Quiz Mon: Calc Review ⊕ HW 1, 2

Model Re-cap

Falling Felix

- No air resistance: $\frac{dv}{dt} = -g$, $v(0) = v_i$
- Air resistance $\sim v$: $\frac{dv}{dt} = -g - \frac{k}{m}v$, $v(0) = v_i$

Population:

- Fraction k reproduces per unit time:

$$\frac{dP}{dt} = kP, \quad P(0) = P_i$$

- --- add harvesting at rate H

$$\frac{dP}{dt} = kP - H, \quad P(0) = P_i$$

- Competition reduces birth rate

$$\frac{dP}{dt} = (a - sP)P = aP - sP^2, \quad P(0) = P_i$$

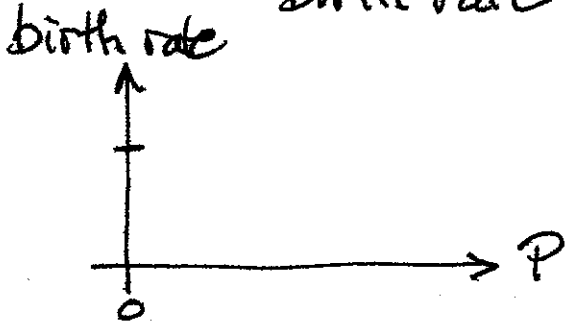
Observations

- "Birth rate" =
- "Harvesting" =
- Decaying population - fixed fraction k disappears

$$\frac{dP}{dt} = kP, \quad k>0$$

- Competition reduces birth rate

$$\frac{dP}{dt} = \underbrace{(a-sP)}_{\text{birth rate}} P$$



- Air resistance & Pop. w/harvesting DE's are "same"

$$\frac{dv}{dt} = -\frac{k}{m}v - g$$

$$\frac{dP}{dt} = kP - H$$

BBC: Beach Beverage Challenge

Bring cold beverage to beach in an ice chest.

How long before it's too warm to drink?

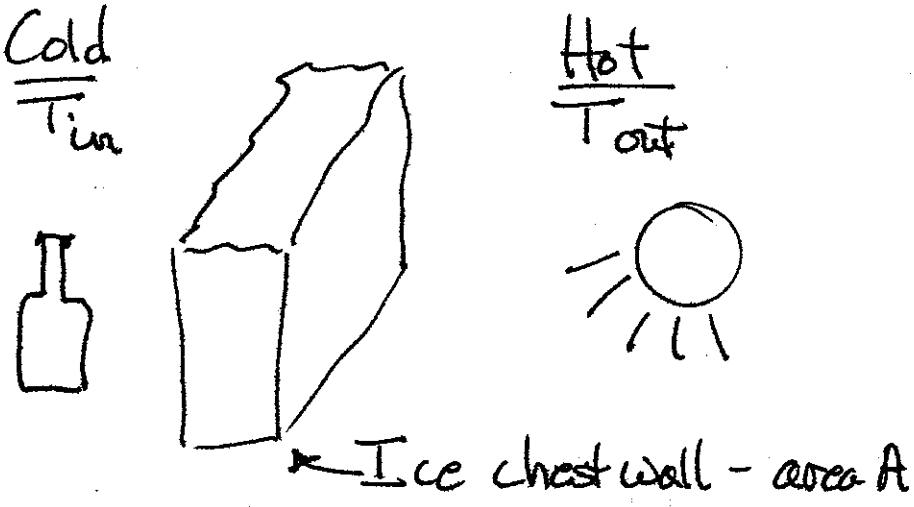
Heat-Flow Model

1. Q = heat (energy) in body is proportional to mass (m) & to absolute temp (T)

$$Q = c\,m\,T$$

c = specific heat

2. Rate of heat flow per unit area is proportional to temp. difference, goes from hot toward cold



Model of beverage temp $T(t)$:

$Q = cmT$ ← For beverage inside ice chest

$$\frac{dQ}{dt} = -Ak(T(t) - T_{out})$$

A = area of ice chest surface (top, sides)

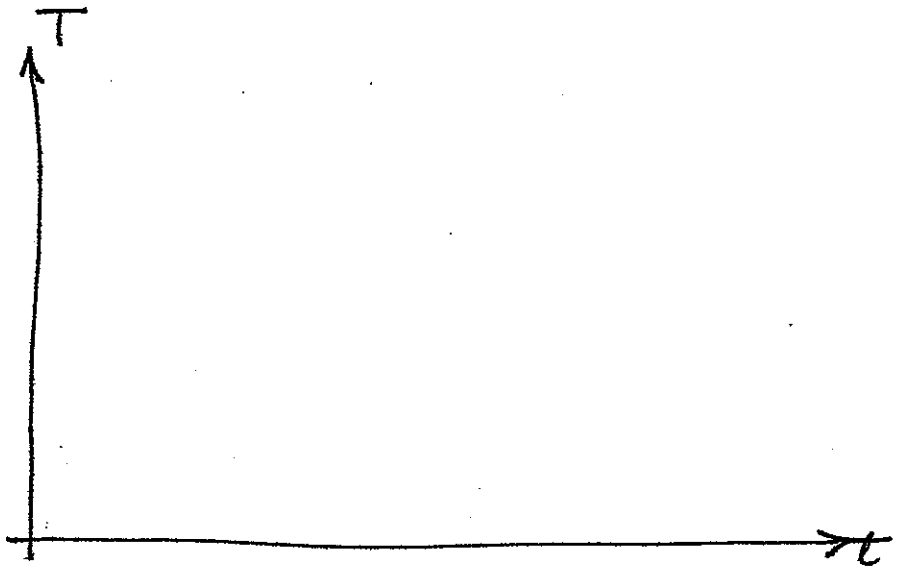
k = thermal conductivity of styrofoam walls, lid

$$Q = cmT \Rightarrow \frac{dQ}{dt} =$$

$$cm\frac{dT}{dt} = -Ak(T - T_{out})$$

$$\frac{dT}{dt} = -\frac{Ak}{cm}(T - T_{out}),\ T(0) = T_i$$

#3-6



Solve by Sep. of Variables

$$\frac{dT}{dt} = -\underbrace{\frac{Ak}{cm}}_{b}(T - T_{out})$$

$$\frac{dT}{-b(T-T_{out})} = dt$$