s}_{ad\, hoc}^2] - E^2[\hat{s}_{ad\, hoc}] = \frac{1}{Q} \left[ E[\hat{s}^2] - E^2[\hat{s} | J \text{ Degrees of Freedom}] \right]
\]
or,

\[ \sigma^2_{\text{ad hoc}} = \frac{1}{Q} \sigma^2_0 \]  

or

The SNR of the ad hoc estimate is

\[ SNR_{\text{ad hoc}} = \left[ \frac{\mathcal{E}[\hat{s}_{\text{ad hoc}}]}{\sigma^2_{\text{ad hoc}}} \right]^{1/2} = \left[ \frac{Q \mathcal{E}[\hat{s} | J \text{ Degrees of Freedom}]}{\mathcal{E}[\hat{s}^2] - \mathcal{E}^2[\hat{s} | J \text{ Degrees of Freedom}]} \right]^{1/2} \]

or,

\[ SNR_{\text{ad hoc}} = \sqrt{Q} \; SNR_{\hat{s} | J \text{ Degrees of Freedom}} \]

Now, to form the MAV processor, let \( J = 1 \) and \( Q = N \). This parameter selection gives \( N \) successive non-overlapping intervals, each interval containing one MSEM0G sample. Thus,

\[ \hat{s}_q = \left( \frac{1}{\bar{M}_t} \sum_{t=q}^{q+t-1} M_t^2 \right)^{1/2} = |M_q| \]

and the ad hoc estimate becomes the MAV processor;

\[ \hat{s}_{\text{ad hoc}} = \frac{1}{Q} \sum_{q=1}^{Q} \hat{s}_q = \frac{1}{N} \sum_{i=1}^{N} |M_i| \equiv \hat{s}_{\text{MAV}} \]

With \( J = 1 \) and \( Q = N \) the statistics of the MAV processor can be computed from the above ad hoc processor. The expected value is

\[ \mathcal{E}[\hat{s}_{\text{MAV}}] = \mathcal{E}[\hat{s} | 1 \text{ Degree of Freedom}] = s \sqrt{\frac{2}{\frac{1}{\Gamma(1)} \Gamma(\frac{1}{2})} \mathcal{E}[\hat{s}_{\text{MAV}}] = s \sqrt{\frac{2}{\pi}} \]

where \( \Gamma(1) = 1 \) and \( \Gamma(\frac{1}{2}) = \sqrt{\pi} \) (see Appendix B). Similarly, the second moment is

\[ \mathcal{E}[\hat{s}_{\text{MAV}}^2] = s^2 \left[ \frac{\pi + 2N - 2}{N\pi} \right] \]
the variance is
\[ \sigma_{\text{MAV}}^2 = \frac{s^2}{N} \left[ \frac{\pi - 2}{\pi} \right] \]
and the SNR is
\[ \text{SNR}_{\text{MAV}} = \sqrt{\left( \frac{2}{\pi - 2} \right) N} \]

Alternatively, the MAV processor can be shown to be optimal if the underlying PDF for the random process is distributed as a Laplace random variable, and successive samples of the MSEMGS signal are independent. (Note that independent is a stronger assumption than uncorrelated.) A zero mean Laplace random variable \( x \), shown in Figure C.17, has the PDF
\[ p_x(X) = \frac{a}{2} e^{-a|X|} \quad -\infty \leq X \leq \infty \]
and the standard deviation
\[ \sigma_x = \frac{\sqrt{2}}{a} \]
for some constant \( a > 0 \). In the discrete-time case, again let a single channel of continuous MSEMGS activity \( m(t) \) be sampled periodically over a finite time duration \( T \), the \( N \) samples of MSEMGS activity being denoted the \( N \) random variables \( m_1, m_2, m_3, ..., m_N \). Given that the EMG amplitude \( s_{\text{Laplace}} \) has the known value \( s_{\text{Laplace}} \), the conditional PDF for the \( i^{th} \) sample is
\[ p_{m_i|s_{\text{Laplace}}}(M_i|s_{\text{Laplace}}) = \frac{\sqrt{2}}{2 s_{\text{Laplace}}} e^{-\frac{\sqrt{2}}{s_{\text{Laplace}}} |M_i|} \quad -\infty \leq X_i \leq \infty \]

Since the random variables are independent, the conditional joint PDF for the \( N \) random variables, arranged as the random vector \( m \), is the product of the
Figure C.17: The Laplace Probability Distribution

The probability density function for the Laplace random variable \( z \) with mean value zero and standard deviation \( \sigma_z = \frac{\sqrt{2}}{a} \) is

\[
p_z(X) = \frac{a}{2} e^{-a |X|} \quad -\infty \leq X \leq \infty
\]
individual conditional PDF's;

\[ p_{\text{ml}|Laplace}(M|\hat{s}_{\text{Laplace}}) = \prod_{i=1}^{N} p_{m_i|s_{\text{Laplace}}}(M_i|\hat{s}_{\text{Laplace}}) \]

\[ = \prod_{i=1}^{N} \frac{\sqrt{2}}{2\hat{s}_{\text{Laplace}}} e^{-\frac{\sqrt{2}}{2\hat{s}_{\text{Laplace}}} |M_i|} \quad -\infty \leq M_i \leq \infty \]

\[ = \left[ \frac{\sqrt{2}}{2\hat{s}_{\text{Laplace}}} \right]^{N} e^{-\frac{\sqrt{2}}{2\hat{s}_{\text{Laplace}}} \sum_{i=1}^{N} |M_i|} \quad -\infty \leq M_i \leq \infty \]

