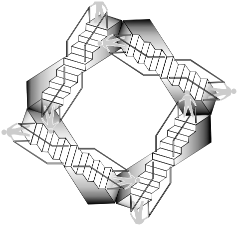
TIMING (CLOCK) RECOVERY

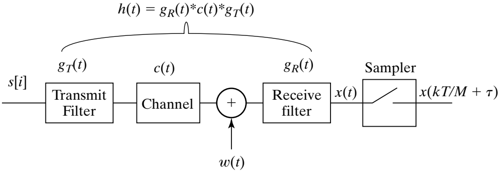
- A Baud-Timing Example
- Decision-Direction
- Output Power Maximization



adaptive components

Baud-Timing

- Consider the situation where the up and down conversion is done perfectly, so we need only consider a baseband model of the communication system.



- We are to select τ in
$x[k] = x(\frac{kT}{M} + \tau)$

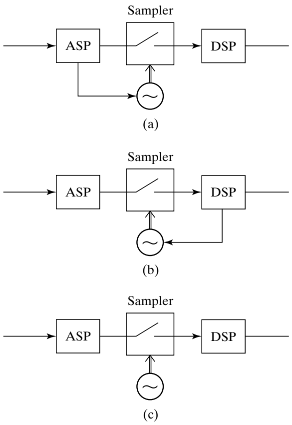
$$= \left(\sum_{i=-\infty}^{\infty} s[i]h(t - iT) + w(t) * g_R(t) \right) \Bigg|_{t=\frac{kT}{M}+\tau}$$

with

$$h(t) = g_T(t) * c(t) * g_R(t)$$

Baud-Timing (cont'd)

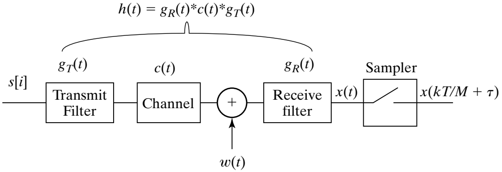
Three possible implementation configurations



We favor the last with its free-running sampler and fine tuning of the baud-timing done in the receiver DSP.

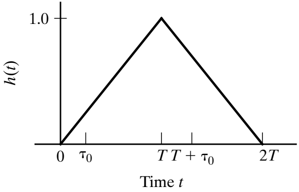
A Baud-Timing Example

We will analyze the special case for



when

- the noise w is absent and
- the analog pulse-shaping filter, the channel transfer function, and the receive filter combine into an impulse response that is a triangle spanning two symbol intervals.



A Baud-Timing Example (cont'd)

- With perfect baud-timing ($\tau = 0$) baud-space-sampled ($M = 1$) combined analog pulse/channel/receive filter impulse response shape is a Nyquist pulse

$$h(kT) = \begin{cases} 1, & k = 1 \\ 0, & k \neq 1 \end{cases}$$

- In general, without perfect baud-timing the sampler output is a weighted combination of several source symbol values

$$x[k] = \sum_i s[i]h(t - iT)\Big|_{t=kT+\tau}$$

- Consider three cases:
  - $\tau = 0$
  - $\tau > 0$
  - $\tau < 0$

A Baud-Timing Example (cont'd)

- $\tau = 0$
  - Only one nonzero point in sampled impulse response
  - Sampled impulse response

$$
\begin{aligned}
h(t - iT)|_{t=kT+\tau} &= h(kT + \tau - iT) \\
&= h((k-i)T + \tau) \\
&= h((k-i)T) \\
&= \begin{cases} 1, & k - i = 1 \\ & \quad \Rightarrow \; i = k - 1 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}
$$

  - $x[k] = s[k-1]$, system is pure delay, and sampler is synchronized with transmitter pulse.

A Baud-Timing Example (cont'd)

- $0 < \tau < T$
  - Two nonzero points in sampled impulse response $h(\tau_0)$ and $h(T + \tau_0)$
  - Sampled impulse response

$$h(t - iT)|_{t=kT+\tau_0} = h((k-i)T + \tau_0)$$

$$= \begin{cases} \frac{\tau_0}{T}, & k - i = 0 \\ 1 - \frac{\tau_0}{T}, & k - i = 1 \\ 0, & \text{otherwise} \end{cases}$$

- $-T < \tau < 0$
  - Two nonzero points in sampled impulse response $h(2T + \tau_0)$ and $h(T + \tau_0)$.
  - Sampled impulse response

$$h(t - iT)|_{t=kT+\tau_0} = \begin{cases} 1 - \frac{|\tau_0|}{T}, & k - i = 1 \\ \frac{|\tau_0|}{T}, & k - i = 2 \\ 0, & \text{otherwise} \end{cases} .$$

A Baud-Timing Example (cont'd)

Any sampled output $x[k]$ is based only on, at most, two symbol-spaced samples for any choice of τ.

