High-order strong stability preserving (SSP) time discretizations for use with spatial discretizations with nonlinear stability properties for the solution of hyperbolic PDEs. In this work, we describe sufficient conditions for a two-derivative multistage method to be SSP, and find some optimal SSP multistage two-derivative methods. While explicit SSP Runge-Kutta methods exist only up to fourth order, we show that this order barrier is broken for explicit multistage two-derivative methods by designing a three-stage fifth-order SSP method. These methods are tested on simple scalar PDEs to demonstrate the sharpness of the SSP time step in many cases.