The Pendulum, a Nonlinear Oscillator

Overview

A pendulum behaves like a simple harmonic oscillator if its swing amplitude is small. For small amplitude, the restoring force F is very nearly proportional to the displacement θ. We say that F is linear in θ. Recall that a simple harmonic oscillator by definition has a restoring force that is proportional to displacement. However, for a pendulum, as amplitude is increased, the restoring force becomes increasingly non-linear in θ (to be precise, F is proportional to $\sin\theta$). As a result, the oscillation frequency varies with amplitude.

You may reasonably ask "Who cares that the pendulum is non-linear"? Let us answer with a hypothetical scenario. Suppose your family business manufactures pendulum-based grandfather clocks, and your market research informs you that your business will thrive if same clocks keep same time. As the staff engineer of your company and a former WPI PH1140 student, you would ensure that all your clock pendulums have precisely the same swing amplitude so that they operate with the same period. In terms of the laptops, cell phones, music pods and GPS units with which you are more familiar, the point is this; these devices have in common internal clocks driven by precision electronic oscillators. None of these devices would work if engineers had not accounted for inherent non-linearity in the oscillators to ensure precise timing.

In this lab, you will demonstrate a pendulum's non-linearity by measuring its frequency dependence on amplitude, and quantify this dependence in terms of ω_{small} , the small amplitude frequency.

Setup

Set your force sensor range to 10N, mount the sensor on the stand, and hang a pendulum from the sensor as shown in the figure.

Procedure

Open Logger Pro template pendulum.cmbl, give the pendulum an initial displacement (amplitude) θ_{max} , measure the displacement, release the pendulum, collect data for several seconds. The force plot is clearly periodic with frequency ω_{force} . Fit the first three cycles of the force plot to a sine function. Read the frequency ω_{force} from the fit parameters.

(W1) Record the initial displacement θ_{max} and the frequency ω_{force} .

(W2) Is ω_{force} the oscillation frequency of the pendulum? Explain.

(W3) Why use only the first three cycles for the fit?

Repeat for different initial displacements ranging from small to large, specifically displacements near 5, 30, 50, 70 and 85 degrees. Be careful with the pendulum bob during large swings! The pendulum precesses (changes its direction), keep an eye on it and catch it if necessary to prevent it from striking the monitor or anything else.

(W4) Record the initial displacements and frequencies.

(R1) Plot the frequency versus the initial displacement $f_{measured}$ (θ_{max}) to show the dependence of frequency on amplitude. For comparison, also plot the frequency f_{theory} (θ_{max}) predicted by Y&F Eq. 13.35. For larger amplitudes, you need to include higher order terms of Eq. 13.35 to ensure f_{theory} (θ_{max}) is computed accurately.

(R2) At what amplitude does the frequency differ from ω_{small} by 10%?

(R3) Why do we use ω_{small} as the reference frequency?

Lab Worksheet for: The Pendulum, a Nonlinear Oscillator

Name: **Section:**

(W1) Record the initial displacement θ_{max} and the frequency ω_{force} .

(W2) Is ω_{force} the oscillation frequency of the pendulum? Explain.

(W3) Why use only the first three cycles for the fit?

(W4) Record other initial displacements and frequencies in the range $0 < \theta_{max} < \pi/2$.