- For example, with $\tau > 0$ for $k = 6$

$$\begin{aligned} x[6] &= \sum_i s[i]h((6-i)T + \tau_0) \\ &= s[6]h(\tau_0) + s[5]h(T + \tau_0) \\ &= s[6]\frac{\tau_0}{T} + s[5](1 - \frac{\tau_0}{T}) \end{aligned}$$

- For example, with $\tau < 0$ for $k = 6$

$$\begin{aligned} x[6] &= \sum_i s[i]h((6-i)T + \tau_0) \\ &= s[5]h(T + \tau_0) + s[4]h(2T + \tau_0) \\ &= s[4]\frac{|\tau_0|}{T} + s[5](1 - \frac{|\tau_0|}{T}) \end{aligned}$$

- For a binary input there are 4 possible symbol pairs $(+1, +1)$, $(+1, -1)$, $(-1, +1)$, and $(-1, -1)$ that are assumed equally likely.

A Baud-Timing Example (cont'd)

- For example, with $\tau > 0$ for $k = 6$
  - $(s[5], s[6]) = (+1, +1) \Rightarrow x[6] = \frac{\tau_0}{T} + 1 - \frac{\tau_0}{T} = 1$
  - $(s[5], s[6]) = (+1, -1) \Rightarrow x[6] = \frac{-\tau_0}{T} + 1 - \frac{\tau_0}{T} = 1 - \frac{2\tau_0}{T}$
  - $(s[5], s[6]) = (-1, +1) \Rightarrow x[6] = \frac{\tau_0}{T} - 1 + \frac{\tau_0}{T} = -1 + \frac{2\tau_0}{T}$
  - $(s[5], s[6]) = (-1, -1) \Rightarrow x[6] = \frac{-\tau_0}{T} - 1 + \frac{\tau_0}{T} = -1$
- Two of the possibilities for $x[6]$ give correct values for $s[5]$, while two are incorrect.
- As long as $2\tau_0 < T$ then the sign$[x(6)]$ matches $s[5]$ for all four possibilities.
- If τ_0 exceeds $T/2$, the sign of $x(6)$ would be associated with an earlier s than $s[5]$.

A Baud-Timing Example (cont'd)

- Similarly, with $\tau < 0$ for $k = 6$, the four equally likely source symbol pairs creating $x[6]$ are $(s[4], s[5])$
  - $(s[4], s[5]) = (+1, +1) \Rightarrow x[6] = \frac{|\tau_0|}{T} + 1 - \frac{|\tau_0|}{T} = 1$
  - $(s[4], s[5]) = (+1, -1) \Rightarrow x[6] = \frac{-|\tau_0|}{T} + 1 - \frac{|\tau_0|}{T} = 1 - \frac{2|\tau_0|}{T}$
  - $(s[4], s[5]) = (-1, +1) \Rightarrow x[6] = \frac{|\tau_0|}{T} - 1 + \frac{|\tau_0|}{T} = -1 + \frac{2|\tau_0|}{T}$
  - $(s[4], s[5]) = (-1, -1) \Rightarrow x[6] = \frac{-|\tau_0|}{T} - 1 + \frac{|\tau_0|}{T} = -1$
- With the addition of the absolute value on τ_0 (which does not effect a positive τ_0) the formulas for the four choices are the same as for positive τ_0.

A Baud-Timing Example (cont'd)

- For $-T/2 < \tau_0 < T/2$, $\mathrm{Q}(x[k]) = s[k-1]$.
- So, source recovery error equals decision error

$$e[k] = s[k-1] - x[k] = \mathrm{Q}(x[k]) - x[k]$$

  when eye is open. (But, if eye is closed, cluster variance does not equal average squared recovery error.)
- We are now in a position to consider some candidate cost functions for this baud-timing example.
- We will compute and sketch the cost functions for two candidates: cluster variance and output power.
- From this example, we will generalize to develop associated approximate gradient ascent/descent schemes for baud-timing.

