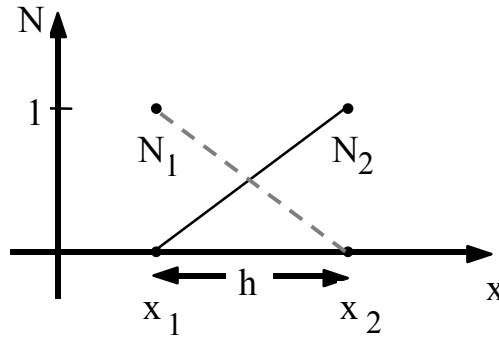


## Integration Formulae for 1-D Linear Elements

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$$N_1 = \frac{x_2 - x}{x_2 - x_1} = \frac{x_2 - x}{h} \qquad \frac{dN_1}{dx} = \frac{-1}{h}$$

$$N_2 = \frac{x - x_1}{h} \qquad \frac{dN_2}{dx} = \frac{1}{h}$$

$$\langle (\text{anything}) \rangle \equiv \int_{x_1}^{x_2} (\text{anything}) dx \qquad \langle 1 \rangle = h$$

$$\langle N_1 \rangle = \langle N_2 \rangle = \frac{h}{2}$$

$$\langle N_1 N_1 \rangle = \langle N_2 N_2 \rangle = \frac{h}{3} \qquad \langle N_1 N_2 \rangle = \frac{h}{6}$$

$$\langle N_1^3 \rangle = \langle N_2^3 \rangle = \frac{h}{4} \qquad \langle N_1^2 N_2 \rangle = \langle N_1 N_2^2 \rangle = \frac{h}{12}$$

$$\left\langle a(x) \frac{dN_1}{dx} \right\rangle = \frac{dN_1}{dx} \langle a(x) \rangle = \frac{-1}{h} \langle a(x) \rangle \text{ and}$$

$$\left\langle a(x) \frac{dN_2}{dx} \right\rangle = \frac{dN_2}{dx} \langle a(x) \rangle = \frac{1}{h} \langle a(x) \rangle$$