The maximum likelihood estimate of the standard deviation is the value of \( \hat{s}_{\text{Laplace}} \) which maximizes the above density. A monotonic transformation of the density does not alter the location of the maximum. Herein, it will prove advantageous to maximize the natural logarithm of the density. Taking the natural logarithm of the density yields

\[ \ln p_{\text{ml}|Laplace}(M|\hat{s}_{\text{Laplace}}) = N \ln \frac{\sqrt{2}}{2} - N \ln \hat{s}_{\text{Laplace}} - \frac{\sqrt{2}}{\hat{s}_{\text{Laplace}}} \sum_{i=1}^{N} |M_i| \]

Differentiating the above with respect to \( \hat{s}_{\text{Laplace}} \) gives

\[ \frac{\partial}{\partial \hat{s}_{\text{Laplace}}} \ln p_{\text{ml}|Laplace}(M|\hat{s}_{\text{Laplace}}) = - \frac{N}{\hat{s}_{\text{Laplace}}} + \frac{\sqrt{2}}{\hat{s}_{\text{Laplace}}} \sum_{i=1}^{N} |M_i| \]

Setting this derivative to zero and solving for \( \hat{s}_{\text{Laplace}} \) gives the desired estimate;

\[ \hat{s}_{\text{Laplace}} = \frac{\sqrt{2}}{N} \sum_{i=1}^{N} |M_i| \]

This is the MAV processor with a constant scaling factor. For completeness, it will be shown that \( \hat{s}_{\text{Laplace}} \) above is, in fact, a maximum (and not, for example, a local minimum). To do so, the second derivative of the maximizing function is taken;

\[ \frac{\partial^2}{\partial \hat{s}_{\text{Laplace}}^2} \ln p_{\text{ml}|Laplace}(M|\hat{s}_{\text{Laplace}}) = \frac{N}{\hat{s}_{\text{Laplace}}^2} - \frac{2\sqrt{2}}{\hat{s}_{\text{Laplace}}^3} \sum_{i=1}^{N} |M_i| \]

455
Now, substitution of the estimate $\hat{s}_{\text{Laplace}}$ into the second derivative gives

$$\frac{\partial^2 \ln p_{M|s_{\text{Laplace}}}(M|\hat{s}_{\text{Laplace}})}{\partial \hat{s}_{\text{Laplace}}^2} = \frac{N^3}{2} \left[ \frac{N^3}{\sum_{i=1}^{N} |M_i|^2} \right]^2 \leq 0$$

Since this second derivative is always less than or equal to zero, $\hat{s}_{\text{Laplace}}$ is a maximum. (Note that, in general, maximization requires evaluating the function at all local maxima and all boundaries. For this problem, $\hat{s}_{\text{Laplace}}$ is bounded at zero, which does not alter the result.)

### C.3.2 Gaussian Model with Additive Gaussian Noise

An additive Gaussian noise term is often a convenient means to represent noise in many physical systems. Such a term has not been included in the previous models of the MSEMg waveform studied within this thesis. General inclusion of an additive Gaussian noise term appears to greatly increase the complexity of the model. Closed form solutions to the optimal processor problem are not readily apparent. However, two simple additive noise models which do have a closed form solution will be discussed.

#### Single Channel Models

A functional model of the MSEMg waveform which includes an additive Gaussian noise term is presented in Figure C.18 for the single channel discrete-time case. The MSEMg waveform is modeled as being formed from a zero mean, WSS, CE, jointly Gaussian white process of unit intensity $w_t$ passed through a stable, causal, inversely stable, inversely causal, linear time-invariant shaping filter $H_{\text{time}}(e^{j\omega})$, multiplied by the EMG amplitude $s$, and added to the zero
A zero mean, wide sense stationary (WSS), correlation-ergodic (CE), jointly Gaussian, white process of unit intensity $w_i$ is passed through the stable, causal, inversely stable, inversely causal, linear, time-invariant shaping filter $H_{time}(e^{j\omega})$, multiplied by the EMG amplitude $s$, and added to the zero mean, WSS, correlation-ergodic (CE), jointly Gaussian noise process $v_i$ to form the measured surface EMG waveform. The processes $w_i$ and $v_i$ are assumed to be independent and jointly Gaussian. The EMG amplitude is constant and the muscle contraction is non-fatiguing.

**Figure C.18:** Discrete-Time Functional Model of a Single Channel of EMG in Additive Gaussian Noise
mean, WSS, CE, jointly Gaussian noise process \( v_i \). The processes \( w_i \) and \( v_i \) are assumed to be independent and jointly Gaussian. If the output of the shaping filter is denoted as \( x_i \), then the \( i^{th} \) sample of the MSEM signal is

\[
m_i = x_i + v_i
\]

For any sequence of \( N \) samples, the random vectors \( x \) and \( v \) are jointly Gaussian. Thus, \( m \) must also be Gaussian distributed. Only the first two moments of \( m \) are needed in order to completely specify its distribution. Since the processes are WSS,

\[
\mu_m = \mu_x + \mu_v
\]

\[
= 0 + 0
\]

\[
\mu_m = 0
\]

Since the processes are WSS and independent,

\[
K_{mm} = K_{xx} + K_{vv}
\]

Thus, the random vector \( m \) has the PDF

\[
p_m(M) = \mathcal{G}(M; \mu_m, K_{mm}) = \mathcal{G}(M; 0, K_{xx} + K_{vv})
\]

This PDF is the general form of the additive Gaussian noise model.

