Syllabus correction: WW PWD Solve Order 1

due Fri 8 Nov - NOT Wed !

Aim to return HW due Thurs. in conf in Fri's lecture - email me if want your HW returned privately

Consult HW Grading Rubric linked from syllabus - meaning of HW, Quiz grade / score

Goal: General sol'n of linear, 1st-order DEs

$$y_g = C y_h + y_p$$

Homo Sol'n: $a_1(x) y_h' + a_0(x) y_h = 0$

Particular Sol'n: $a_1(x) y_p' + a_0(x) y_p = f(x)$

Gen'l sol'n covers all possible sol'ns of

$$a_1(x) y' + a_0(x) y = f(x)$$

Focus: special case: const. coefficient linear DEs

$a_1(x) = b_1 = \text{const}$, $a_0(x) = b_0 = \text{const.}$

Tactic: Make educated guess for y_h - sub. & check
" " " " y_p - " " "

Warning: be alert in case guess
blows up / goes nonsensical

Const. coeff. linear homogeneous DE

Educated guessing called Characteristic Eqn. Method

$$b_1 y' + b_0 y = 0$$

(b_1, b_0: Const's)

Ex: $P' = 0.015P$

$$1P' - 0.015P = 0$$

(b_1, b_0)

Educated guess: $P = e^{rt}$, $r = \text{const.} = ?$

Test guess: $\frac{dP}{dt} = re^{rt} = 0.015e^{rt}$

$e^{rt} \neq 0 \Rightarrow \boxed{r = 0.015}$

Charac. eqn.

Ex: $v' = -9.8$ $\quad 1\,v' + 0\,v = -9.8$

✓? b_1 b_0 f - ind. of v

(No) XH? $v=0 \Rightarrow 0 \neq -9.8$

BAD IDEA ~~Try anyway: guess $v = e^{rt}$, r=const.~~

Plug in: $v=e^{rt} \Rightarrow v' = re^{rt} = -9.8$

$\therefore r = -9.8/e^{rt}$? ← not r=const.

Ex: $v' = -9.8 - \frac{k}{m}v$

$v' + \frac{k}{m}v = -9.8$ $\quad$ L ✓ $\quad$ HX $\Rightarrow$ can't use charac. eqn.

New Problem Look at homo. version: $v_h' + \frac{k}{m}v_h = 0$

Guess $v_h = e^{rt}$, r=const.

$v_h' = re^{rt} \Rightarrow re^{rt} + \frac{k}{m}e^{rt} = 0$

Charac. Eqn: CE: $r + \frac{k}{m} = 0 \Rightarrow r = -\frac{k}{m}$

$$\boxed{v_h = e^{-kt/m}}$$

Gen'l Sol'n of $v_p' + \frac{k}{m}v_p = -9.8$

$v_{gen} = \underbrace{Ce^{-kt/m}}_{v_h} + v_p$

v_p=const. $\overset{A}{\Rightarrow} \frac{k}{m}A = -9.8$

Gen'l Sol'n $\boxed{v_g = Ce^{-kt/m} - \frac{9.8m}{k}}$

Ex: $P' = aP - sP^2 \Rightarrow \underbrace{1}_{b_1}P' \underbrace{-a}_{b_0}P = \underbrace{sP^2}_{f}$ f depends on P

X? L

No

Ex: $y' = y \sin 2\pi t \Rightarrow \underbrace{1}_{b_1}y' - (\sin 2\pi t)y = 0$

L?

H? $y = 0 \Rightarrow 0 = 0$ ✓

CC? $a_0(t) = \sin 2\pi t \neq$ const.

Preview: Educated guessing for y_p:

$$b_1 y_p' + b_0 y_0 = f(x)$$

called Method of Undetermined Coeff's

- y_p "looks like" $f(x)$

Ex: $v' = -9.8 - \frac{k}{m} v$

Found $v_h = e^{-(k/m)t}$: $v_h' + \frac{k}{m} v_h = 0$

Need v_p : $v_p' + \frac{k}{m} v_p = -9.8$

- RHS f = const $\Rightarrow$ guess y_p = const. = A

$v_p = A \Rightarrow v_p' = 0$

$$v_p' + \frac{k}{m} v_p = 0 + \frac{k}{m} A = -9.8 \Rightarrow A = \frac{-9.8m}{k}$$

$$\therefore v_p = A = \frac{-9.8m}{k}$$

$$\therefore \boxed{v_g = C v_h + v_p = C e^{-(k/m)t} + \left(\frac{-9.8m}{k}\right)}$$

Ex: $y' - y = 7x$

Homo: $y_h' - y_h = 0$

Gui: $y_h = e^{rx}$, r = const. $\Rightarrow re^{rx} - e^{rx} = 0$

$\Rightarrow r = 1$

$\therefore y_h = e^x$

Parti: $y_p' - y_p = 7x$

Guess: $y_p = A_1 x + A_0$, A_i = const.

$y_p' - y_p = A_1 - (A_1 x + A_0) = 7x$

$-A_1 x + (A_1 - A_0) = 7x + 0$

$-A_1 = 7$ $\quad$ $A_1 - A_0 = 0$

$\Rightarrow A_1 = -7$ $\quad$ $A_0 = A_1 = -7$

$\therefore y_p = -7x - 7$

$$\therefore \boxed{y_g = \underbrace{Ce^x}_{y_h} + \underbrace{(-7x-7)}_{y_p}}$$