

MA 2051 B2013 — Quiz 1

Name: Solutions

Section (circle): 9 a.m. (Nathan) 10 a.m. (Ruofan) 11 a.m. (Alex) noon (Tuan)
noon (Hector) 3 p.m. (Jesus)

Instructions: Work neatly. Show your work. Do your work on this paper. Use the back if needed.

"Source"
in HW 7
1.5/1

1. (5 pts)

- (a) Show that $v(t) = -gt + v_i$ is a solution of the initial-value problem $\frac{dv}{dt} = -g$, $v(0) = v_i$.

$$\text{DE: } \frac{d}{dt}(-gt + v_i) = -g + 0 = -g \checkmark$$

$$\text{IC: } v(0) = -g \cdot 0 + v_i = v_i \checkmark$$

1.5/4

- (b) This initial-value problem models the velocity $v(t)$ of a ball thrown vertically upwards. Use the solution $v(t) = -gt + v_i$ to find the time T when the ball reaches its peak. Is the sign of v_i important?

$$v(T) = 0 \Rightarrow 0 = -gT + v_i \Rightarrow T = v_i/g$$

Need $v_i > 0$ - otherwise, ball doesn't go up.

- 2.2/3 2. (5 pts) The differential equation $\frac{dP}{dt} = 0.015P - H$ models a population of cells (P is measured in millions) that is reproducing with a constant birth rate 0.015 day^{-1} and being harvested at a constant rate H cells/day. The culture begins with a population of $P(0) = 1.2$ million cells. Find the harvesting rate H that will keep the population constant at 1.2 million.

$$\begin{aligned} P \text{ const.} &\Rightarrow \frac{dP}{dt} = 0 = 0.015P - H \Rightarrow H = 0.015P \\ &= 0.015 \cdot 1.2 = \boxed{.18 \text{ million cells/day} = H} \end{aligned}$$