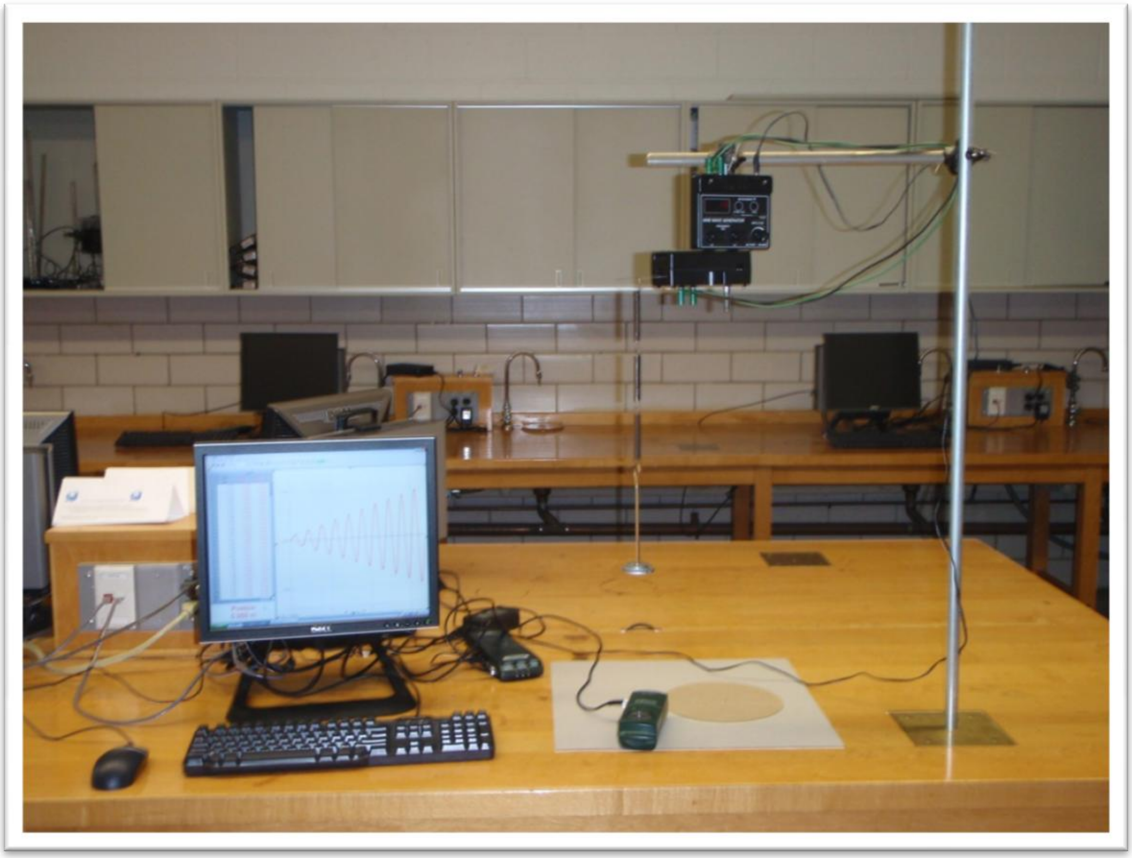
The Forced Mass-Spring Oscillator and Resonance



Overview

In the two previous labs you gained experience with free and damped mass-spring oscillators. In this third and final adventure with the mass-spring system, you will use a vibrator to apply a small, periodic force to the system, very much like the periodic nudge you would give to a child on a swing. If the forcing frequency matches the frequency of the free system, the system will resonate. Its amplitude will grow with each nudge, and continue to grow until limited by inherent damping. If a system such as our mass-spring system has very little damping, the magic of resonance can turn a mouse into a moose. In our setup, the amplitude of the vibrator is quite small, about a millimeter, yet it is able to stimulate amplitudes in the mass-spring system over 50 times greater! But a little bit of damping stifles the resonance dramatically.

Setup

Mount the vibrator to the bench post as shown in the figure, and connect the sine wave generator to the vibrator. Connect 3 springs in series, place a 50g mass on the hanger, attach the hanger to the spring and hang the assembly from the tongue of the vibrator. Center the position sensor directly beneath the hanger. Adjust the height of the hanger so that its bottom is 25 cm above the position sensor. Open Logger Pro template forced_mk.cmbl, stabilize the hanger and zero the position sensor.

Procedure

Measure the free oscillation frequency. Turn off the sine wave generator, induce small vertical oscillations in the mass-spring system (small enough to avoid "clicking" of the vibrator tongue). Collect data and fit a sine curve to the position plot. From the fit parameters, read the frequency ω_0. It should be $f = \omega_0/2\pi = 1.6 \pm 0.04\ Hz$. If necessary, adjust the mass to achieve this frequency.

(W1) Record the frequency. Show the calculations (if any) you made to adjust the mass.

Force oscillations. Set the generator amplitude to minimum, turn on the generator, set the generator frequency to 1.6 Hz, start collection, then set the generator amplitude to maximum. The mass-spring system will begin to oscillate, and the oscillations will quickly grow to a maximum.

(W3) From the position plot, estimate and record the maximum amplitude.

(W4) How many cycles does it take for the system to reach its maximum amplitude?

(W5) Based on the rate of growth of the amplitude, estimate the average power delivered by the vibrator to the mass-spring system.

Measure the resonance quality. Vary the generator frequency, selecting two values above (1.7 and 1.8 Hz) and two values below (1.4 and 1.5 Hz) the resonance frequency. Force the system as before and collect data. Allow the system to oscillate for awhile and observe its behavior carefully.

(W6) Record the maximum amplitudes for each of the frequencies.

(W7) For each of the five frequencies, does the system remain at maximum amplitude?

(W8) Describe how the maximum amplitude varies with time.

(W9) Plot maximum amplitude versus frequency and estimate the resonance Q factor.

Even though our intent is to employ the hanging mass-spring system as a one dimensional vertical oscillator, the system is capable of moving in three dimensions and sustaining three independent oscillation modes simultaneously.

(W10) Aside from the linear vertical oscillations of the mass, what other oscillatory modes appear?

(W11) What changes could you make to the system to avoid the other modes?

Force oscillations with damping. Place 65g mass on the hanger, induce oscillations and obtain the free frequency ω_0. The frequency should be $f = \omega_0/2\pi = 1.5 \pm 0.04\ Hz$. Force the system and obtain the forced frequency ω_f and maximum amplitude A_f. Remove 15g and place the 20cm diameter, 15g damping disk on the hanger, induce oscillations and obtain the damped frequency ω_d and damping factor γ. Force the system to obtain the forced/damped frequency ω_{fd} and maximum amplitude A_{fd}.

(W12) Record ω_0, $\omega_d, \gamma, \omega_f$, ω_{fd}, A_f, and A_{fd}.

(W13) Compare the forced/damped case to the forced/un-damped case. Comment on the differences and the effect of damping on frequencies and amplitudes of the forced system.