A Baud-Timing Example (cont'd)

- Cluster variance

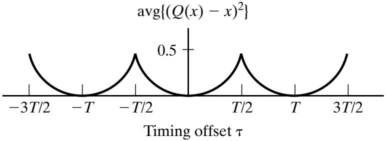
$$\text{avg}\{(\text{Q}(x[k]) - x[k])^2\}$$

- $\text{avg}\{(\text{Q}(x[6]) - x[6])^2\} =$

$$\left\{(1-1)^2 + (1-(1-\tfrac{2|\tau_0|}{T}))^2 + (-1-(-1+\tfrac{2|\tau_0|}{T}))^2 + (-1-(-1))^2\right\}$$

$$= \left(\frac{1}{4}\right)\left(\frac{4\tau_0^2}{T^2} + \frac{4\tau_0^2}{T^2}\right) = \frac{2\tau_0^2}{T^2}$$

- The same result occurs for other k.
- Desired offset of $\tau = 0$ $(\pm nT)$ occurs with minimization of average squared sampler output

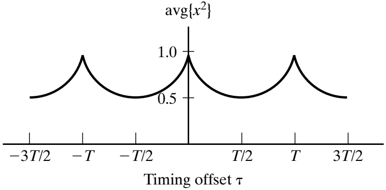


A Baud-Timing Example (cont'd)

- Average squared sampler output (or output power) $\text{avg}\{x^2[k]\}$

$$\begin{aligned}&= (1/4)[(1)^2 + (1 - (2|\tau|/T))^2 \\ &+(-1 + (2|\tau|/T))^2 + (-1)^2] \\ &= (1/4)[2 + 2(1 - (2|\tau|/T))^2] \\ &= 1 - (2|\tau|/T) + (2|\tau|^2/T^2)\end{aligned}$$

- Desired offset of $\tau = 0$ ($\pm nT$) occurs with maximization of average squared sampler output



Cluster Variance

- As a cost function for baud-timing, we first consider a moving average of squared decision error (aka cluster variance)

$$J_{CV}(\tau) = \frac{1}{N} \sum_{k=k_0}^{k_0+N-1} \{(\mathrm{Q}(x[k]) - x[k])^2\}$$
$$= \mathrm{avg}\{(\mathrm{Q}(x[k]) - x[k])^2\}$$

- To minimize J_{CV} using a gradient descent

$$\tau[k+1] = \tau[k] - \bar{\mu}\frac{\partial}{\partial\tau}[\mathrm{avg}\{(\mathrm{Q}(x[k]) - x[k])^2\}]|_{\tau=\tau[k]}$$

Using the interchangability of averaging and differentiation (see Appendix G) and dropping the "outer" average yields

$$\tau[k+1] = \tau[k] - \bar{\mu}\frac{\partial[(\mathrm{Q}(x[k]) - x[k])^2]}{\partial\tau}|_{\tau=\tau[k]}$$

Cluster Variance (cont'd)

- Using the chain rule (A.59) and the derivative of a signal raised to a power (A.61) and presuming that the derivative of $Q[x]$ with respect to x is zero yields

$$\tau[k+1] = \tau[k] - \bar{\mu}\frac{\partial[(\mathrm{Q}(x[k]) - x[k])^2]}{\partial\tau}|_{\tau=\tau[k]}$$

$$= \tau[k] + 2\bar{\mu}\left((Q(x[k]) - x[k])\frac{dx[k]}{d\tau}\right)|_{\tau=\tau[k]}$$

where for small δ we can approximate the derivative of x with respect to τ via

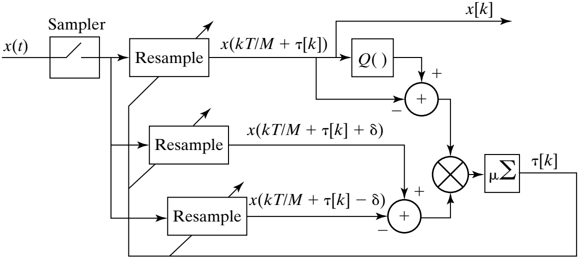
$$\frac{dx[k]}{d\tau} = \frac{dx(\frac{kT}{M}+\tau)}{d\tau}$$

$$\approx \frac{x(\frac{kT}{M}+\tau+\delta) - x(\frac{kT}{M}+\tau-\delta)}{2\delta}$$

- For positive, small $\bar{\mu}$, we can replace $2\bar{\mu}$ with a positive, small μ.

