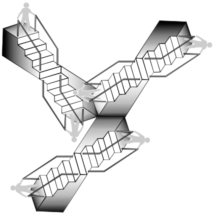
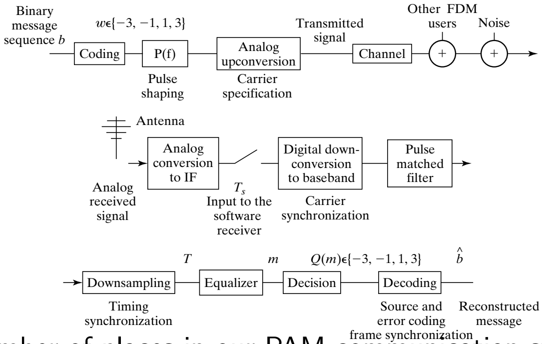
DIGITAL FILTERING AND THE DFT

- Digital Linear Filters in the Receiver
- Discrete-time Linear System Tidbits
- DFT Tidbits
- Filter Design Tidbits



idealized system

Digital Linear Filters in the Receiver



There are a number of places in our PAM communication system receiver after the sampler where a digital linear filter is needed, including:

- lowpass filter in digital downconversion
- pulse-matched filter
- timing interpolator
- equalizer
- correlator for decoder frame synchronization

Discrete-time Linear Systems Tidbits

- *Discrete-time impulse*:

$$\delta[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

- *Signal as weighted sum of delayed impulses*:

$$\{w[k]\} = \{1,\ 2,\ -1, ...\}$$

$$w[k] = \delta[k] + 2\delta[k-1] - \delta[k-2] \ldots$$

- *Discrete-time linear system response*:
  - input: $w[k] = \delta[k] \Rightarrow$ output: $y[k] = h[k]$
  - input: $w[k] = \delta[k] + 2\delta[k-1] = w[0]\delta[k] + w[1]\delta[k-1]$
    $\Rightarrow$ output:$y[k] = w[0]h[k] + w[1]h[k-1]$
  - input: $w[k] = \sum_{j=-\infty}^{\infty} w[j]\ \delta[k-j]$
    $\Rightarrow$ output:$y[k] = \sum_{j=-\infty}^{\infty} w[j]\ h[k-j] \equiv w[k] * h[k]$

DFT Tidbits

- *DFT*: Given

$$\{w[k]\} = \{w[0],\ w[1],\ ...,\ w[N-1]\}$$

  we define

$$\begin{aligned} W[n] &= \text{DFT}\left(\{w[k]\}\right) \\ &= \sum_{k=0}^{N-1} w[k] e^{-j(2\pi/N)nk} \quad n = 0, 1, 2, ..., N-1 \end{aligned}$$

- *IDFT*: Given

$$\{W[n]\} = \{W[0],\ W[1],\ ...,\ W[N-1]\}$$

  we define

$$\begin{aligned} w[k] &= \text{IDFT}\left(\{W[n]\}\right) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} W[n] e^{j(2\pi/N)nk} \quad k = 0, 1, 2, ..., N-1 \end{aligned}$$

DFT Tidbits (cont'd)

Define

$$\mathbf{w} = [w[0] \;\; w[1] \;\; w[2] \;\; \ldots \;\; w[N-1]]^T$$

$$M^{-1} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & e^{\frac{j2\pi}{N}} & e^{\frac{j4\pi}{N}} & \ldots & e^{\frac{j2\pi(N-1)}{N}} \\ 1 & e^{\frac{j4\pi}{N}} & e^{\frac{j8\pi}{N}} & \ldots & e^{\frac{j4\pi(N-1)}{N}} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & e^{\frac{j2(N-1)\pi}{N}} & e^{\frac{j4(N-1)\pi}{N}} & \ldots & e^{\frac{j2(N-1)^2\pi}{N}} \end{bmatrix}$$

$$\mathbf{W} = [W[0] \;\; W[1] \;\; W[2] \;\; \ldots \;\; W[N-1]]^T$$

Then, for the IDFT

$$\mathbf{w} = \left(\frac{1}{N}\right) M^{-1}\mathbf{W}$$

and for the DFT

$$\mathbf{W} = NM\mathbf{w}$$

DFT Tidbits (cont'd)

$$N\mathbf{w} = W[0]\begin{bmatrix}1\\1\\1\\\vdots\\1\end{bmatrix} + W[1]\begin{bmatrix}1\\e^{j2\pi/N}\\e^{j4\pi/N}\\\vdots\\e^{j2\pi(N-1)/N}\end{bmatrix} + \ldots + W[N-1]\begin{bmatrix}1\\e^{j2(N-1)\pi/N}\\e^{j4(N-1)\pi/N}\\\vdots\\e^{j2(N-1)^2\pi/N}\end{bmatrix}$$

$$= W[0]\ C_0 + W[1]\ C_1 + \ldots + W[N-1]\ C_{N-1}$$

$$= \sum_{n=0}^{N-1} W[n] C_n$$

$\mathbf{w}$ is a linear combination of the columns C_n.

