OH Thurs: ~~3-5 p.m.~~ → 5-6 p.m.

Fri: Quiz - chap. 6
- CE / UDC / Gen'l Sol'n
- Oscillating systems

(Sat: WW due)

Mon: Review for Test 2 (WW due)

Tues: → Test 2

Help for Quiz Fri/22 Nov

noon - 1:00 p.m. SH 304

tutoring 1:00 - 2:00 SH 002A

Help for Test

Sun 24 Nov: 4-6 pm SH 002A tutoring
5-6 " EPC (Daniels) MASH

Mon 25 Nov: 2-3 pm SL 115 - class
Noon-1 pm SH002A

Tues 26 Nov: Conf. @ 9, 10, noon, 1:00
(Crash any one!)

p. 264-268

Charac. eqn - complex roots

$$r = \alpha \pm i\beta$$

$\Rightarrow$ LI sol'ns are $e^{\alpha t}\sin\beta t,\ e^{\alpha t}\cos\beta t$

How from $e^{(\alpha \pm i\beta)t}$?

1. $e^{(\alpha \pm i\beta)t} = e^{\alpha t} e^{\pm i\beta t}$

2. $e^{\pm i\beta t} = \cos\beta t \pm i\sin\beta t$ (Euler)

3. $\dfrac{e^{i\beta t} + e^{-i\beta t}}{2} = \cancel{2}\cos\beta t$

$$\frac{e^{i\beta t} - e^{-i\beta t}}{2i} = \cancel{2i}\sin\beta t$$

(Superposition)

Physically

1. Growth / decay in real part α

2. Oscillation period/frequency in imaginary part β

Ex: Spring-mass in class

- Mass of 100 gm stretched spring ~4cm

$$F_g \text{ on } 100 \text{ gm} = mg = (0.1 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= .98 \text{ N}.$$
$$\text{Extension} = 4 \text{ cm} = 0.04 \text{ m}$$
$$\Rightarrow k = \frac{.98}{.04} = \frac{98}{4} \approx 25 \text{ N/m}$$

- Suspend mass of 400 gm

$\underbrace{\text{Ignore damping}}_{\text{No decay}}$. Period = ?

Natural frequency = ?

Ans: Model: $mx'' + kx = 0$

$$\begin{matrix} m = .4 \text{ kg} \\ k \cong 25 \text{ N/m} \end{matrix} \Rightarrow .4x'' + 25x = 0$$

CE: $x = e^{rt} \Rightarrow .4r^2\cancel{e^{rt}} + 25\cancel{e^{rt}} = 0$

$$r^2 = -\frac{25}{.4} \Rightarrow r = \pm i\sqrt{\frac{25}{.4}} \approx 0 \pm 8i$$

α β

$$x_g = C_1 \sin 8t + C_2 \cos 8t$$

Repeats when $8t = 2\pi \Rightarrow \boxed{\text{period } t = \frac{2\pi}{8} = \frac{\pi}{4}}$

Nat'l freq. $\omega_n = 8$ rad/s {Cyclic freq. $\frac{8}{2\pi}$ Hz

Ex: Same spring, $m = 100$ gm

Ex: Same spring, $m = 400$ gm.

Start from rest, up 4 cm

Ex: Same spring, m = 400 gm
Start from equil. w/push down of 2 cm/s

<u>Ex:</u> Same spring, $m = 400$ gm, damping p

Oscillate?

<u>Model:</u> $.4x'' + px' + \overset{25}{\cancel{32}}x = 0$

$x = e^{rt} \Rightarrow \ldots$

<u>CE:</u> $.4r^2 + pr + \overset{25}{\cancel{32}} = 0$

$$r = \frac{-p \pm \sqrt{p^2 - 4(.4)(\cancel{32})(25)}}{2(.4)}$$

$$r = \underbrace{\frac{-p}{.8}}_{\alpha} \underbrace{\pm \frac{1}{.8}\sqrt{p^2 - \underset{40.0}{\cancel{51.2}}}}_{i\beta}$$

Need <u>complex</u> r to oscillate

" $\boxed{p^2 - \overset{40.0}{\cancel{51.2}} < 0}$ " "

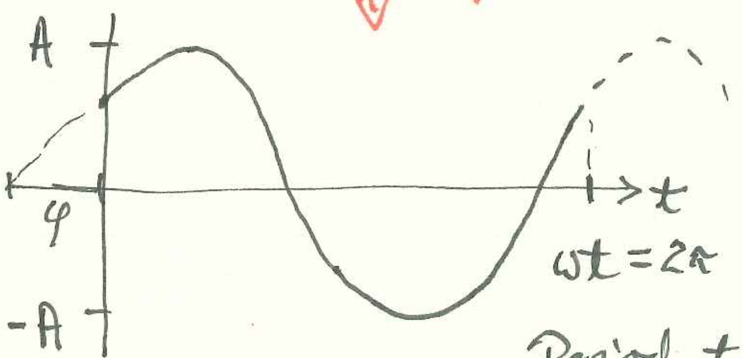
$\therefore$ Oscillates if $p^2 < 40$ or $p < \sqrt{40} = 2\sqrt{10}$

Alternate form of Gen'l Sol'n – oscillations

$$\underbrace{A}_{\text{arbitrary}} \sin(\omega t + \underbrace{\varphi}_{\text{arbitrary}}) \quad \underline{\underline{vs.}} \quad \underbrace{C_1}_{\text{arb.}} \sin\omega t + \underbrace{C_2}_{\text{arb.}} \cos\omega t$$

arbitrary ⇓

arb. ⇓ ?Sinusoidal ?