The first simple additive noise model is formed by assuming that the shaping filter becomes an all-pass filter and that the additive noise is white of intensity \( q \). The covariance matrix of the MSEM samples becomes

\[
K_{mm} = s^2 I + q I
\]

\[
= (s^2 + q) I
\]

This covariance matrix now has a simple inverse, namely

\[
K_{mm}^{-1} = \frac{1}{s^2 + q} I
\]
The PDF for the Gaussian random vector \( m \) given that \( s \) has the known value \( \hat{s} \) can now be written as

\[
p_{m|s}(M|\hat{s}) = \frac{e^{-\frac{(M-\mu_m)^T K_{mm}^{-1} (M-\mu_m)}{2}}}{(2\pi)^{N/2} |K_{mm}|^{1/2}} \]

\[
= \frac{1}{[2\pi(\hat{s}^2 + q)]^{N/2}} e^{-\frac{\sum_{i=1}^{N} M_i^2}{2(\hat{s}^2 + q)}}
\]

The maximum likelihood estimate of the standard deviation is the value of \( \hat{s} \) which maximizes the above density. A monotonic transformation of the density does not alter the location of the maximum. Herein, it will prove advantageous to maximize the natural logarithm of the density. Taking the natural logarithm of the density yields

\[
\ln p_{m|s}(M|\hat{s}) = -\frac{N}{2} \ln 2\pi - \frac{N}{2} \ln(\hat{s}^2 + q) - \frac{\sum_{i=1}^{N} M_i^2}{2\hat{s}^2 + 2q}
\]

Differentiating the above with respect to \( \hat{s} \) gives

\[
\frac{\partial}{\partial \hat{s}} \ln p_{m|s}(M|\hat{s}) = -\frac{N}{2} \frac{2\hat{s}}{[\hat{s}^2 + q]} + \frac{4\hat{s} \sum_{i=1}^{N} M_i^2}{[2\hat{s}^2 + 2q]^2}
\]

Setting this derivative to zero gives the equation

\[
\frac{N\hat{s}}{\hat{s}^2 + q} = \frac{4\hat{s} \sum_{i=1}^{N} M_i^2}{4\hat{s}^4 + 8\hat{s}^2q + 4q^2}
\]

If the above equation is cross multiplied and common terms collected;

\[
\hat{s}^4[N] + \hat{s}^2[2Nq - \sum_{i=1}^{N} M_i^2] + [Nq^2 - q \sum_{i=1}^{N} M_i^2] = 0
\]

Direct application of the binomial formula will provide two possible solutions for \( \hat{s}^2 \). The square root of these solutions gives two possible values for \( \hat{s} \). (Note that only the positive square root must be considered since \( \hat{s} \) must be non-negative.)

\[
\hat{s} = \left[ \frac{\sum_{i=1}^{N} M_i^2 - 2Nq \pm \left\{ [2Nq - \sum_{i=1}^{N} M_i^2]^2 - 4N[Nq^2 - q \sum_{i=1}^{N} M_i^2] \right\}^{1/2}}{2N} \right]^{1/2}
\]
The inner square root term can be simplified and the estimate written as
\[
\hat{s} = \left[ \frac{\sum_{i=1}^{N} M_i^2 - 2Nq \pm \sum_{i=1}^{N} M_i^2}{2N} \right]^{1/2}
\]

The two possible solutions are then
\[
\hat{s} = \left[ \frac{2 \sum_{i=1}^{N} M_i^2 - 2Nq}{2N} \right]^{1/2} \quad \text{or} \quad \hat{s} = \left[ \frac{-2Nq}{2N} \right]^{1/2}
\]

which can be written more compactly as
\[
\hat{s} = \left[ \left\{ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right\} - q \right]^{1/2} \quad \text{or} \quad \hat{s} = [-q]^{1/2}
\]

The solution on the right is not possible since \( q \) must be non-negative. Thus, the desired estimate is
\[
\hat{s} = \left[ \left\{ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right\} - q \right]^{1/2}
\]

This estimator is shown in Figure C.19. For completeness, it will be shown that \( \hat{s} \) above is, in fact, a maximum (and not, for example, a local minimum). To do so, first rewrite the first derivative of the maximizing function as
\[
\frac{\partial \ln p_{m|s}(M|\hat{s})}{\partial \hat{s}} = \frac{-\hat{s}^3 N - \hat{s} N q + \hat{s} \sum_{i=1}^{N} M_i^2}{[\hat{s}^2 + q]^2}
\]

Next, the second derivative of the maximizing function is evaluated;
\[
\frac{\partial^2 \ln p_{m|s}(M|\hat{s})}{\partial \hat{s}^2} =
\]
\[
= -4\hat{s} \left[ -\hat{s}^3 N - \hat{s} N q + \hat{s} \sum_{i=1}^{N} M_i^2 \right] + \frac{-3\hat{s}^2 N - N q + \sum_{i=1}^{N} M_i^2}{[\hat{s}^2 + q]^2}
\]
\[
= \frac{\hat{s}^4 [N] + \hat{s}^2 \left[ -3 \sum_{i=1}^{N} M_i^2 \right] + \left[ q \left\{ \sum_{i=1}^{N} M_i^2 \right\} - N q^2 \right]}{[\hat{s}^2 + q]^3}
\]
Estimates \( \hat{s}_i \) of the EMG amplitude are formed from the measured surface myoelectric sequence \( m_i \) as shown in the figure above. Note that if \( \frac{1}{N} \sum_{i=1}^{N} m_i^2 < q \), then the optimal estimate is \( \hat{s}_i = 0 \).

Now, substitution of the estimate \( \hat{s} \) into the second derivative gives

\[
\frac{\partial^2 \ln p_{m|\hat{s}}(M|\hat{s})}{\partial \hat{s}^2} =
\]

\[
= \frac{N \left[ \left\{ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right\} - q \right]^2 + \left[ \left\{ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right\} - q \right]}{[\hat{s}^2 + q]^3} \left[ -3 \sum_{i=1}^{N} M_i^2 \right] + \]

\[
+ \frac{q \left( \sum_{i=1}^{N} M_i^2 \right) - Nq^2}{[\hat{s}^2 + q]^3}
\]

\[
= \left[ \frac{2 \sum_{i=1}^{N} M_i^2}{[\hat{s}^2 + q]^3} \right] \left[ q - \left\{ \frac{1}{N} \sum_{i=1}^{N} M_i^2 \right\} \right]
\]

This second derivative is less than or equal to zero according to;

\[
\frac{\partial^2 \ln p_{m|\hat{s}}(M|\hat{s})}{\partial \hat{s}^2} \leq 0 \]

\[
\iff \frac{1}{N} \sum_{i=1}^{N} M_i^2 \geq q
\]
The left term in the second relation above is essentially the sample variance of the observed signal within the observed time frame. Thus, the optimal estimate is viable only if this variance estimate is greater than or equal to the additive noise intensity. This condition will almost always be satisfied since the observed signal contains at a minimum the variance of the noise term itself. However, certain realizations of the observed signal can violate the above condition. For such a case, maximization with respect to \( \hat{s} \) of the probability density will occur at the boundary constraint where \( \hat{s} = 0 \).

