Consider the general quadratic equation:

$$a\gamma^2 + b\gamma + c = 0 \tag{1}$$

Where it is assumed that (a) is positive. The solution is:

$$\gamma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or

$$\gamma = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} \tag{2}$$

For $\frac{c}{a} > \left(\frac{b}{2a}\right)^2$, γ will be complex and its magnitude is given by:

$$|\gamma|^2 = \frac{c}{a} \tag{3}$$

The necessary and sufficient condition for the stability of the complex roots is:

$$\frac{c}{a} \leq 1 \tag{4}$$

If $\frac{c}{a} \leq \left(\frac{b}{2a}\right)^2$, γ will be real and one of the roots is:

$$|\gamma| \geq \left|\frac{b}{2a}\right| \geq \sqrt{\frac{c}{a}} \tag{5}$$

The condition (4) is also necessary for the stability of the real roots. From (5) a second condition is that:

$$\left|\frac{b}{2a}\right| \leq 1 \qquad (6)$$

Assuming that (6) is satisfied, the stability constraint for the real roots is:

$$\left|\frac{b}{2a}\right| + \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} \leq 1 \qquad (7a)$$

or

$$\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} \leq 1 - \left|\frac{b}{2a}\right| \qquad (7b)$$

Since both sides of (7b) are non-negative, they can be squared:

$$\left(\frac{b}{2a}\right)^2 - \frac{c}{a} \leq 1 + \left(\frac{b}{2a}\right)^2 - \left|\frac{b}{a}\right| \qquad (8)$$

Rearrangement of (8) gives the stability constraint for the real roots:

$$\left|\frac{b}{a}\right| \leq 1 + \frac{c}{a} \tag{9a}$$

or

$$|b| \leq a + c \tag{9b}$$

As long as (4) is satisfied, (9b) implies (6).

It can be shown that when (4) is satisfied, but (9b) is not, the roots of (1) are real. From (9b)

$$b^2 > a^2 + c^2 + 2ac = 4ac + (a - c)^2 \tag{9a}$$

and thus,

$$b^2 > 4ac \tag{9b}$$

Under these conditions the roots are real and unstable.

Consequently, the necessary and sufficient conditions for the stability of γ are:

$$\frac{c}{a} \leq 1 \tag{4}$$

and

$$|b| \leq a + c \tag{9b}$$