Solve w/ IVPs

1. $\mathcal{L}\{DE\}$; use ICs. Solve for
   $\bar{Y}(s) = \mathcal{L}\{y(t)\}$

2. $y(t) = \mathcal{L}^{-1}\{\bar{Y}\}$

Answer(s) submitted:

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(incorrect)

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6. (1 pt)

(1) Set up an integral for finding the Laplace transform of $f(t) = 4$.

$$F(s) = \mathcal{L}\{f(t)\} = \int_A^B \underline{\hspace{3em}}$$

where $A =$ ___ and $B =$ ___.

(2) Find the antiderivative (with constant term 0) corresponding to the previous part.

____

(3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ __________

(4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

____

Answer(s) submitted:

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(incorrect)

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7. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 9t^5 + 9t + 4$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\left\{9t^5 + 9t + 4\right\} =$ __________________

(2) For what values of s does the Laplace transform exist?

____

Answer(s) submitted:

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(incorrect)

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8. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 7 + \sin(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{7 + \sin(6t)\} =$ __________________

(2) For what values of s does the Laplace transform exist?

____

Answer(s) submitted:

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(incorrect)

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9. (1 pt) Consider the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

__________________ = __________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ __________________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ __________________________

Answer(s) submitted:

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(incorrect)

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10. (1 pt) Consider the initial value problem

$$y' + 4y = 64t, \qquad y(0) = 4.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

__________________ = __________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ______________________

Answer(s) submitted:

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(incorrect)

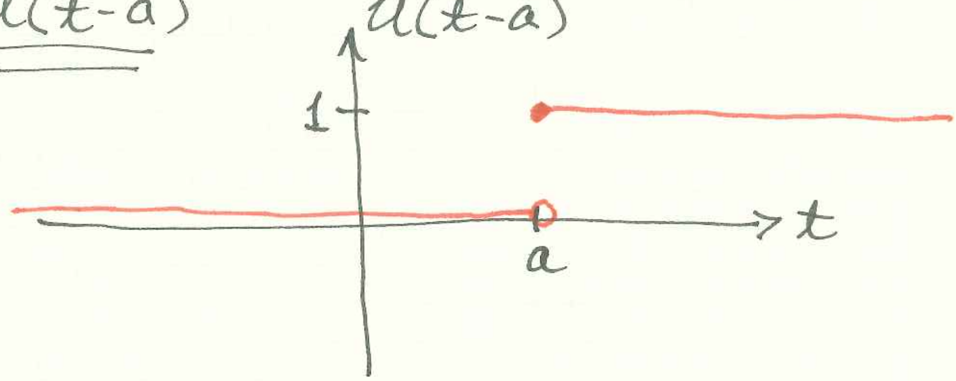
Unit Step Func:

ww: step(t)

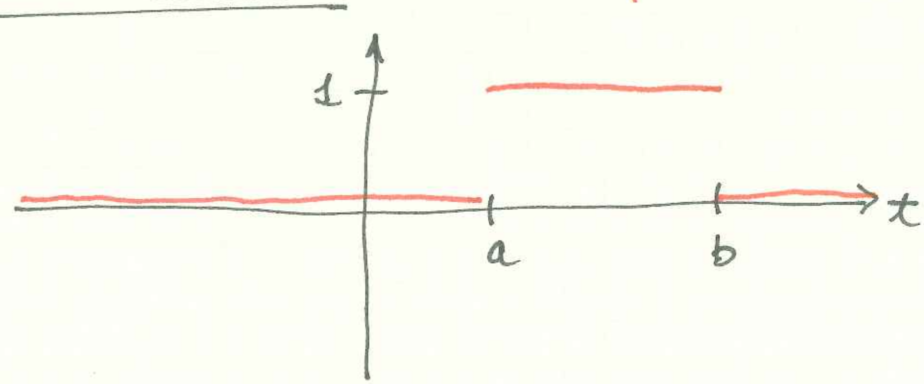
$$\mathcal{U}(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$$

$$\mathcal{U}(\ominus) = 0, \quad \mathcal{U}(+) = 1$$

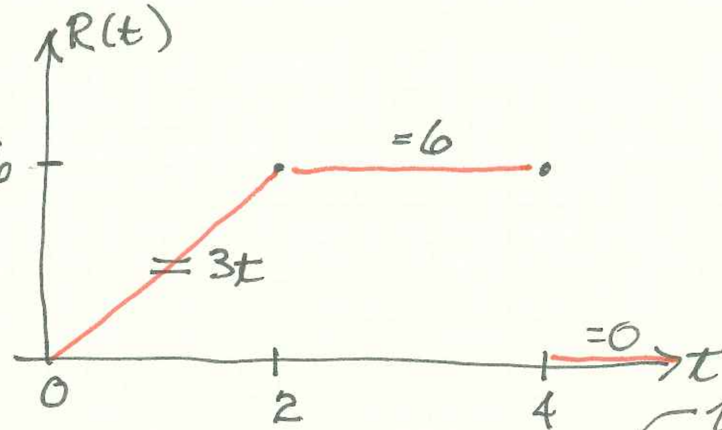
$\mathcal{U}(t-a)$



$\mathcal{U}(t-a) - \mathcal{U}(t-b)$ ⇒ Bump between $a \leq t \leq b$



Ex: Write ramp func. using $\mathcal{U}$:



$\mathcal{U}(t-0)$

Bump

$$R(t) = \begin{cases} 3t, & 0 \le t \le 2 \\ 6, & 2 \le t \le 4 \\ 0, & 4 \le t \end{cases} \qquad \begin{matrix} 1 - \mathcal{U}(t-2) \\ \mathcal{U}(t-2) - \mathcal{U}(t-4) \\ \mathcal{U}(t-4) \end{matrix}$$

$$R(t) = 3t\left[1 - \mathcal{U}(t-2)\right] + 6\left[\mathcal{U}(t-2) - \mathcal{U}(t-4)\right] + 0\left[\mathcal{U}(t-4)\right]$$

$$\boxed{R(t) = 3t + (6 - 3t)\,\mathcal{U}(t-2) - 6\,\mathcal{U}(t-4)}$$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = \int_0^{\infty} f(t-a)\mathcal{U}(t-a)e^{-st}\,dt$$

$$= \int_{t=a}^{\infty} f(\underbrace{t-a}_{u})\, e^{-st}\,dt$$

$$[\![\; u = t-a \Rightarrow t=a \Rightarrow u=0$$

$$t=\infty \Rightarrow u=\infty$$

$$t = u+a \;,\; dt = du \;]\!]$$

$$= \int_{u=0}^{\infty} f(u)\underbrace{e^{-s(u+a)}}_{e^{-su}e^{-sa}}\,du$$

$$= e^{-sa}\int_{\overset{}{\underset{t}{\cancel{u}}}=0}^{\infty} f(\overset{t}{\cancel{u}})\,e^{-s\overset{t}{\cancel{u}}}\,d\overset{t}{\cancel{u}} = e^{-sa}\mathcal{L}\{f\}$$

Ex: $\mathcal{L}\{R(t)\}$ using $\mathcal{L}\{f(t-a)\,\mathcal{U}(t-a)\} = e^{-as}F(s)$

$$R(t) = 3t\left[1 - \mathcal{U}(t-2)\right] + 6\left[\mathcal{U}(t-2) - \mathcal{U}(t-4)\right]$$

$$= \overset{(1)}{3t} + \overset{(2)}{(-3t+6)\,\mathcal{U}(t-2)} - \overset{(3)}{6\,\mathcal{U}(t-4)}$$

$\mathcal{L}\{R\}$ term-by-term:

- $\mathcal{L}\{3t\} = \boxed{\dfrac{3}{s^2}} = 3\mathcal{L}\{t\}$ (1)

- $\mathcal{L}\{\underbrace{(6-3t)}\,\mathcal{U}(t-\overset{a}{2})\} = \boxed{e^{-2s}\underbrace{\left(-3\frac{1}{s^2}\right)}_{F(s)}}$ (2)

$$f(\underbrace{t-2}_{u}) = 6 - 3t \Rightarrow f(u) = 6 - 3(u+2)$$

$$= 6 - 3u - 6$$

$$= -3u$$

$[\![\, u = t-2 \Rightarrow t = u+2 \,]\!]$

$$F(s) = \mathcal{L}\{f(u)\} = \mathcal{L}\{-3\cancel{u}\,t\} = \underbrace{-3\frac{1}{s^2}}_{F(s)}$$

- $\mathcal{L}\{-6\,\mathcal{U}(t-4)\} = -6\,\mathcal{L}\{\underbrace{1}_{f(t-4)}\,\mathcal{U}(t-\overset{a}{4})\}$

$$\Rightarrow F(s) = \mathcal{L}\{1\} = 1/s$$

$$\therefore \mathcal{L}\{-6\,\mathcal{U}(t-4)\} = \boxed{-6e^{-4s}\frac{1}{s}}$$ (3)

Paul Davis **MA2051B13**

WeBWorK assignment number PD2051Laplace is due : 12/16/2013 at 10:00pm EST.

