Old Terms: in / dependent variable
t v, P - unknown function
sol'n of ODE, IVP

New Terms:

Trivial sol'n: $\equiv 0$ sol'n

Homogeneous ODE: has trivial sol'n

Non " " : $\equiv 0$ NOT a sol'n

Linear 1^{st}-order ODE has form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

"forcing func."

y = dependent variable = unknown
x = in " "

Otherwise, ODE is nonlinear

(PS - when is Linear 1^{st}-order DE also Homo?

Ans: when $f \equiv 0$

$y \equiv 0 \Rightarrow a_1 y' + a_0 y = a_1 \cdot 0 + a_0 \cdot 0 = 0 \overset{?}{=} f \Leftrightarrow f \equiv 0$

Examples: H or NH? L or NL?

Test: check if 0 is sol'n

Test: write as $a_1(x)y' + a_0(x)y = f(x)$

1. $\dfrac{dv}{dt} = -g$

$\underline{H?}$: $v \equiv 0 \Rightarrow 0 \neq -g \Rightarrow$ NH

$\underline{L?}$: $\left.\underbrace{1}_{a_1} v' + \underbrace{0}_{a_0} \cdot v = \underbrace{-g}_{f}\right\} \Rightarrow$ L

← const's/func. of t

2. $\dfrac{dv}{dt} = -g - \dfrac{k}{m} v$

$\underline{H?}$: $v \equiv 0 \Rightarrow 0 \stackrel{?}{=} -g - \frac{k}{m} \cdot 0 \Rightarrow 0 \neq -g \Rightarrow$ NH

$\underline{L?}$: $\left.\underbrace{1}_{a_1(t)} v' + \underbrace{\frac{k}{m}}_{a_0(t)} v = \underbrace{-g}_{f(t)}\right\} \Rightarrow$ L

← func's of t only

3. $\dfrac{dP}{dt} = kP$

$\underline{H?}$: $P \equiv 0 \Rightarrow \dfrac{d0}{dt} \stackrel{?}{=} k0 \Rightarrow 0 = 0 \checkmark \Rightarrow$ H

$\underline{L?}$: $\left.\underbrace{1}_{a_1(t)} P' - \underbrace{k}_{a_0(t)} P = \underbrace{0}_{f(t)}\right\} \Rightarrow$ L

- OK t only

4. $\dfrac{dP}{dt} = aP - sP^2$

$\underline{H?}$: $P \equiv 0 \Rightarrow \dfrac{d0}{dt} \stackrel{?}{=} a \cdot 0 - s \cdot 0^2 \Rightarrow 0 = 0 \checkmark \Rightarrow$ H

$\underline{L?}$: $\underbrace{\underbrace{1}_{a_1(t)} \cdot P' - \underbrace{a}_{a_0(t)} \cdot P}_{\text{OK - func's of } t \text{ only}} = \underbrace{\underbrace{-sP^2}_{?f(t)?}}_{\text{Func. of } P, \text{ not } t!} \Rightarrow$ NL

Forcing term f depends on unknown P!

Linear DEs permit superposition

$$a_1(x)\,y' + a_0(x)\,y = f(x)$$
$$a_1(x)\,z' + a_0(x)\,z = g(x)$$
$$\Rightarrow C_1 y + C_2 z \text{ solves DE} = C_1 f + C_2 g$$

Check: ~~Check:~~ $a_1(x)\left[C_1 y + C_2 z\right]' + a_0(x)\left[C_1 y + C_2 z\right]$

$$= a_1\left[C_1 y' + C_2 z'\right] + a_0\left[C_1 y + C_2 z\right]$$
$$= C_1\left(a_1 y' + a_0 y\right) + C_2\left(a_1 z' + a_0 z\right)$$
$$= C_1 f \qquad\qquad + C_2\, g \quad \checkmark$$

Special case: $a_1 y_p' + a_0 y_p = f \quad (*)$

$a_1 y_h' + a_0 y_h = 0$ (H version of DE)

Superposition $\Rightarrow y_g = C y_h + y_p$ solves $(*)$

$\Rightarrow$ way to add arb. const. to sol'n

Check: $a_1 y_g' + a_0 y_g = a_1\left[C y_h + y_p\right]' + a_0\left[C y_h + y_p\right]$

$$= a_1\left[C y_h' + y_p'\right] + a_0\left[C y_h + y_p\right]$$
$$= C\left(a_1 y_h' + a_0 y_h\right) + \left(a_0 y_p' + a_0\right)$$
$$= C \cdot 0 + \qquad\qquad f \quad = f \checkmark$$

