MA 2051 - #20 | Th / 5 Dec 13

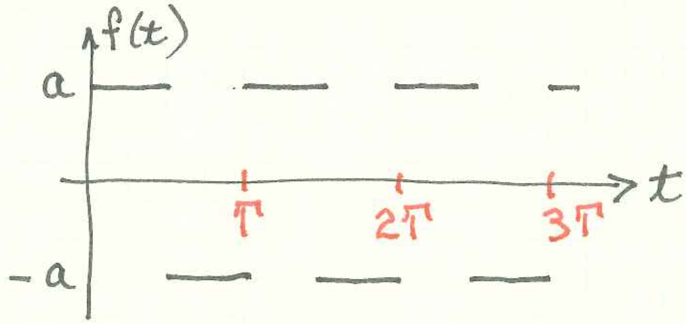
Quiz Fri - 10.1 - 10.2

More $\mathcal{L}$ properties

period

Periodic func: $f(t+T) = f(t)$

$$\Rightarrow \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-st}} \int_0^T e^{-st} f(t)\,dt$$



Square Wave

Period T

Ex. 13-15, p. 535-539

Read on your own.

Derivative of $\boxed{\text{trans}}$: $F(s) = \mathcal{L}\{f(t)\}$

1st: $\dfrac{dF}{ds} = \mathcal{L}\{-t f(t)\}$

nth: $\dfrac{d^{(n)}F}{ds^{(n)}} = \mathcal{L}\{(-t)^n f(t)\}$

Ex: $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-a)^2}\right\}$

Notice: $\dfrac{1}{(s-a)^2} = (s-a)^{-2} = -\dfrac{d}{ds}(s-a)^{-1}$

$= -F'(s)$, $F(s) = \mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$

$\therefore F(s) = \dfrac{1}{s-a}$, $f(t) = e^{at} \Rightarrow$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2}\right\} = \mathcal{L}^{-1}\{-F'(s)\}$$

$$= -\mathcal{L}^{-1}\{F'(s)\}$$

$$= -(-t f(t)) = \boxed{t e^{at}}$$

Trans. of Integral: $F(s) = \mathcal{L}\{f(t)\}$

$$\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = \frac{1}{s}F(s)$$

Ex: $\mathcal{L}^{-1}\left\{\frac{8}{s(s-4)}\right\} = 8\,\mathcal{L}^{-1}\left\{\frac{1}{s(s-4)}\right\}$

Scribble

$$\frac{1}{s(s-4)} = \frac{1}{s}F(s)\,,\quad F(s) = \mathcal{L}\{\cancel{}\,e^{4t}\} = \frac{1}{s-4}$$

$$F(s) = \frac{1}{s-4},\quad f(t) = e^{4t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{s-4}\right\} = \int_0^t e^{4r}\,dr = \frac{1}{4}e^{4r}\Big|_{r=0}^{r=t}$$

$$= \underbrace{\frac{1}{4}(e^{4t}-1)}$$

$$\therefore\ \mathcal{L}^{-1}\left\{\frac{8}{s(s-4)}\right\} = 8\times\ \ = 2e^{4t}-2$$

Check: $\mathcal{L}\{2e^{4t}-2\} = 2\mathcal{L}\{e^{4t}\} - 2\mathcal{L}\{1\}$

$$= 2\frac{1}{s-4} - 2\frac{1}{s} = 2\,\frac{s-(s-4)}{s(s-4)} = \frac{8}{s(s-4)}\ ✓$$

Integral of Trans: $F(s) = \mathcal{L}\{f(t)\}$

$$\mathcal{L}\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty F(r)\,dr$$

We'll use less often – tricky at $t=0$:

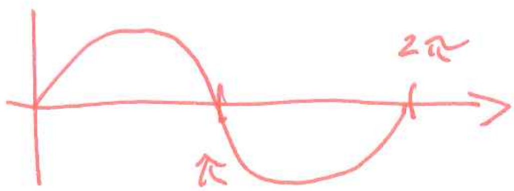
- $\dfrac{\sin t}{t} \to 1$ as $t \to 0$ so

$$\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \int_0^\infty \frac{\sin t}{t} e^{-st}\,dt \text{ exists}$$

- $\dfrac{\cos t}{t} \to \infty$ as $t \to 0$ so

$$\mathcal{L}\left\{\frac{\cos t}{t}\right\} = \int_{\underline{\underline{0}}}^\infty \frac{\cos t}{t} e^{-st}\,dt$$

problematic



EXERCISE GUIDE (Continued)

To gain experience ...	Try exercises
Verifying existence of transforms	16–17
Solving initial-value problems	20–21
Deriving transform properties from the definition of Laplace transform	23–27
Analyzing and interpreting behavior of solutions	20

In exercises 1–5, use the indicated transform property to evaluate the Laplace transform of the given function. If the transform can be evaluated by a second method, do so to verify your answer. If a transform does not exist, explain why. When possible, compare your result with that of DELAB.