The second additive noise model is formed by assuming that the output of the shaping filter \( \varphi \) and the noise sequence \( \nu \) have covariance matrices which are identical in shape, differing only in proportion. From Figure C.18 it is clear that the covariance matrix of the sequence \( \varphi \) is

\[
K_{\varphi \varphi} = s^2 K_{nn}
\]

where \( K_{nn} \) is the covariance matrix of the sequence \( n \). Thus, the covariance matrix of the noise sequence \( \nu \) can be denoted as

\[
K_{\nu \nu} = q K_{nn}
\]

where \( q \) denotes the noise intensity. The covariance matrix of the MSEMGI samples becomes

\[
K_{mm} = K_{\varphi \varphi} + K_{\nu \nu} = s^2 K_{nn} + q K_{nn} = (s^2 + q) K_{nn}
\]

Since no simple form exists for the inverse of \( K_{mm} \), consider passing the sequence \( m \) through the whitening filter \( H_{time}^{-1}(e^{j\omega}) \). Since \( H_{time}(e^{j\omega}) \) has a stable causal inverse, \( H_{time}^{-1}(e^{j\omega}) \) exists. From previous whitening arguments, it should be clear that both the signal sequence \( \varphi \) and the additive noise sequence \( \nu \) are whitened.
Estimates \( \hat{s}_i \) of the EMG amplitude are formed from the measured surface myoelectric sequence \( m_i \) as shown in the figure above. Note that if \( \frac{1}{N} \sum_{j=0}^{N-1} y_{i-j}^2 < q \), then the optimal estimate is \( \hat{s}_i = 0 \).

by the inverse filter. The output of the inverse filter is, therefore, a zero mean white Gaussian process of intensity \( s^2 + q \). The optimal estimate of \( s \) can now be derived as in the first additive noise model discussed above. The resultant estimator is shown in Figure C.20.

Multiple Channel Models

The Gaussian model with additive Gaussian noise can be expanded to the multiple channel case as shown in Figure C.21. \( L \) independent, zero mean, JWSS, white, CE, jointly Gaussian processes of unit intensity are passed through an \( L \)-input, \( L \)-output, LTI shaping filter, which is stable, causal, and whose inverse exists and is stable and causal. This multi-dimensional shaping filter is restricted to account \textit{only} for the spatial dependence between channels, including differences in signal strength. Hence, the multi-dimensional filter has no dynamics.
Figure C.21: Discrete-Time Functional Model of EMG in Additive Noise

$L$ independent, zero mean, jointly wide sense stationary (JWSS), correlation-ergodic (CE), jointly Gaussian, white processes of unit intensity are passed through the multi-dimensional filter $H_{\text{space}}$ which accounts only for the spatial dependence between channels. These filter outputs are each passed through a shaping filter $H_{\text{time},j}(e^{j\omega})$, multiplied by the EMG amplitude $s$, and added to one of $L$ independent zero mean, JWSS, CE, jointly Gaussian noise processes $v_{j,i}$ to form the $L$ channels of measured surface EMG waveform. The processes $w_{j,i}$ and $v_{j,i}$ are assumed to be independent and jointly Gaussian. The EMG amplitude is constant and the muscle contraction is non-fatiguing.
and can be represented as a linear transformation. The $L$ outputs from the multi-dimensional shaping filter are passed through a bank of LTI shaping filters to form $L$ dependent, zero mean, JWSS, non-white, CE, jointly Gaussian processes. These shaping filters are stable, causal, and have an inverse which is stable and causal. The output of each shaping filter is multiplied by the EMG amplitude $s$, and added to the zero mean, WSS, CE, jointly Gaussian noise process $v_{j,i}$ to form the MSEMG waveform. The processes $w_i$ and $v_j$ are assumed to be independent and jointly Gaussian for all $i$ and $j$. If the output of the $j^{th}$ shaping filter is denoted as $x_{j,i}$, then the $i^{th}$ sample of the $j^{th}$ MSEMG channel is

$$m_{j,i} = x_{j,i} + v_{j,i}$$

As in the single channel case, for any sequence of $N$ samples, the random vector $m_j$ is a jointly Gaussian random vector with mean vector

$$\mu_{m_j} = 0$$

and covariance matrix

$$K_{m_j, m_j} = K_{x_{j}, x_{j}} + K_{x_{j}, v_{j}}$$

Since the MSEMG signals are all mutually jointly Gaussian, only the cross-covariances are needed to completely specify their distribution. Since all of the random vectors are zero mean,

$$K_{m_j, m_j} = \mathbb{E}[m_j m_j]$$

$$= \mathbb{E}[(x_{i} + v_{i})(x_{j} + v_{j})]$$

$$= \mathbb{E}[x_{i} x_{j}] + \mathbb{E}[v_{i} x_{j}] + \mathbb{E}[x_{i} v_{j}] + \mathbb{E}[v_{i} v_{j}]$$
Since the processes \( \psi_i \) and \( \psi_j \) are independent for all \( i, j \) (which implies that \( \varphi_i \) and \( \varphi_j \) are independent for all \( i, j \)), then
\[
\mathcal{E}[\psi_i \varphi_j] = \mathcal{E}[\varphi_i \psi_j] = 0
\]
and
\[
K_{m_i m_j} = K_{\varphi_i \varphi_j} + K_{\psi_i \psi_j}
\]