DFT Tidbits (cont'd)

- The all ones C_0 is a vector of samples of a zero frequency sinusoid.
- The entries of C_1 maintain the same (unit) magnitude but have an angle that increases from 0 to $2\pi(N-1)/N$, i.e. traversing one period over the data record length.
- The entries of C_2 traverse the full unit circle twice in the positive (counterclockwise) direction.
- The DFT re-expresses the time vector as a linear combination of sinusoids with periods equal to the data record length, half this length, one-third this length, and so forth up to $(1/(N-1))$th of this length.

DFT Tidbits (cont'd)

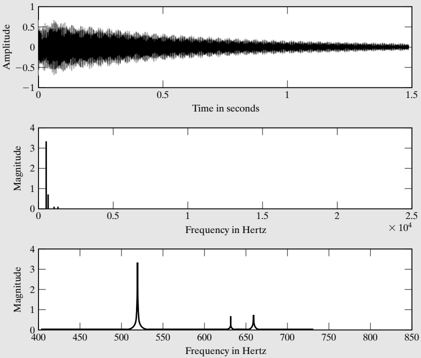
The key factors in a DFT based frequency analysis are:

- The sampling interval T_s is the time resolution, the shortest time over which any event can be observed.
- The sampling rate is $f_s = \frac{1}{T_s}$. As the sample rate increases, the time resolution decreases.
- The total time is $T = NT_s$ where N is the number of samples in the analysis.
- The frequency resolution is $\frac{1}{T} = \frac{1}{NT_s} = \frac{f_s}{N}$. Sinusoids closer together (in frequency) than this value are indistinguishable.
- As the time resolution increases, the frequency resolution decreases, and vice versa.

DFT Tidbits (cont'd)

Example: In `specgong.m`

- sampling interval: $T_s = \frac{1}{44100}$
- number of samples: $N = 2^{16}$
- total time: $NT_s = 1.48$ seconds
- frequency resolution: $\frac{1}{NT_s} = 0.67$ Hz

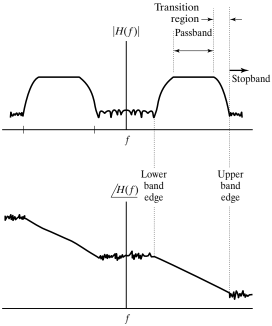


Filter Design Tidbits

Types:

- lowpass filter
- high pass filter
- bandpass filter
- bandstop (notch) filter

Bandpass filter spectra specification:



Filter Design Tidbits (cont'd)

From `help firpm`:
FIRPM performs Parks-McClellan optimal equiripple FIR filter design.
i.e. a linear phase (real, symmetric coefficients) FIR filter which has the best approximation to the desired frequency response in the minimax sense.
`b = firpm(fl,fbe,damps)`

- `b` is the output vector containing the impulse response of the designed filter.
- `fl` is (one less than) the number of terms in `b`.
  - `fl` $\uparrow \Rightarrow$ fit to design specs improves
  - `fl` $\uparrow \Rightarrow$ computational costs increase
  - `fl` $\uparrow \Rightarrow$ throughput delay increases

Filter Design Tidbits (cont'd)

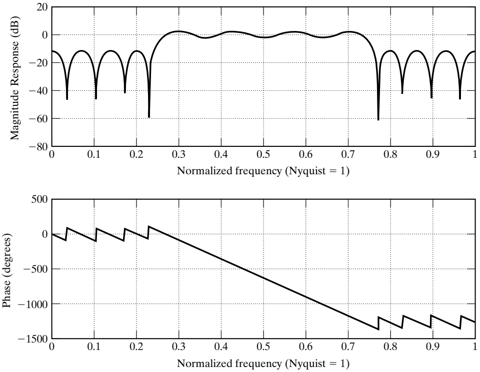
- `fbe` is a vector of frequency band edge values as a fraction of the prevailing Nyquist frequency. For a basic bandpass filter:
  - bottom of stopband (presumably zero)
  - top edge of lower stopband (which is also the lower edge of the lower transition band)
  - lower edge of passband
  - upper edge of passband
  - lower edge of upper stopband
  - upper edge of upper stopband (generally the last value will be 1).
- `damps` is the vector of desired amplitudes of the frequency response at each band edge.