Example of superposition

$$(*) \quad \frac{dT}{dt} = -\left(\frac{Ak}{cm}\right)(T - T_{out})$$

OR

$$\frac{dT}{dt} + \left(\frac{Ak}{cm}\right)T = \left(\frac{Ak}{cm}\right)T_{out}$$

Consider

$$H) \quad \frac{dT}{dt} + \left(\frac{Ak}{cm}\right)T = 0$$

Check: $(e^{-(\,)t})' + (\,)e^{-(\,)t} = 0$ ✓

has sol'n $T_H = e^{-(Ak/cm)t}$

$$NH) \quad \frac{dT}{dt} + \left(\frac{Ak}{cm}\right)T = \left(\frac{Ak}{cm}\right)T_{out}$$

Check: $\underbrace{\frac{dT_{out}}{dt}}_{=0} + (\,)T_{out} = (\,)T_{out}$ ✓

has sol'n $T_{NH} = T_{out}$

Super. says $T_g = Ce^{-(Ak/cm)t} + T_{out}$ solves $(*)$

Sol'n of IVP: $(*)$, $T(0) = T_i$?

$T_g(0) = Ce^0 + T_{out} = T_i \Rightarrow C = T_i - T_{out}$

$\therefore$ sol'n of IVP is $\boxed{T(t) = (T_i - T_{out})e^{-(Ak/cm)t} + T_{out}}$

General sol'n of linear, 1^{st}-order DE:

$$a_1(x)y' + a_0(x)y = f(x)$$

$$y_g = C\,y_h + y_p$$

Nontrivial sol'n of homo. DE $(a_1y' + a_0y = 0)$ — Gives arbitrary const. for IC

Any particular sol'n of given DE $(a_1y' + a_0y = f)$ — Gives proper RHS in DE

∴ can solve Linear 1^{st}-order DE in 2 phases

I. Find some/any particular sol'n y_p of given NH DE: $a_1y_p' + a_0y_p = f$

II. Find nontrivial sol'n y_h of homo. version of DE: $a_1y_h' + a_0y_h = 0$

Obtain general sol'n: $\boxed{y_g = C y_h + y_p}$

Quiz ?

~~Heat loss~~

HW 2, Ques'n 3, part 2 -

Pop'n model - chose E rate to make

$P(t) = \text{const}.$

#6: Harvest yeast

Quiz - 2

3): $$\frac{dP}{dt} = \underset{\underset{0.015}{\parallel}}{k} P - \underset{\underset{0.209}{\parallel}}{E}, \quad \cancel{P(0) = 0.209}, \quad P(1847) = 8$$

Choose E : $P(t) = \text{const.}$

Seek sol'n $P(t) = \text{const.}$

Plug $P = I = \text{const.}$ into DE

$$0 = I' = .015 I - E$$

$$E = .015 \cdot \underset{\underset{8}{\parallel}}{I}$$

$$\therefore \boxed{E = .015 \cdot 8 = .12 \text{ mil/yr}}$$

6) Harvest yeast:

a) $\left.\frac{dP}{dt}\right|_{repro} = RP$ - repro. rate

b) $\left.\frac{dP}{dt}\right|_{Harvest} = H$

c) Only changes are repo., harvest

Total $\frac{dP}{dt} = +\left.\frac{dP}{dt}\right|_R - \left.\frac{dP}{dt}\right|_H$

$$\boxed{\frac{dP}{dt} = RP - H, \quad P(0) = P_i}$$

Radioactive decay of I

decay rate k, created at rate S

$$\frac{dI}{dt} = -kI + S$$

1.5/4) v_i for slingshot ?

Aim up. Time to peak t_p

$$v' = -g \iff \boxed{\begin{array}{l} \dfrac{dv}{dt} = -9.8\,m/s^2 \\ v(0) = v_i \end{array}}$$

$$v(t) = -9.8t + v_i$$

At peak, $v(t_p) = 0 \Rightarrow 0 = -9.8t_p + v_i$

$$\Rightarrow \boxed{v_i = 9.8t_p}$$