The multiple channel additive noise model corresponding to the first previously discussed simple single channel additive noise model is formed by assuming that the spatial filter and all whitening filters become the all-pass filter, and that each additive noise source is both white of intensity \( q \) and independent of each other additive noise source. With these assumptions,
\[
K_{\varphi_i \varphi_j} = K_{\psi_i \psi_j} = 0 \quad \text{for } i \neq j
\]
and, thus,
\[
K_{m_i m_j} = \begin{cases} (s^2 + q)I, & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases}
\]

If an \( N \cdot L \) dimension composite random vector \( m_{j,i} \) is formed from all \( N \) samples of all \( L \) channels, then this random vector is a jointly Gaussian random vector with mean vector equal to zero and covariance matrix equal to
\[
K_{m_{j,i} m_{j,i}} = (s^2 + q)I
\]

Hence, this multiple channel model is characterized in an identical manner as the corresponding single channel model, except for the dimension of the random vector. The desired estimate can thus be written as
\[
\hat{s} = \left[ \frac{1}{N \cdot L} \sum_{j=1}^{L} \left( \sum_{i=1}^{N} M_{j,i}^2 \right) - q \right]^{\frac{1}{2}}
\]
Figure C.22: Discrete-Time Multiple Channel Optimal EMG Processor — Additive Noise, First Case

Estimates $\hat{s}_i$ of the EMG amplitude are formed from the measured surface EMG waveform $m_i$ as shown in the figure above. Note that if $\frac{1}{L} \sum_{j=1}^{L} y_{j,i} < q$, then the optimal estimate is $\hat{s}_i = 0$. 
where if the term within the curly brackets is greater the $q$, then $\delta = 0$. This estimator is shown in Figure C.22.

The multiple channel additive noise model corresponding to the second previously discussed simple single channel additive noise model is formed by assuming that (1) the output of each shaping filter $x_i$ and its corresponding noise sequence $v_i$ have covariance matrices which are identical in shape, differing only by the common proportion $q$ (i.e. $K_{v_i,v_i} = q K_{n_i,n_i}$), and (2) the ensemble random vector $v_{*,i}$, where

$$v_{*,i} = \begin{bmatrix} v_{1,i} \\ v_{2,i} \\ v_{3,i} \\ \vdots \\ v_{L,i} \end{bmatrix}_{L \times 1}$$

has a covariance matrix equal to that of the ensemble random vector $n_{*,i}$ (defined previously). To find the amplitude estimate, consider temporally whitening each channel followed by spatial uncorrelation of the $L$ channels. At the output of the temporal whitening filter, both the signal sequences and the additive noise sequences have been whitened (they share a common temporal whitening filter). At the output of the spatial uncorrelation filter, both the signal sequences and the additive noise sequences have been spatially uncorrelated (they share a common spatial uncorrelation filter). Thus, the $L$ channels are now uncorrelated (independent since they are jointly Gaussian) white processes with intensity equal to $s^2 + q$, the sum of the intensities of the signal sequence and the additive noise sequence. The optimal estimate of $s$ can now be found as in the first multiple channel additive noise model. The resultant estimator is shown in Figure C.23.
Figure C.23: Discrete-Time Multiple Channel Optimal EMG Processor — Additive Noise, Second Case

Estimates $\hat{s}_i$ of the EMG amplitude are formed from the measured surface EMG waveform $m_i$ as shown in the figure above. Note that if $\frac{1}{L} \sum_{j=1}^{L} y_{j,i} < q$, then the optimal estimate is $\hat{s}_i = 0$. 
Appendix D

Experimental Apparatus
D.1 Introduction

The experimental apparatus consisted of an instrumented chair for measuring the torque generated about the elbow, and a set of ten electrode-amplifiers with associated amplification for measuring the surface EMG. The instrumented chair allowed a seated subject to elevate his/her arm such that the upper and lower arm were perpendicular to the floor (shoulder ab ducted 90 degrees from the anatomic position), and the upper arm was directed laterally outward from the shoulder (normal to the sagital plane). The subject’s wrist was rigidly attached to a flexible beam. Strain gauges, mounted on the beam, with associated electronic circuitry measured bending of the beam. Commercial electrode-amplifiers were placed on the skin above muscles to detect the surface EMG. Electronics required to power the commercial electrode-amplifiers as well as to further amplify the EMG signal were constructed. Mechanical and electrical details of the instrumented chair and the EMG acquisition are described below. Additionally, the Informed Consent and Subject Interview documents used for the experiments are provided.

D.2 Instrumented Chair

D.2.1 Mechanical Design

A straight-back metal chair, shown in Figure D.1, was acquired and instrumented to measure torque about the elbow. The instrument was designed for comfortable use by average individuals ranging in height from approximately five feet three inches to six feet three inches, according to the design criteria of Diffrient et al. (1983). Measurement of torque about the right elbow, only, was instrumented.
Figure D.1: Straight-Back Metal Chair

The chair back rest was removed and the back rest plate was secured to the chair at the five locations marked with the symbol “x”. The chair was bolted to a wooden platform to prevent tipping.
The chair was bolted to a wood platform to prevent tipping. The wood platform had dimensions of 27.5 x 32 x 1.5 inches and was comprised of two pieces of plywood, each of dimension 27.5 x 32 x 0.75 inches, nailed together. The chair was fixed to the platform by four inch long corner irons secured through the platform and through the legs of the chair with $\frac{1}{4}$ inch diameter bolts. Corner irons were oriented so as to prevent both side to side and back to front movement of the chair.

