MA 2051 - #24 Th/12 Dec 13

HW/WW needing $\mathcal{U}$

Due Monday at midnight

$\mathcal{U}$ h step

Ex. of inversion tactics / expectations

$$y' - 0.015y = -\begin{cases} 0.209, & 0 \le t < 4 \\ 0, & 4 \le t \end{cases}, \quad y(0) = 4$$

$$\Rightarrow Y(s) = \mathcal{L}\{y(t)\} = \overset{①}{\frac{4}{s-0.015}} - \overset{②}{\frac{0.209}{s(s-0.015)}}$$

$$+ \overset{③}{\underbrace{\frac{0.209}{s(s-0.015)}}_{F(s)}} e^{-4s}$$

$y(t) = ?$

$$\mathcal{L}^{-1}\{①\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s-0.015}\right\} =$$

Expectations

① has form $\frac{1}{s-a}$, $a = 0.015 \to e^{0.015t}$

OK for growth

Partial Frac's

② has form $\frac{A}{s} + \frac{B}{s-0.015} \to A + Be^{0.015t}$

$$0.015 = a$$

③ has form $F(s)e^{-4s}$, $4 = a$, $= \mathcal{L}\{f(t-4)u(t-4)\}$

$f(t) = \mathcal{L}^{-1}\{F(s)\} \leftarrow$ Know from ②

PFs ②:
$$\frac{0.209}{s(s-0.015)} = \frac{A}{s} + \frac{B}{s-0.015}$$
$$= \frac{A(s-0.015)+Bs}{s(s-0.015)}$$
$$= \frac{(A+B)s - 0.015A}{s(\quad)}$$

s^1: $0 = A+B \Rightarrow A = -B$

s^0: $0.209 = -0.015A \Rightarrow A = -\frac{.209}{.015} = -B$

$$\boxed{\therefore y(t) = 4e^{0.015t} + \frac{.209}{.015}\left[-1 + e^{0.015t}\right] - \frac{.209}{.015}\left[e^{.015(t-4)} - 1\right]u(t-4)}$$

11:59 p.m.

Paul Davis **MA2051B13**

WeBWorK assignment number PD2051Laplace is due : 12/16/2013 at 10:00pm EST.

The link

http://users.wpi.edu/ pwdavis/Courses/MA2051B13/MA2051B13syllabus.htm

leads to the syllabus for the course. It contains the homework and test schedule, grading policy, and other information.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2\wedge 3$ instead of 8, $sin(3*pi/2)$instead of -1, $e\wedge(ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5))\wedge 6-7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

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1. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{8e^{-6s}}{s^2+64}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{8e^{-6s}}{s^2+64}\right\} =$ ____________

Answer(s) submitted:

-

(incorrect)

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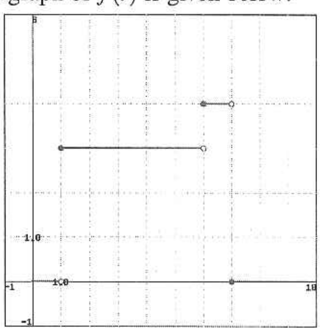
2. (1 pt) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

(1) Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$u(t-a)$

$f(t) =$ ____________

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ ____________

Answer(s) submitted:

-
-

(incorrect)

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3. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{5s-12}{s^2-6s+25}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{5s-12}{s^2-6s+25}\right\} =$ ____________

Answer(s) submitted:

-

(incorrect)

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4. (1 pt) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^{2t}\cos(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{e^{2t}\cos(6t)\} =$ ____________

Answer(s) submitted:

-

(incorrect)

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5. (1 pt) Consider the initial value problem

$$y' + 5y = \begin{cases} 0 & \text{if } 0 \leq t < 3 \\ 11 & \text{if } 3 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty, \end{cases} \qquad y(0) = 6.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

____________ = ____________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________

Answer(s) submitted:

-
-
-
-

(incorrect)

6. (1 pt)

(1) Set up an integral for finding the Laplace transform of $f(t) = 4$.

$$F(s) = \mathcal{L}\{f(t)\} = \int_A^B \underline{\hspace{3em}}$$

where $A =$ ___ and $B =$ ___.

(2) Find the antiderivative (with constant term 0) corresponding to the previous part.

_____

(3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ _____

(4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

_____

Answer(s) submitted:

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-
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-
-

(incorrect)

7. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 9t^5 + 9t + 4$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\left\{9t^5 + 9t + 4\right\} =$ _____

(2) For what values of s does the Laplace transform exist?

_____

Answer(s) submitted:

-
-

(incorrect)

8. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 7 + \sin(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{7 + \sin(6t)\} =$ _____

(2) For what values of s does the Laplace transform exist?

_____

Answer(s) submitted:

-
-

(incorrect)

9. (1 pt) Consider the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

_____ = _____

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ _____

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ _____

Answer(s) submitted:

-
-
-
-

(incorrect)

10. (1 pt) Consider the initial value problem

$$y' + 4y = 64t, \qquad y(0) = 4.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

_____ = _____

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ______________

Answer(s) submitted:

-
-
-
-

(incorrect)

10.6

In exercises 1–2, use the definition of Laplace transform to find the transform of the given function. Sketch the graph of each function before you attempt to find its transform.

