

Laplace transform pairs

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
$t^n, n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}, s > a$
$\sin at$	$\frac{a}{s^2 + a^2}, s > 0$
$\cos at$	$\frac{s}{s^2 + a^2}, s > 0$
$\sinh at$	$\frac{a}{s^2 - a^2}, s > a$
$\cosh at$	$\frac{s}{s^2 - a^2}, s > a$
$\delta(t-a)$	$e^{-as}, a > 0$

Integral of a transform:

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(r) dr$$

Transform of a shifted function using unit step function $\mathcal{U}$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s), \quad a > 0$$

Properties of Laplace Transforms

$$F(s) = \mathcal{L}f(t), \quad G(s) = \mathcal{L}g(t)$$

Transform of a derivative:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

Convolution theorem:

$$\mathcal{L}\left\{\int_0^t f(r)g(t-r) dr\right\} = F(s)G(s)$$

Shifted transform:

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

Transform of a periodic function of period T :

$$\mathcal{L}\{f\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st}f(t) dt$$

Derivative of a transform:

$$\mathcal{L}\{-tf(t)\} = F'(s)$$

$$\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s), \quad n = 1, 2, \dots$$

Transform of an integral:

$$\mathcal{L}\left\{\int_0^t f(r) dr\right\} = \frac{F(s)}{s}$$