The chair back rest was removed and replaced with a back rest plate. The back rest plate was made of a cast aluminum of dimensions $\frac{1}{2} \times 16 \times 32.5$ inches. The back rest plate was machined as shown in Figure D.2. The back rest plate was secured to the chair with five $\frac{3}{8}$ inch diameter bolts whose locations are marked with the symbol “X” in Figures D.1 and D.2. These five bolts were custom aligned by first clamping the back rest plate into position (in the plane of the chair back rest) on the chair. This position is shown in Figure D.3. Next, $\frac{3}{8}$ inch holes were drilled through the back rest plate and chair simultaneously. The bolts were inserted and tightened, and then the positioning clamps released. This manner of custom fitting avoided detailed planning and alignment of the securing bolt locations. For subject comfort, the back rest plate provides approximately $3\frac{1}{2}$ inches of clearance from the chair seat. Foam padding for the subject’s back was provided.

The subject was secured to the back rest plate via five belts. Each belt was comprised of two segments, the segments mating with quick-release connectors. The belts were secured to the instrument by $\frac{3}{8}$ inch bolts in four locations. Two bolts were passed through holes in the back rest plate (see Figure D.2). Two bolts were placed in the chair legs, one in each rear leg, just under the chair seat.
The back rest plate was constructed of \( \frac{1}{2} \) inch thick cast aluminum plate. Slots were centered on the locations marked and were \( \frac{3}{8} \) inch in diameter. Holes were centered on the locations \((2,27)\) and \((14,27)\) and were \( \frac{3}{8} \) inch in diameter. The back rest plate was secured to the chair at the five locations marked with the symbol "$x$". Location of the securing bolts was determined as described in the text. Figure locations are marked in inches.
Figure D.3: Subassembly One

Front view of the assembled straight-back chair, wood platform, back rest plate and crossbar.
All bolts were oriented front to back with the threaded end extending to the rear of the instrument. A belt was held in place by drilling a hole in the belt material and bolting the belt to the rear of the instrument. Two belts criss-crossed the front of the body from shoulder to opposite hip (from lower bolt to opposite upper bolt). A belt was oriented from the shoulder to respective hip, one belt for each side of the body (from lower bolt to respective upper bolt). The upper segments of these belts connected to their securing bolts by passing through their respective slot in the back rest plate. The fifth belt was worn across the lap (from lower bolt to opposite lower bolt).

A crossbar, shown in Figure D.4, was secured to the back rest plate. Figure D.3 shows the orientation of the crossbar. Two bolts were passed through the slots in the back rest plate, through the slots in the crossbar, and through holes in the crossbar spacer. The crossbar spacer is shown in Figure D.5. When the securing nuts were tightened to the bolts, the crossbar spacer prevented deformation of the walls of the crossbar. The slots in the back rest plate allowed vertical adjustment of the crossbar. The slots in the crossbar allowed horizontal adjustment of the crossbar.

A pivot plate, shown in Figure D.6 was mounted to the crossbar via two \( \frac{3}{8} \) inch bolts passing through slots in the pivot plate and holes in the crossbar. A pivot plate mounting spacer, shown in Figure D.7, was slid into the crossbar such that the two bolts traveled through the spacer. The pivot plate mounting spacer prevented deformation of the walls of the crossbar when the securing nuts were tightened. The pivot plate was oriented in the plane of the floor, as shown in Figure D.8. The slots in the pivot plate allowed front to back adjustment of the pivot plate.
The crossbar was constructed of 2x2 inch 6061 aluminum square channel of thickness $\frac{1}{8}$ inch. Slots were centered on the locations marked and were $\frac{3}{8}$ inch in diameter. Holes were centered on the locations marked and were $\frac{3}{8}$ inch in diameter. Figure locations are marked in inches.
Figure D.5: Crossbar Spacer

The crossbar spacer was constructed of scrap aluminum bar. Holes were centered 8 inches apart and were $\frac{3}{8}$ inches or greater in diameter. The crossbar spacer was $\frac{3}{8}$ inches thick with planar dimensions of $3\frac{1}{4} \times 10$ inches. The semi-circular cutout provided clearance for the top bolt which secured the back rest plate to the chair.

An aluminum beam was mounted to the pivot plate via an angle bracket. The angle bracket, shown in Figure D.9, was secured to the pivot plate at the common pivot point with a $\frac{3}{8}$ inch diameter bolt. The angle bracket could be rotated about the pivot point, with one remaining hole in each respective face of the angle bracket and pivot plate becoming aligned every $15^\circ$ (over a span of approximately $180^\circ$). An aluminum beam, shown in Figure D.10, was mounted to the angle bracket, oriented as shown in Figure D.11. The beam was mounted via two $\frac{3}{8}$ diameter bolts which passed through a beam spacer (shown in Figure D.12), through the angle bracket, through a beam spacer, through the beam, through a beam spacer, and through the rear beam shield (shown in Figure D.13). The portion of the rear beam shield most distant from the mounting location extended towards the slotted end of the beam to protect the
The pivot plate was constructed of $\frac{1}{2}$ inch thick cast aluminum plate. Slots were centered on the locations marked and were $\frac{3}{8}$ inch in diameter. The two located holes were $\frac{3}{8}$ inch in diameter, centered on the locations marked. Remaining holes were $\frac{3}{8}$ inch in diameter and centered as follows: A guide ray was drawn from (3,14) (denoted the pivot point) through (3,12). Holes were then placed at radii of 2 inches from the pivot point along rays originating from the pivot point with angle of $\pm 30^\circ$, $\pm 60^\circ$ and $\pm 90^\circ$ from the guide ray. Figure locations are marked in inches.
Figure D.7: Pivot Plate Mounting Spacer

The pivot plate mounting spacer was constructed of scrap solid aluminum square of dimensions $4.5 \times 1\frac{3}{4} \times 1\frac{3}{4}$ inches. Holes were centered 3 inches apart and were $\frac{3}{8}$ inch or greater in diameter.