1. $g(t) = \begin{cases} mt, & 0 \le t \le b \\ 0, & b < t < \infty \end{cases}$

2. $g(t) = \begin{cases} 0, & 0 \le t \le 2 \\ t-2, & 2 \le t \le 4 \\ 6, & 4 < t < \infty \end{cases}$

In exercises 3–10, find the indicated transform; a and ω are constants. Confirm each result with DELAB.

3. $\mathcal{L}\{\cos 2t \sin 4t\}$
4. $\mathcal{L}\{\cos^2 3t\}$
5. $\mathcal{L}\{e^{-4t}\cos 2t \sin 4t\}$
6. $\mathcal{L}\{e^{-2t}\cos^2 3t\}$
7. $\mathcal{L}\{t(1-t)\}$
8. $\mathcal{L}\{e^{at}\cos \omega t\}$
9. $\mathcal{L}\{t\cos \omega t\}$
10. $\mathcal{L}\{\sin \omega t + \omega t \cos \omega t\}$

In exercises 11–12,

(i) Graph the forcing term and write it in terms of the unit step function $\mathcal{U}$.

(ii) Solve the initial-value problem, using Laplace transforms.

(iii) Give the interval(s) within which the highest derivative of the solution appearing in the equation is continuous.

(iv) Verify that your solution satisfies the differential equation within each interval of continuity.

(v) As appropriate, confirm each transform and solution result with DELAB.

11. $y' + 2y = g(t)$, $y(0) = 5$,

$$g(t) = \begin{cases} -2, & 0 \le t < 4 \\ 4, & 4 \le t < 8 \\ 0, & 8 \le t < \infty \end{cases}$$

12. $y'' - 9y = g(t)$, $y(0) = 4$, $y'(0) = 0$,

$$g(t) = \begin{cases} 0, & 0 \le t < 2 \\ 2, & 2 \le t < \infty \end{cases}$$

13. Assuming that it exists, find the Laplace transform of the solution of $x' + x = g(t)$, $x(0) = -3$, where $g(t)$ is the isolated pulse and cosine wave of exercise 7, page 555,

$$g(t) = \begin{cases} 1, & 0 \le t \le 2\pi \\ \cos t, & 2\pi \le t \le 7\pi/2 \\ 0, & 7\pi/2 < t < \infty \end{cases}$$

Work directly from the differential equation.

14. Draw a graph like that of figure 10.10, page 547, illustrating the "turn-on" property of the unit step function product $f(t-a)\mathcal{U}(t-a)t - a$ for the function appearing in example 24, page 549,

$$g(t) = \begin{cases} 0, & 0 \le t \le 2 \\ \sin t, & 2 < t < \infty \end{cases}$$

In exercises 15–24, predict the general behavior of the function of t whose transform is given, then find the function and compare it with your prediction. Comment on differences. As appropriate, confirm each result with DELAB.

15. $\dfrac{4}{s^2+5s+6}$

16. $\dfrac{4e^{-2s}}{s^2+5s+6}$

17. $\dfrac{2s}{s^2-4}$

18. $\dfrac{2s}{s^2+4}$

19. $\dfrac{2se^{-5s}}{s^2+4}$

20. $\dfrac{s-1}{(s+1)(s^2-4)}$

21. $\dfrac{4}{s^2+2s+5}$

22. $\dfrac{3}{s^3+3s^2+2s}$

23. $\dfrac{2s+6}{s^2+6s+18}$

24. $\dfrac{3s}{s^2-3s+10}$

In exercises 25–28, the transform of the response of system is given. In each case,

(i) Determine whether the response contains oscillatory terms. If so, give their frequencies.

(ii) Determine whether the response includes exponential decay or growth. If so, give the factor in the exponential (or the corresponding time constant).

(iii) Determine if the response contains any terms that switch on or off. If so, give the switching time(s).

Do *not* invert the transforms. (As appropriate, use DELAB to check your results.)

25. $\dfrac{s^2+1}{s^4-16}$

26. $\dfrac{s^2e^{-2s}}{(s^2+4)(s^2+4}$

27. $\dfrac{2-4e^{-3s}}{(s-1)^3}$

28. $\dfrac{3}{(s-3)(s^2+1}$

Laplace transforms
coefficient different
roduce variables f
$Y(s) = \mathcal{L}\{y(t)\}$,
solve for $Y(s)$, $Z($
exercises 29–36. C

29. $y' = y + z$,
$z' = -y + z$,

30. $y' = z$, $y(0) =$
$z' = y$, $z(0) =$

31. $y' = -4y + 2z$
$z' = -3y + 3$

32. $y' = -4z$, $y($
$z' = -4y$, $z($

33. $y' = y/2 + \sqrt{}$
$z' = \sqrt{3}\,y/2 -$

34. $y' = z + 4\cos$
$z' = y$, $z(0) =$

35. $y' = y + 3z +$
$z' = 3y + z -$

36. $y' = y - z -$
$z' = 2y + 4z$

10.7 ■ CH