1. Use

$$\mathcal{L}\{f\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt,$$

where f has period T, to find the Laplace transform of

(a) $\sin \pi t$ $\pi T = 2\pi \Rightarrow T =$

(b) The square wave of period 4, which is defined for $0 \le t \le 4$ by

$$g(t) = \begin{cases} 1, & 0 \le t \le 2, \\ -1, & 2 < t \le 4. \end{cases}$$

2. Use $\mathcal{L}\{-tf(t)\} = F'(s)$ to find the Laplace transform of

(a) t^2 with $f(t) = -t$ $F(s) = \mathcal{L}\{-t\}$

(b) t^3 with $f(t) = -t^2$

(c) $t \cos 2t$

3. Use $\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s)$ with an appropriate choice of f to find the Laplace transform of

(a) $t^2 e^{at}$

(b) $2t^3 \cos \pi t$

(c) $5t^2 \sinh at$

4. Use $\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = F(s)/s$ to find the Laplace transform of

(a) $\dfrac{\sin \pi t}{\pi}$ with $f(t) = \cos \pi t$

(b) $\dfrac{e^{at}}{a}$ with $f(t) = e^{at}$

(c) $\dfrac{t^5}{5}$ with $f(t) = t^4$

5. Use $\mathcal{L}\{f(t)/t\} = \int_s^\infty F(s)\,ds$ with an appropriate choice of f to find the Laplace transform of

(a) $t^{-1} \sinh 2t$

(b) $t^{-1} \sin 2t$

(c) $\dfrac{1-\cos t}{t}$

In exercises 6–8, use the indicated transform property (and other properties as needed) to find the inverse of the given transform. When possible, compare your result with that of DELAB.

6. Use $\mathcal{L}\{-tf(t)\} = F'(s)$ to invert the Laplace transforms

(a) $\dfrac{4}{(s-1)^2}$

(b) $\dfrac{2s}{(s^2+16)^2}$

(c) $\dfrac{3s}{(1+s^2)^3}$

7. Use $\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = F(s)/s$ to invert the Laplace transforms

(a) $\dfrac{1}{s(s-1)}$

(b) $\dfrac{6(s-1)}{s^2(s+2)}$

8. Use $\mathcal{L}\{f(t)/t\} = \int_s^\infty F(s)\,ds$ with an appropriate choice of f to invert the Laplace transforms

(a) $\dfrac{2}{s-2}$

(b) $\dfrac{4s}{(s^2+a^2)^2}$

9. Sketch a graph of the nonsymmetric square wave of period T

$$f(t) = \begin{cases} a, & 0 \le t < b, \\ -a, & b \le t < T, \end{cases}$$

and find its Laplace transform.

10. Sketch a grap

f

and find its L

In exercises 11–1 constants. When DELAB.

11. $\mathcal{L}\{te^{at}\}$

12. $\mathcal{L}\{t^n e^{at}\}$

13. $\mathcal{L}\{e^{at} \sin \omega t\}$

14. $\mathcal{L}\{t \sin \omega t\}$

15. $\mathcal{L}\{\sin \omega t - a$

16. Example 20 t

Use l'Hôpita this limit.

17. Verify the cla

is of exponen

18. Supply the d

omitted from

19. Work throug problem forc which were

20. Example 15

$x'' + 4$

where f is a

It concludes whether this this system's quency?

Paul Davis **MA2051B13**

WeBWorK assignment number PD2051Laplace is due : 12/16/2013 at 10:00pm EST.

The link

http://users.wpi.edu/ pwdavis/Courses/MA2051B13/MA2051B13syllabus.htm

leads to the syllabus for the course. It contains the homework and test schedule, grading policy, and other information.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2\wedge 3$ instead of 8, $sin(3*pi/2)$ instead of -1, $e\wedge(ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5))\wedge 6-7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

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1. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \frac{8e^{-6s}}{s^2+64}$. $= \frac{8}{s^2+64}\cdot e^{-6s}$

$f(t) = \mathcal{L}^{-1}\left\{\frac{8e^{-6s}}{s^2+64}\right\} =$ ________________

Answer(s) submitted:

Exponential in s, not t!