Filter Design Tidbits (cont'd)

From bandex with

- fbe=[0 0.24 0.26 0.74 0.76 1]
- damps=[0 0 1 1 0 0]
- fl=30

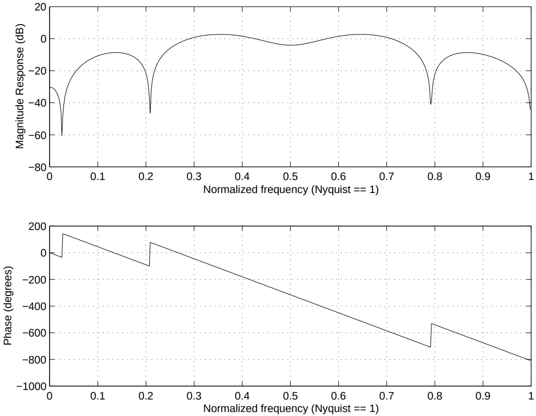
b=firpm(fl,fbe,damps); freqz(b) produces



Filter Design Tidbits (cont'd)

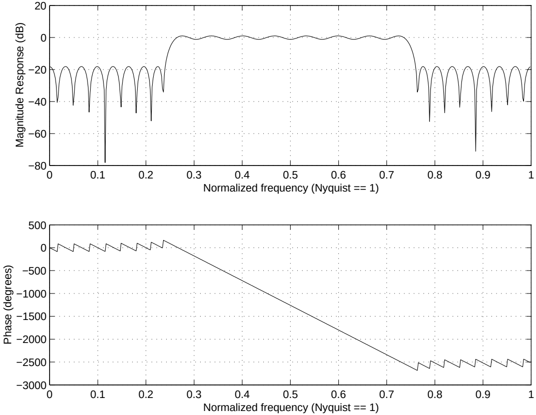
To demonstrate criteria fit impact of filter length, repeat preceding example with `fl` halved and doubled.

- `fl=15`



Filter Design Tidbits (cont'd)

- `fl=60`

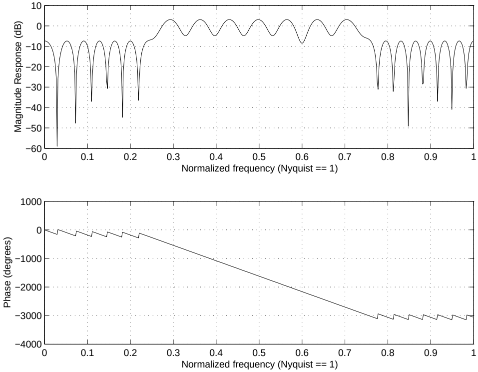


Note improved fit to design specifications with increase in filter length (`fl`).

Filter Design Tidbits (cont'd)

Adding an in-band notch with

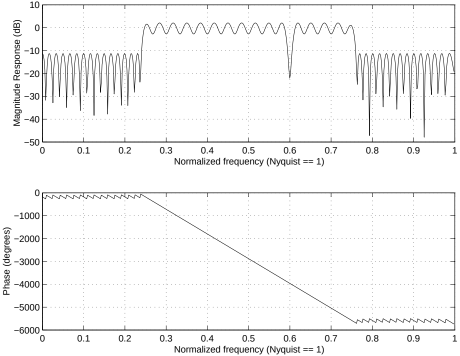
- `fbe=[0 0.24 0.26 0.59 0.595 0.605 0.61 0.74 0.76 1]`
- `damps=[0 0 1 1 0 0 1 1 0 0]`
- `fl=60`



Desired notch (at normalized frequency = 0.6) is barely perceptible.

Filter Design Tidbits (cont'd)

- `fl=120`



Note improved fit to design specifications with increase in filter length (`fl`). Desired notch (at normalized frequency $= 0.6$) is quite pronounced.
NEXT... We discuss the conversion of bits to symbols to pulse-amplitude modulated signals and the reversal with correlation used to locate the frame break.