

MA 2051 B 2013 — Quiz 4

Name: Solutions

Section (circle): 9 a.m. (Nathan) 10 a.m. (Ruofan) 11 a.m. (Alex) noon (Tuan)
noon (Hector) 1:00 p.m. (Pi) 3 p.m. (Jesus)

Instructions: Work neatly. Show your work. Do your work on this paper. Use the back if needed. Justify your answers.

10.1.3/1

1. (5 pts) Use the definition of Laplace transform to find $\mathcal{L}\{9e^{-4t}\} = \int_0^{\infty} (9e^{-4t})(e^{-st}) dt$

$$= \lim_{b \rightarrow \infty} 9 \int_0^b e^{-(4+s)t} dt = 9 \lim_{b \rightarrow \infty} \left. \frac{-1}{4+s} e^{-(4+s)t} \right|_0^b$$

$$= \frac{-9}{4+s} \lim_{b \rightarrow \infty} (e^{-(4+s)b} - 1) = \boxed{\frac{9}{4+s} \text{ if } 4+s > 0 \text{ (or } s > -4\text{)}}$$

$\underbrace{\hspace{10em}}_{\rightarrow 0 \text{ if } 4+s > 0}$
1 point

10.2.5/1

2. (5 pts) Find the function $f(t)$ whose Laplace transform is $\frac{4}{s(s+4)}$. Indicate clearly which transform table entries you use.

$$\frac{4}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4} \quad (\text{Partial Fractions})$$

Will use $\mathcal{L}\{1\} = \frac{1}{s}$, $\mathcal{L}\{e^{-4t}\} = \frac{1}{s+4}$

$$\frac{4}{s(s+4)} = \frac{A(s+4) + Bs}{s(s+4)} \Rightarrow 4 = \underbrace{(A+B)}_{=0} s + \underbrace{4A}_{=4} \Rightarrow A=1$$

$$\mathcal{L}\{f(t)\} = \frac{4}{s(s+4)} = \frac{1}{s} - \frac{1}{s+4} \Rightarrow B = -A = -1$$

$$= \mathcal{L}\{1 - e^{-4t}\} \Rightarrow \boxed{f(t) = 1 - e^{-4t}}$$