•

(incorrect)

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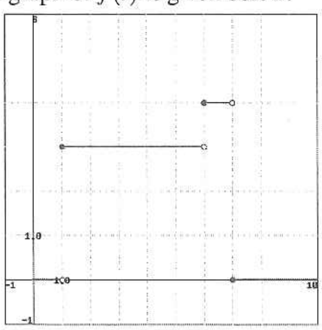
2. (1 pt) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

(1) Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$f(t) =$ ________________

(2) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ ________________

Answer(s) submitted:

•

•

(incorrect)

Partial Frac's or Complete Squ.?

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3. (1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \frac{5s-12}{s^2-6s+25}$.

$f(t) = \mathcal{L}^{-1}\left\{\frac{5s-12}{s^2-6s+25}\right\} =$ ________________

Answer(s) submitted:

•

(incorrect)

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4. (1 pt) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^{2t}\cos(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{e^{2t}\cos(6t)\} =$ ________________

Answer(s) submitted:

•

(incorrect)

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5. (1 pt) Consider the initial value problem

$$y' + 5y = \begin{cases} 0 & \text{if } 0 \leq t < 3 \\ 11 & \text{if } 3 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty, \end{cases} \qquad y(0) = 6.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

____________ = ____________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ________________

○ = you can solve now

Answer(s) submitted:

-
-
-
-

(incorrect)

6. (1 pt)

(1) Set up an integral for finding the Laplace transform of $f(t) = 4$.

$$F(s) = \mathcal{L}\{f(t)\} = \int_A^B \_\_\_\_\_\_$$

where $A =$ ___ and $B =$ ___.

(2) Find the antiderivative (with constant term 0) corresponding to the previous part.

______

(3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ ______

(4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

______

Answer(s) submitted:

-
-
-
-
-
-

(incorrect)

7. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 9t^5 + 9t + 4$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\left\{9t^5 + 9t + 4\right\} =$ ______

(2) For what values of s does the Laplace transform exist?

______

Answer(s) submitted:

-
-

(incorrect)

8. (1 pt)

(1) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = 7 + \sin(6t)$, defined on the interval $t \geq 0$.

$F(s) = \mathcal{L}\{7 + \sin(6t)\} =$ ______

(2) For what values of s does the Laplace transform exist?

______

Answer(s) submitted:

-
-

(incorrect)

9. (1 pt) Consider the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \leq t < 7 \\ 0 & \text{if } 7 \leq t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

______ = ______

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ______

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ______

Answer(s) submitted:

-
-
-
-

(incorrect)

10. (1 pt) Consider the initial value problem

$$y' + 4y = 64t, \qquad y(0) = 4.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

______ = ______

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________

Answer(s) submitted:

-
-
-
-

(incorrect)

#20-5

WW #1

$$\frac{8e^{-6s}}{s^2+64} = \underbrace{\frac{8}{s^2+64}}_{= F(s)} \cdot \underbrace{e^{-6s}}_{= G(s)}$$

$\Downarrow$

? $f(t)$?

Wait until

δ

States as of Th/5 Dec

Laplace transform pairs

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
* 1	$\dfrac{1}{s}$
$t^n, n = 1, 2, \ldots$	$\dfrac{n!}{s^{n+1}}$
* e^{at}	$\dfrac{1}{s-a}, s > a$
$\sin at$	$\dfrac{a}{s^2+a^2}, s > 0$
$\cos at$	$\dfrac{s}{s^2+a^2}, s > 0$
$\sinh at$	$\dfrac{a}{s^2-a^2}, s > a$
$\cosh at$	$\dfrac{s}{s^2-a^2}, s > a$
$\delta(t-a)$	$e^{-as}, a > 0$

* Developed from defn. of $\mathcal{L}$ in class on Mon.

Others - see text

δ - still to come

Introduced **Properties of Laplace Transforms**

$$F(s) = \mathcal{L}f(t), \quad G(s) = \mathcal{L}g(t)$$

Mon Transform of a derivative:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$
$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

Tu Convolution theorem:

$$\mathcal{L}\left\{\int_0^t f(r)g(t-r)\,dr\right\} = F(s)G(s)$$

Tu Shifted transform:

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

Th Transform of a periodic function of period T:

$$\mathcal{L}\{f\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$$

Th Derivative of a transform:

$$\mathcal{L}\{-tf(t)\} = F'(s)$$
$$\mathcal{L}\{(-t)^n f(t)\} = F^{(n)}(s), \quad n = 1, 2, \ldots$$

Th Transform of an integral:

$$\mathcal{L}\left\{\int_0^t f(r)\,dr\right\} = \frac{F(s)}{s}$$

Th Integral of a transform:

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(r)\,dr$$

Transform of a shifted function using unit step function $\mathcal{U}$

$$\mathcal{L}\{f(t-a)\,\mathcal{U}(t-a)\} = e^{-as}F(s), \quad a > 0$$