Period $t = \frac{2\pi}{\omega}$

p.279 -280

Phase angle (φ) – amplitude (A) form of GS

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\Rightarrow A\sin(\omega t + \varphi)$$

$$= \underbrace{(A\cos\varphi)}_{C_1} \sin\omega t + \underbrace{(A \sin\varphi)}_{C_2} \cos\omega t$$

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EXERCISE GUIDE (Continued)

To gain experience . . .	Try exercises
Analyzing and interpreting damped response	4, 5(a), 6–11, 24, 25, 31, 32
Analyzing and interpreting periodic response	2–3, 5(b–c), 8, 12, 15–16, 18, 23, 30, 32(a)
With analogies among various physical systems	22, 26–29

1. Derive the phase angle-amplitude form of the solution of the initial-value problem modeling each of the undamped systems described here. Assume a general solution of the form $x_g(t) = A\sin(\omega t + \phi)$, and determine the constants A and ϕ directly from the initial conditions.
   (a) A mass of 600 g is suspended from a spring that extended 4 cm when a mass of 100 g was hung from it. The 600-g mass is set in motion by releasing it from rest 6 cm above its equilibrium position. $x'(0) = 0$ $x(0) = +.06$
   (b) The spring of part (a) is now set horizontally, attached to a fixed anchor as in figure 6.8. A 2-kg air puck is attached to the free end of the spring. (An air puck rides on a cushion of air so that it can slide with essentially no friction.) The puck is pushed 7 cm toward the fixed anchor and released.

2. Consider the system described in part (a) of exercise 1.
   (a) Suppose the mass is released from rest from an arbitrary position x_i. How does the amplitude of the resulting motion depend upon x_i? How does the phase angle depend upon x_i? Do these relationships seem physically reasonable?
   (b) Suppose the mass is started in motion from its equilibrium position with a velocity v_i. How does the amplitude of the resulting motion depend upon v_i? How does the phase angle depend upon v_i? Do these relationships seem physically reasonable?

3. The air-puck system described in part (b) of exercise 1 is set in motion from its equilibrium position with an unknown initial velocity.
   (a) The period of its oscillations is observed to be 6.39 s. Can you determine the initial velocity of the puck?
   (b) The maximum displacement of the puck is observed to be 14.1 cm. Can you determine its initial velocity?

4. The system described in part (b) of exercise 1 is not truly undamped, although that exercise suggested that you could model it that way. In fact, during a period of 4 min, the maximum displacement of the oscillations of the puck was observed to decrease from 10 cm to 5.6 cm. What is the damping coefficient?

5. Example 27 shows that the general solution of the undamped model equation

$$mx''(t) + kx(t) = 0$$

contains no exponential factors to cause either growth or decay.
   (a) Explain why no solution of the initial-value problem

$$mx''(t) + kx(t) = 0,$$
$$x(0) = x_i, \quad x'(0) = v_i,$$

   will exhibit growth or decay, regardless of the choice of initial values x_i, v_i.
   (b) Show that all solutions of this initial-value problem oscillate with period $2\pi\sqrt{m/k}$.
   (c) What happens when $x_i = v_i = 0$?

6. A 12-kg mass is suspended from a spring. Air resistance and internal friction in the spring resist the motion of the mass with a force whose magnitude is 0.02 times the velocity of the mass. Find the range of spring-constant values for which the mass will *not* oscillate.

7. A mass of 4 kg is suspended from a spring in a liquid that offers a resistance force whose magnitude is eight times the velocity of the mass.
   (a) Does decreasing the magnitude of the spring constant cause the mass to oscillate slower or faster? $\sqrt{(-)}$
   (b) What range of values of the spring constant will prevent the mass from oscillating? $(\ \ ) \geq 0$

8. You are given the characteristic roots r_1, r_2 of a model of a spring-mass system. Give a step-by-step recipe (an algorithm) for determining from r_1, r_2 whether the system is damped or undamped. If it is damped, determine whether it is underdamped, critically damped, or overdamped.

Ex: $A \sin(\omega t + \varphi) = x_g$ $\qquad (\omega \leftarrow \beta)$

$x(0) = .06, \quad x'(0) = 0$

$x(0) = A \sin\varphi = .06$

$x'(0) = A\omega\cos(0 + \varphi) = 0$

$\cos\varphi = 0 \Rightarrow \varphi = \pi/2$