The link

http://users.wpi.edu/ pwdavis/Courses/MA2051B13/MA2051B13syllabus.htm

leads to the syllabus for the course. It contains the homework and test schedule, grading policy, and other information.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $sin(3 * pi/2)$instead of -1, $e \wedge (ln(2))$ instead of 2, $(2+tan(3)) * (4-sin(5)) \wedge 6 - 7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

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1. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{8e^{-6s}}{s^2+64}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{8e^{-6s}}{s^2+64}\right\} =$ ______

Answer(s) submitted:

-

(incorrect)

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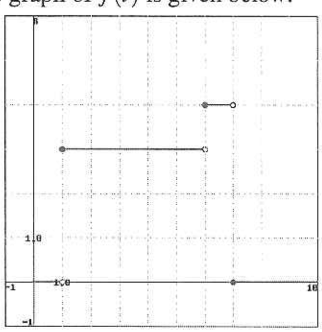
2. (1 pt) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

(1) Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$f(t) =$ ______

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ ______

Answer(s) submitted:

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(incorrect)

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3. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{5s-12}{s^2-6s+25}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{5s-12}{s^2-6s+25}\right\} =$ ______

Answer(s) submitted:

-

(incorrect)

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4. (1 pt) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^{2t}\cos(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{e^{2t}\cos(6t)\} =$ ______

Answer(s) submitted:

-

(incorrect)

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5. (1 pt) Consider the initial value problem

$$y' + 5y = \begin{cases} 0 & \text{if } 0 \leq t < 3 \\ 11 & \text{if } 3 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty, \end{cases} \qquad y(0) = 6.$$

(0)> u write RHS w/step

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

______ = ______

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ______

Ex: Find transform of sol'n of IVP:

$$y' - 0.015y = -\begin{cases} 0.209, & 0 \le t \le 4 \\ 0, & 4 < t \end{cases}$$

$y(0) = 4$

- RHS in terms of $\mathcal{U}$

$$E(t) = \begin{cases} 0.209, & 0 \le t \le 4 \\ 0, & 4 < t \end{cases} \qquad \begin{matrix} \underline{\text{Bump}} \\ 1-\mathcal{U}(t-4) \\ \mathcal{U}(t-4) \end{matrix}$$

$$= 0.209[1-\mathcal{U}(t-4)] + 0 \cdot \mathcal{U}(t-4)$$

- $\mathcal{L}\{RHS\} = \mathcal{L}\{-E(t)\} \underset{a}{=} -0.209\mathcal{L}\{1-\mathcal{U}(t-4)\}$

$$\overset{1}{=} -0.209\left[\frac{1}{s} - \mathcal{L}\{\underset{\text{in}}{1} \cdot \mathcal{U}(t-4)\}\right]$$

② $\to f(t-4) = 1 \Rightarrow f(t) = 1 \Rightarrow F(s) = \frac{1}{s}$

$$= -0.209\left[\frac{1}{s} - \frac{e^{-4s}}{s}\right] = \frac{0.209(e^{-4s}-1)}{s}$$

- $\mathcal{L}\{LHS\} = sY(s) - \underbrace{y(0)}_{=4} - 0.015Y$

$$= Y(s)(s-0.015) - 4$$

Solve for Y: $\boxed{Y = \frac{4}{s-0.015} + \frac{0.209(e^{-4s}-1)}{s(s-0.015)}}$

#22-5 #23

Ex: Solve $y' - 0.015y = -\begin{cases} 0.209, & 0 \le t < 4 \\ 0, & 4 \le t \end{cases}$.

$y(0) = 4$

- $\underbrace{Y(s) = \mathcal{L}\{y(t)\}}$ $= {}^{①}\,\dfrac{4}{s - 0.015} - {}^{②}\,\dfrac{0.209}{s(s-0.015)}$

$$③ \; + \underbrace{\frac{\overset{F(s)}{0.209}}{s(s-0.015)}}_{f(t-4) = g(t)} e^{-\overset{=a}{4}s}$$

- $\underbrace{\text{Key ideas to invert}}$

Table ① $= 4\,\dfrac{1}{s - \underset{a}{0.015}} \Rightarrow \cancel{\mathcal{L}\{e^{4t}\}}\; 4\,\mathcal{L}\{e^{0.015t}\}$

Partial Frac's ② $= 0.209\left[\dfrac{A}{s} + \dfrac{B}{s - 0.015}\right]$

$= 0.209\left[A\,\mathcal{L}\{1\} + B\,\mathcal{L}\{e^{0.015t}\}\right] \Rightarrow \cancel{f}\,g(t)$

$\mathcal{L}\{f(t-a)\, u(t-a)\} = e^{-as}F(s)$ ③

$\cancel{f(t-4)} =$

$g(t) = 0.209\left[A + Be^{0.015t}\right]$

$\cancel{\Rightarrow f(t-4)}$

$\left.\begin{array}{l} u = t - 4 \\ t = u + 4 \end{array}\right\} \Rightarrow f(u) = 0.209\left[A + Be^{0.015(u+4)}\right]$

$\Rightarrow \cancel{F(s) = \mathcal{L}\{f(u)\}} = \ldots$

$f(u) = \mathcal{L}^{-1}\{F(s)\}$