strain gauges. The wrist cuff was secured through the slot in the beam with a $\frac{10}{32}$ inch diameter bolt. The slot allowed the location of the cuff to be adjusted. The complete mechanical assembly is shown in Figure D.14.
Figure D.8: Subassembly Two

View from above of the assembled pivot plate and crossbar.
The angle bracket was constructed of a 4 inch long, 4 x 4 inch 6061 aluminum angle. All holes were \( \frac{3}{8} \) inch in diameter. The two holes on the same face as the pivot point were located as follows: A guide ray was drawn 1 inch from the angle edge, originating from the pivot point, as shown in the figure. Holes were then placed at a radius of 2 inches from the pivot point along rays originating from the pivot point at angles 30° above the guide ray and 15° below the guide ray. Figure distances are marked in inches.
The beam was constructed of $\frac{3}{8}$ inch thick 2024 aluminum rectangle. Slot was centered on the location marked and was $\frac{10}{32}$ inch in diameter. Holes were centered on the locations marked and were $\frac{3}{8}$ inch in diameter. Figure locations are marked in inches.
Figure D.11: Subassembly Three

View from chair seat of the beam, angle bracket and beam spacer.
Figure D.12: Beam Spacer

Beam spacers were constructed of $\frac{3}{8}$ inch thick scrap aluminum. Holes were $\frac{3}{8}$ inch in diameter and centered 1 inch apart. Planar dimensions of the spacers were $2.5 \times 1.5$ inches.

Figure D.13: Rear Beam Shield

The rear beam shield was constructed of $\frac{3}{8}$ inch thick scrap aluminum. Holes were $\frac{3}{8}$ inch in diameter and centered 1 inch apart. Planar dimensions of the rear beam shield were $4.25 \times 2.5$ inches.
Figure D.14: Complete Mechanical Assembly of the Instrumented Chair
D.2.2 Strain Gauges and Associated Electronics

A Wheatstone bridge, with each of four legs an active strain gauge, was used to measure deflection of the beam. Figure D.15 shows the excitation circuit for the strain gauges. The strain gauges were BLH Electronics Part Number FAE-50-35-S13EL. Each strain gauge had a nominal resistance of 350Ω. Strain gauges $R_1$ and $R_4$, shown in Figure D.11, were mounted on one side of the beam and strain gauges $R_2$ and $R_3$ were mounted on the opposite side of the beam. ($R_2$ was mounted opposite $R_1$, and $R_3$ was mounted opposite $R_4$.) With a flexion torque about the elbow, $R_1$ and $R_4$ were strained in compression while $R_2$ and $R_3$ were strained in expansion. Outputs of the strain gauge excitation circuit were differentially amplified by the strain gauge amplification circuit shown in Figure D.16.

The hardware for the above circuits was located in three locations. Strain gauges $R_1$, $R_2$, $R_3$ and $R_4$ were mounted on the beam. A remote electronics box contained the power supply and components $R_f$ and $C_f$. The remaining components were assembled in a small electronics box and secured close to the strain gauges (secured to bolts extending from the angle bracket). This arrangement allowed the differential amplification stage to be physically close to the strain gauges (short lead wires promoted improved noise rejection), but did not require challenging miniaturizing of the electronic circuitry.

D.2.3 Performance of the Instrumented Chair

A static evaluation of the performance of the Instrumented Chair was performed. The emphasis of the evaluation was on the linearity of the elbow torque measurement. The subassembly shown in Figure D.11 was removed from the complete
Figure D.15: Strain Gauge Excitation Circuit

Each strain gauge had a nominal resistance of 350Ω. $V_+ = 15V$, $V_- = -15V$. 
Figure D.16: Strain Gauge Amplification Circuit

Circuit elements were: \( V_+ = 15V, V_- = -15V, R_{\text{Gain}} = 680\Omega, C_+ = C_- = 3.3\mu F, R_{\text{Scale}} = 100K\Omega, R_{\text{Null}} = 0-10K\Omega, R_f = 3K\Omega, C_f = 0.33\mu F. \)
assembly. The angle bracket was mounted to a table (via clamps), oriented such that weights hung from the beam would deflect the beam in the same sense as would elbow flexion. (Orientation was performed manually, since errors in orientation would not effect the evaluation of linearity.) Five weights were attained and weighed on a Douglas Homs Corp. scale (Temperature Compensated, Model 20, 20 pound by 1 ounce). The scale was specified as accurate to one ounce. The five weights ranged from approximately five to twenty pounds. Various combinations of the weights were hung from the beam. Results are plotted in Figure D.17. (Voltage was measured with a Fluke 8020A Multimeter set to the 20V setting, and expressed in hundredths of volts.) Additionally, at each of the weight combinations an "offset" weight (approximately three pounds) was added and the offset voltage recorded. In all cases, the offset voltage was 0.23V.

The angle bracket was then oriented such that weights hung from the beam would deflect the beam in the same sense as would elbow extension. Results of attached weight versus amplifier output voltage are shown in Figure D.17. When the offset weight was added, the offset voltage was 0.23V or 0.24V.

To evaluate the influence of forces applied to the beam in directions other than desired elbow flexion and extension, the angle bracket was oriented in its assembled orientation such that weights hung from the beam should not have been measured by the strain gauge system. Weights of approximately 20 and 40 pounds has a cross sensitivity of approximately 8%. This measurement represented a maximum cross sensitivity since improper orientation for this measurement could have effected the results.

