MA 2051 - #1 | 29 Oct 13

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__All__ in syllabus: www.wpi.edu/~pwdavis/Courses/MA2051B13/MA2051B13syllabus.~~pdf~~ htm

Course Goals - Tests - Quizzes - HW ...
Grades - Academic Honesty - Accomodations ...

~~Memorize~~ Think Understand

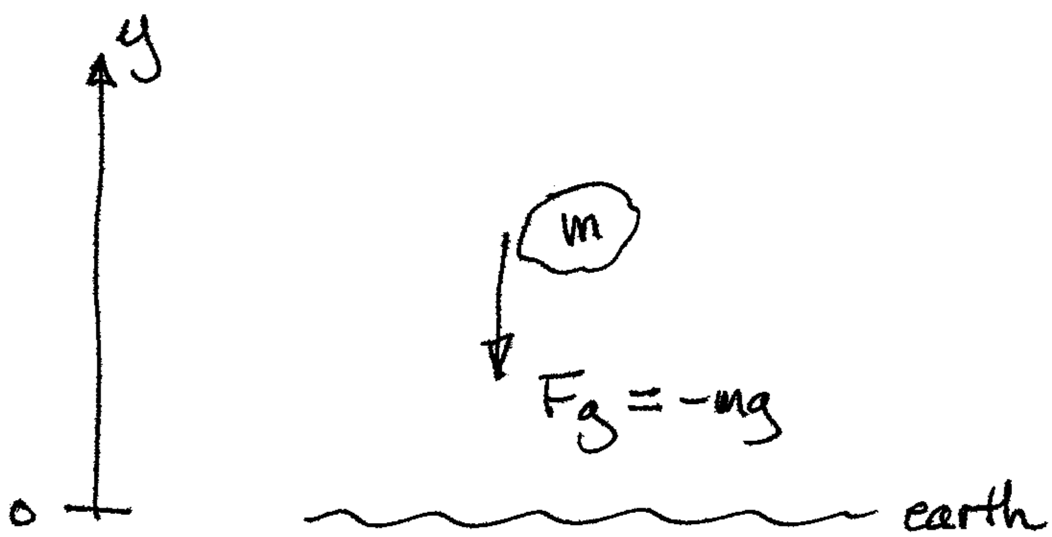
Differential Equation is ...

- Mathematical Model
- " tool to aid understanding

⇒ Goal: understand $v(t)$

**Example:** model of object falling due to gravity

- Basic physical law: $F_{total} = ma$
- Experimental "fact": $F_{gravity} = mg$, $g = 9.8\,m/s^2$
- Ignore air resistance




Newton: $\underbrace{F_{total} = F_g = ma \quad \text{Fact: } F_g = -mg}_{\cancel{m}a = -\cancel{m}g}$

$$a = \frac{dv}{dt} \Rightarrow \boxed{\frac{dv}{dt} = -g}$$

$$\boxed{\frac{dv}{dt} = -9.8, \; v(0) = 0}$$

$$\Rightarrow v' < 0 \Rightarrow v \text{ dec'g} \Rightarrow v \xrightarrow{?} -\infty$$

Model w/out air resistance

$$\frac{dv}{dt} = -g = -9.8, \quad v(0) = 0$$

Ordinary Differential Eqn. (ODE)

$v$ De ~~Ind~~ependent Variable = unknown function:

$t$ In~~De~~pendent Variable = variable we "control":

Order of ODE = highest deriv. : 1

Solution of ODE = func. that makes "=" true

$v(t) = \frac{1}{2}\cancel{g}t^2$ Check: $(\frac{1}{2}gt^2)' \overset{?}{=} -g$ No

$= -gt + C$ Check: $(-gt)' \overset{?}{=} -g$ ✓

⇒ Initial Value Problem (IVP)

= ODE ⊕ value of unknown func. at some time (e.g., $t=0$)

Solution of IVP = func. that solves ODE ⊕ ~~right~~ correct initial value

Solve for C: $v(0) = -g \cdot 0 + C = C$

$$\underset{0}{\overset{\shortparallel}{v(0)}} \Rightarrow C = 0$$

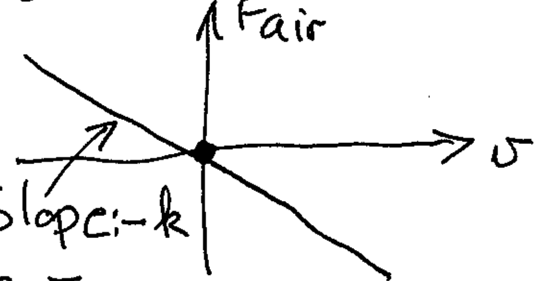
$$\boxed{\text{Sol'n of IVP: } v(t) = -gt}$$