Cluster Variance (cont'd)

- Decision-directed, cluster-variance-minimizing baud-timing adaptation algorithm (with $x[k] = x((kT/M) + \tau[k])$)
$\tau[k+1] = \tau[k] + \mu(Q(x[k]) - x[k])$
$$\cdot \left(x(\frac{kT}{M} + \tau[k] + \delta) - x(\frac{kT}{M} + \tau[k] - \delta) \right)$$
- Decision-directed baud-timing adjusted oversampler schematic

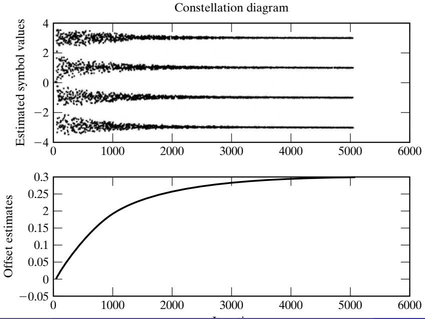


- Resample can be done as a linear filter with a truncated sinc response as in `interpsinc`.

Cluster Variance (cont'd)

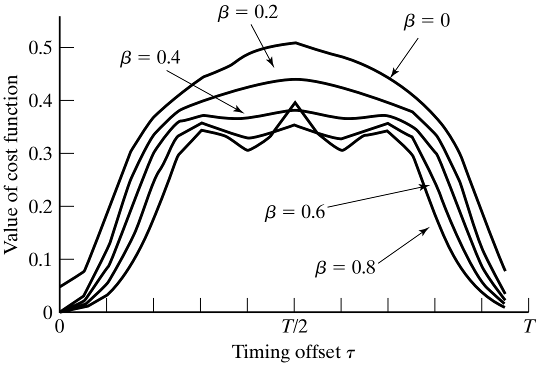
Example (from `clockrecdd`*)*:

- Source: 4-PAM
- Baud-timing adaptor stepsize: $\mu = 0.01$
- Derivative approximation increment: $\delta = 0.1$
- Pulse shape: SRRC with $\beta = 0.5$
- Free-running receiver sampler offset: -0.3 ($\Rightarrow$ desired baud-timing adjustment of $\tau = 0.3$)



Cluster Variance (cont'd)

Example (from `clockrecddcost`*)*:
Decision-directed cluster variance cost functions for desired τ of zero with various SRRC pulse shape roll-off factors β



- Local minima occur for larger β
- Plots constructed for specific (hopefully generic) dataset.

Output Power

- Moving average of square of sampler output

$$J_{OP}(\tau) = \frac{1}{N} \sum_{k=k_0}^{k_0+N-1} \{x^2[k]\} = \text{avg}\{x^2[k]\}$$

- To maximize J_{OP} using a gradient ascent

$$\tau[k+1] = \tau[k] + \bar{\mu} \frac{\partial}{\partial \tau} [\text{avg}\{x^2[k]\}]|_{\tau=\tau[k]}$$

we interchange the average and the differentiation (see Appendix G) and drop the "outer" average yielding

$$\tau[k+1] = \tau[k] + \bar{\mu} \frac{\partial (x^2[k])}{\partial \tau}|_{\tau=\tau[k]} = \tau[k] + 2\bar{\mu} \left(x[k] \frac{\partial x[k]}{\partial \tau} \right) |_{\tau=\tau[k]}$$

where for small δ

$$\frac{dx[k]}{d\tau} = \frac{dx(\frac{kT}{M} + \tau)}{d\tau} \approx \frac{x(\frac{kT}{M} + \tau + \delta) - x(\frac{kT}{M} + \tau - \delta)}{2\delta}$$

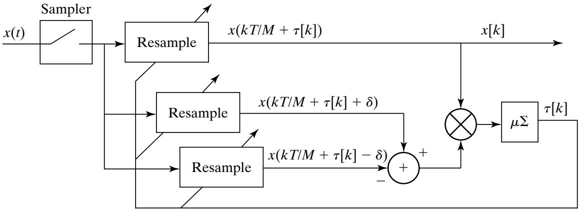
Output Power (cont'd)

