Test Mon - 50 min

Sec 7? Pi

Overview

- "Model" means DE or $\overbrace{\text{DE + IVP}}^{\text{IVP}}$
  - Newton: $F_{total} = ma$
  - Conservation:
    rate of change
    = rate in − rate out
- Analyze
  - Const. sol'ns?
    Steady States: $dy/dt = 0$
  - Sol'n graph sketch
    IC gives one point
    $dy/dt = \cdots$ gives inc'g/dec'g

    Ex: $\dfrac{dP}{dt} = kP \begin{cases} >0, & P>0 \\ =0, & P=0 \text{ (SS)} \\ <0, & P<0 \end{cases}$

• "Solve" = find formula for sol'n

1st order DE

- NL
  - ? Sep of Var ?
- L
  - $y_g = C y_h + y_p$
    - Const. Coeff
      - Homo: Charac. Eqn
        - $y_h = e^{rt}$
      - Part: RHS = $\begin{cases} \text{poly} \\ \text{exp} \\ \text{cos/sin} \end{cases}$
        - y_p "looks like" RHS
        - Undet'd Coeff's
    - Other
      - Homo: S of V
      - Part:

Ex: $3y' + 9y = 2t - 3e^{4t} + 12\sin\pi t$ $+14e^{-2t}$

Find Gen'l Sol'n

$$y_g = C y_h + y_p$$

Homo: $3y' + 9y = 0 \Rightarrow y_h = e^{-3t}$

Part: $3y' + 9y$

Guess

1. " $= 2t$ $\quad y_{p_1} = A_1 t^1 + A_0$
2. " $= -3e^{4t}$ $\quad y_{p_2} = Ae^{4t}$
3. " $= 12\sin\pi t$ $\quad y_{p_3} = E\sin\pi t + F\cos\pi t$
4. " $= 14e^{-2t}$ $\quad y_{p_4} = Ge^{-2t}$

$$y_p = y_{p_1} + y_{p_2} + y_{p_3} + y_{p_4}$$

$$\therefore y_g = C y_h + y_{p_1} + y_{p_2} + y_{p_3} + y_{p_4}$$

Solve 1 - 20, 19, 3, 6, 7, 14, 12, 2, 8

19/20 $\frac{dp}{dt} = p' = kp(Q-p) = \underbrace{[k(Q-p)]}_{\text{depends on } p \Rightarrow NL} p$

S of V: $\displaystyle\int \frac{dp}{p(Q-p)} = \int k\,dt = kt + C$

Partial Fracts: $\displaystyle\frac{1}{p(Q-p)} = \frac{A}{p} + \frac{B}{Q-p}$

$$= \frac{A(Q-p) + Bp}{p(Q-p)} = \frac{AQ + (B-A)p}{p(Q-p)}$$

$\therefore$ $AQ = 1$, $B - A = 0$ $\Rightarrow$ $A = 1/Q$, $B = 1/Q$

$$\int \frac{dp}{p(Q-p)} = \frac{1}{Q}\int\Big(\underbrace{\frac{1}{p}}_{= \ln p} + \underbrace{\frac{1}{Q-p}}_{= -\ln(Q-p)}\Big)dp$$

$$\therefore\ e^{\ln\frac{p}{Q-p}} = e^{(Qkt + C_1)}$$

$$\frac{p}{Q-p} = \underbrace{e^{C_1}}_{C_2} e^{Qkt}$$

$\vdots$

7. $y' - y = 3e^t$

<u>H:</u> $y_h = e^t$

<u>P:</u> $\underline{y_p = Ae^t}$ $\Rightarrow (Ae^t)' - (Ae^t) = 3e^t$

BAD $\qquad 0 \neq 3e^t$

<u>Better:</u> $y_p = t(Ae^t)$

$(tAe^t)' - (tAe^t) = 3e^t$

$Ae^t + \cancel{tAe^t} - \cancel{tAe^t} = 3e^t$

$\therefore A = 3$

$\underline{y_g = Ce^t + \cancel{A}\, 3te^t}$

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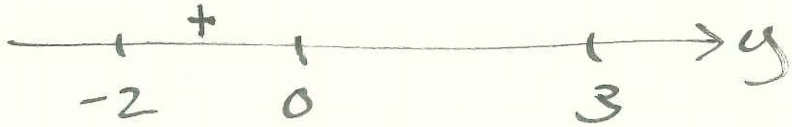
14) $y' = y(y-3)(y+2)$

$y_s = 0, 3, -2$

y $\quad -$

$y-3$ $\quad -$

$y+2$ $\quad +$



$y' > 0$