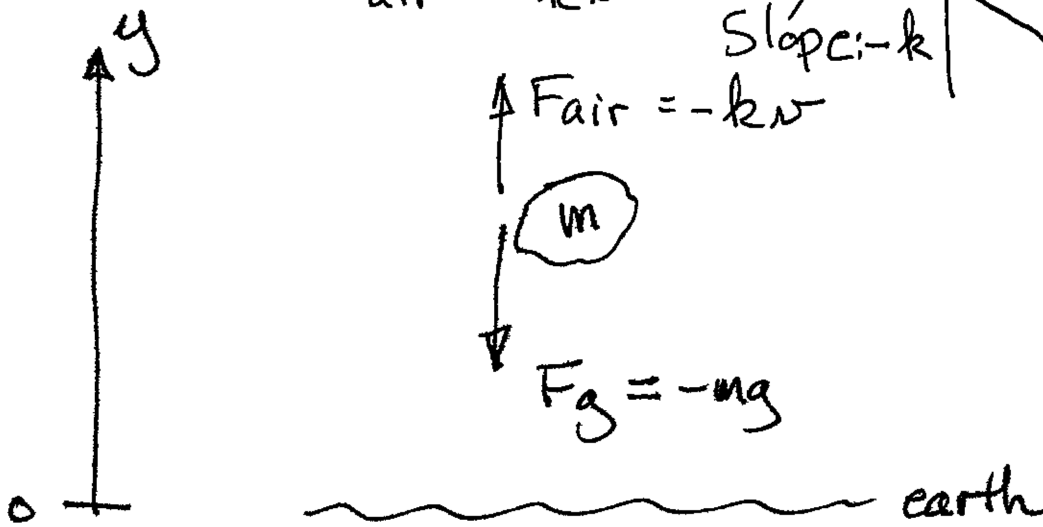
Check: Check DE ✓, Check $v(0) = 0$ ✓

Example: model of object falling due to gravity

- Basic physical law: $F_{total} = ma$
- Experimental "fact": $F_{gravity} = mg$, $g = 9.8\,m/s^2$
- ~~Ignore~~ Include air resistance

$F_{air} = -kv$




Newton: $\underbrace{F_{total} = ma \qquad \text{Facts: } F_g = -mg}$, $F_{air} = -kv$

$$\cancel{m}a = -\cancel{m}g - \frac{kv}{m}$$

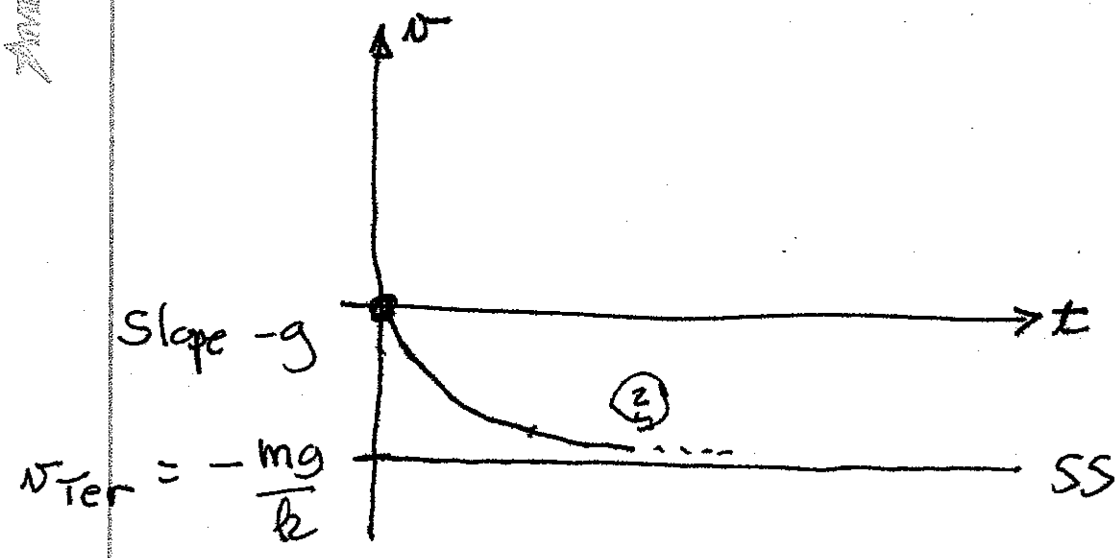
$$\frac{dv}{dt} = -g - \left(\frac{k}{m}\right)v$$

Analyze model = get info from ODE

Ex: falling w/air resistance

ODE: $$\frac{dv}{dt} = -\cancel{m}g - \frac{k}{m}v$$

I.C.: $v(0) = 0$




$\frac{dv}{dt} = 0$ when not falling faster - at $v_{Terminal}$

$$0 = -g - \frac{k}{m} v_{Ter} \Rightarrow \boxed{v_{Ter} = -\frac{mg}{k}}$$

Steady-state sol'n of DE

Look at Ex. 11, p. 16 to solve DE