- Output-power-maximizing baud-timing adaptation algorithm (with $x[k] = x((kT/M) + \tau[k]))$

$$\tau[k+1] = \tau[k] + \mu x[k]$$

$$\cdot \left(x(\frac{kT}{M} + \tau[k] + \delta) - x(\frac{kT}{M} + \tau[k] - \delta) \right)$$

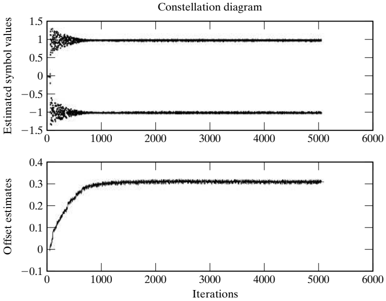
- Output-power-maximizing baud-timing adjusted oversampler schematic



Output Power (cont'd)

Example (from `clockrecOP`*)*:

- Source: 2-PAM
- Baud-timing adaptor stepsize: $\mu = 0.05$
- Derivative approximation increment: $\delta = 0.1$
- Pulse shape: SRRC with $\beta = 0.5$
- Free-running receiver sampler offset: -0.3 ($\Rightarrow$ desired baud-timing adjustment of $\tau = 0.3$)

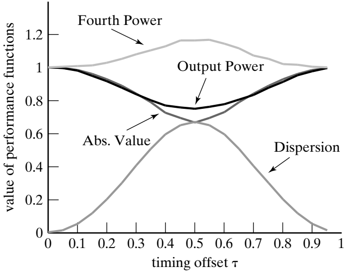


Output Power (cont'd)

Example (from `clockrecOPcost`*)*:

Cost functions for desired τ of zero with SRRC pulse shape with roll-off factors $\beta = 0.5$

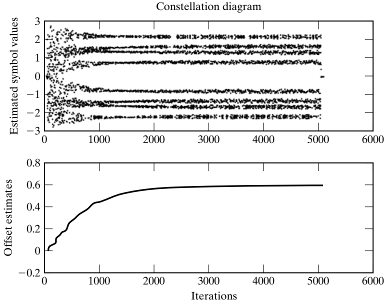
- absolute value: $J_{AV} = \frac{1}{N}\sum_{k=k_0}^{k_0+N-1}\{|x[k]|\}$
- fourth power: $J_{FP} = \frac{1}{N}\sum_{k=k_0}^{k_0+N-1}\{x^4[k]\}$
- output power (aka output energy): $J_{OP}(\tau) = \frac{1}{N}\sum_{k=k_0}^{k_0+N-1}\{x^2[k]\}$
- dispersion (aka constant modulus): $J_D(\tau) = \frac{1}{N}\sum_{k=k_0}^{k_0+N-1}\{(x^2[k]-1)^2\}$



Output Power (cont'd)

What happens with ISI? (using `clockrecOP`*)*:

- Channel: [1, 0.7, 0, 0, 0.5]
- All else same. (2-PAM source; $\mu = 0.05$; $\delta = 0.1$; SRRC pulse with $\beta = 0.5$; Free-running receiver sampler offset: -0.3; and $M = 2$)

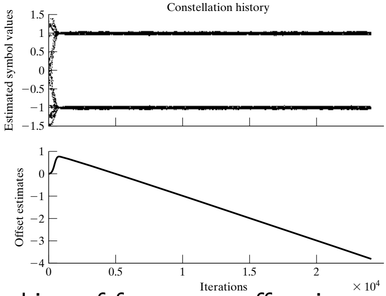


- Initially closed eye is opened within 500 iterations.
- Asymptotic offset not same as without ISI.

Output Power (cont'd)

What happens with symbol period offset? (from `clockrecperiod`*)*:

- Free-running receiver sampler: period 1.0001 times $T/2$; initial offset -1
- All else same as initial set-up. (2-PAM; no-ISI; $\mu = 0.05$; $\delta = 0.1$; SRRC with $\beta = 0.5$; and $M = 2$)



- Similar to phase tracking of frequency offset in carrier recovery.

NEXT... We present various adaptive algorithms for a baud-spaced linear equalizer.