MA 2051 B 2013 — Quiz 5

Name: Solutions

Section (circle): 9 a.m. (Nathan) 10 a.m. (Ruofan) 11 a.m. (Alex) noon (Tuan) noon (Hector) 1:00 p.m. (Pi) 3 p.m. (Jesus)

Instructions: Work neatly. Show your work. Do your work on this paper. Use the back if needed. **Don't skip steps. Justify your answers.**

10.2.5/19 1. (3 pts) Find the Laplace transform $X(s)$ of the solution of $\overbrace{x'' + 4x}^{LHS} = \overbrace{\cos \pi t}^{RHS}$, $x(0) = 1$, $x'(0) = 4$. **Do not** invert the transform.

$$\mathcal{L}\{x'(t)\} = sX(s) - x(0) = sX - 1;\ \mathcal{L}\{x''(t)\} = s^2X - s\underbrace{x(0)}_{=1} - \underbrace{x'(0)}_{=4}$$

$$\Rightarrow \mathcal{L}\{LHS\} = s^2X - s - 4 + 4X = \mathcal{L}\{RHS\} = \frac{s}{s^2+\pi^2}$$

$$X(s^2+4) = 4 + s + \frac{s}{s^2+\pi^2} \Rightarrow \boxed{X = \frac{4+s}{s^2+4} + \frac{s}{(s^2+4)(s^2+\pi^2)}}$$

Not HW (yet!) Extra credit (1 pt): Look at the terms in $X(s)$. Without inverting, identify the frequencies, if any, you would expect to see in the solution $x(t)$

$$\frac{\cdots}{(s^2+a^2)} \to \begin{matrix}\sin\\ \cos\end{matrix}(at),\ a = 2, \pi \text{ in } X \Rightarrow \boxed{\text{expect oscillations of } 2, \pi \text{ rad/s}}$$

ω_n

10.3.5/6(b) 2. (4 pts) Use $\mathcal{L}\{-tf(t)\} = F'(s)$ (where $F(s) = \mathcal{L}\{f(t)\}$) to invert $\dfrac{2s}{(s^2+16)^2}$

$$F'(s) = \frac{2s}{(s^2+16)^2} \Rightarrow F(s) = \int 2s(s^2+16)^{-2}\,ds = \frac{-1}{(s^2+16)} + \cancelto{0}{C}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s^2+16}\right\} = -\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{4}{s^2+16}\right\} = -\frac{1}{4}\sin 4t$$

$$\therefore \boxed{\mathcal{L}^{-1}\left\{\frac{2s}{(s^2+16)^2}\right\} = -tf(t) = -\frac{t}{4}\sin 4t}$$

10.3.5/11 3. (3 pts) Find $\mathcal{L}\{te^{at}\}$. Use $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ w/ $f = t \Rightarrow F(s) = \frac{1}{s^2}$

$$\therefore \boxed{\mathcal{L}\{te^{at}\} = \frac{1}{(s-a)^2}}$$

(OR use property in #2)