Noise performance of the instrumented chair was not explicitly characterized, but could be evaluated visually from plots of the measured torque (found in the
Figure D.17: Instrumented Chair Performance

Top plot shows strain gauge amplifier circuit output voltage versus attached weight for the same sense as elbow flexion. Bottom plot shows strain gauge amplifier circuit output voltage versus attached weight for the same sense as elbow extension. Lines are an aid to the eye only.
text). The large aluminum chair back rest plate unfortunately served as a noise antenna. Future instruments should consider non-conducting materials for the structural members of the chair.

D.3 EMG Recording Apparatus

The EMG recording apparatus was discussed in detail in Section 5.2. Performance was discussed in Section 10.4.2. This section augments the previous descriptions with two figures. Figure D.18 shows the wiring assignment for the Liberty Mutual MYO111 electrode-amplifier. After amplification on the Liberty Mutual electrode-amplifier, each EMG signal was further amplified by an inverting amplifier with a variable gain of 0–10. Figure D.19 shows the circuit diagram for this variable gain amplifier.

D.4 Informed Consent

Each experimental subject (except the present author) was required to read and sign an Informed Consent Document and complete a Subject Interview each day experimental data were collected. Figure D.20 shows the three-page Informed Consent Document and Figure D.21 shows the Subject Interview form.
Note: A ground reference contact is required for proper acquisition of EMG.

(Supply +) Red

Black
(Supply -)

Electrode Contacts

Yellow (not used)

Green (Ground or Common)

White (Signal Output)

Figure D.18: Liberty Mutual MYO111 Electrode-Amplifier Wiring Assignments

[From The Liberty Mutual Research Center, Hopkinton, MA]
Figure D.19: Inverting Amplifier Circuit

Circuit elements were: $R_{\text{in}} = 10K\Omega$, $R_f = 0-100K\Omega$. The circuit was powered at $\pm 15V$. 
INFORMED CONSENT DOCUMENT

Title of Study: Relating the Surface Electromyogram to Muscle Torque

Principal Investigator: Prof. Neville Hogan

Associated Investigator: Edward A. Clancy

PURPOSE OF STUDY

Presently, there exists no reliable means to relate the electrical activity of a muscle (as observed non-invasively at the body surface) to the mechanical activity produced. Knowledge of this relationship (assuming such a direct relationship exists) is desired for the control of cybernetic prosthesis. Further, a non-invasive tool for the measurement of muscle force would be an indispensable tool in the study of human movement and biomechanics. This study will utilize the flexor and extensor muscles of the human elbow joint to investigate an EMG to torque relationship.

EXPERIMENTAL PROTOCOL

A subject will be seated in a straight-back chair and held in place with three quick-release belts. Two belts will criss-cross the front of the body (from shoulder to opposite hip) and the third belt will be worn across the lap. The belts will be secured taught so as to limit motion of the subject’s trunk. The subject’s right arm will be fitted into a wrist cuff which is rigidly attached to a fixed beam connected to the chair. The cuff eliminates the need for active grasp of the force-measuring device. The beam is instrumented to measure flexion/extension torques generated about the elbow joint of the right arm. Commercial electrodes will be secured to regions of the biceps and triceps muscles via an elastic arm band. The electrodes monitor the natural electrical activity of muscle, but do not disturb the muscle in any way.
During experimental trials, the subject will be instructed to pull/push on the beam to maintain specified levels of torque and/or specified levels of biceps/triceps contraction. The generated beam torque and the electrical activity monitored by the electrodes will be recorded on a digital computer. Additionally, instantaneous measures of generated beam torque and/or muscular electrical activity can be provided to the subject as feedback to assist in performing the specified trial task.

RISKS AND BENEFITS

Risks to the experimental subject are minimal. The belts which secure the subject to the chair should feel similar to tightly worn seat belts in an automobile. There is some possibility of minor discomfort due to wearing the wrist cuff – much as a shoe encloses the foot, the cuff encloses the wrist snugly while the wearer is exerting muscular force against it. The monitoring electrodes are commercial devices which have been used safely for several years now and pose no known risk to the experimental subject.

There are no benefits to the experimental subject beyond the satisfaction of contributing to advances in the understanding of the physiology pertinent to prosthetics, human movement and biomechanics.

Throughout the experiments, we welcome and encourage any questions, comments, or suggestions that you might have regarding the experiments.
The subject agrees to the following:

I am free at any time to seek further information regarding the experiment. In addition, I am also free to withdraw consent and discontinue participation at any time.

The subject will remain anonymous in all publications of the results of this experiment.

In the unlikely event of physical injury resulting from participation in this research, I understand that medical treatment will be available from the M.I.T. Medical Department, including first aid emergency treatment and follow-up care as needed, and that my insurance carrier may be billed for the cost of such treatment. However, no compensation can be provided for medical care apart from the foregoing. I further understand that making such medical treatment available, or providing it, does not imply that such injury is the Investigator's fault. I also understand that by my participation in this study I am not waiving any of my legal rights. Further information may be obtained by calling the Institute's Insurance and Legal Affairs Office at 253-2822.

I understand that I may also contact the Chairman of the Committee on the Use of Humans as Experimental Subjects, M.I.T. 253-6787, if I feel I have been treated unfairly as a subject.

I have read the above consent document and understand the experiments described in the document. I agree to participate in the experiments as a subject.

The project investigators retain the right to cancel or postpone the experimental procedures at any time they see fit.

Date: ____________________________

Subject's Name: ____________________________

Subject's Signature: ____________________________

Figure D.20 (Final Page): Informed Consent Document

Page 3.
SUBJECT INTERVIEW

Title of Study: Relating the Surface Electromyogram to Muscle Torque

Principal Investigator: Prof. Neville Hogan

Associated Investigator: Edward A. Clancy

QUESTIONS

1) Subject's Name: ______________________

2) Subject's Age: ________

3) Subject's Sex: Male Female

4) Subject's Handedness: Right Left

5) Does the subject have any neuromuscular condition effecting the function of the right shoulder, arm or hand?

Figure D.21: Subject Interview Form
References


