PH 1120: Summary Homework 4 Solutions   Term B08

1.(a) The path of the ion is a semicircle curving to the right from the point of entry, and terminating at the point X on the plate.
(b) From the right hand rule, the direction of the B field must be out of the paper, or along **k**, for the particle to curve to the right. Since $R = \frac{mv}{qB}$, one can solve for the magnitude of the magnetic field as $B = \frac{mv}{qR}$. The distance of the point of entry of the particle from X, which is $2R$, is given to be 0.075m. Using this and the other data we can calculate B as

$$B = \frac{mv}{qR} = \frac{\left(1.4\bullet 10^{-25}\right)\left(3\bullet 10^{4}\right)}{\left(1.6\bullet 10^{-19}\right)\left(.0375\right)} = .71\,\text{T}$$

(c) Time = $\frac{\pi R}{v} = \frac{\pi\left(.0375\right)}{3\bullet 10^{4}} = 3.93\bullet 10^{-6}\text{s}$
(d) The heavier Kr-86 ions will hit the plate to the right of X.

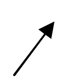
2.(a) The E field will exert a force on the ions in the downward (-**j**) direction. If the magnetic field points into the paper (i.e. along –**k),** it will exert an upward force on the ion and counteract the effect of the E field. Since $E = vB$, the magnitude of the required B field can be found as

$$B = \frac{E}{v} = \frac{3000}{3\bullet 10^{4}} = 0.1\text{T} \qquad \text{(along } -\mathbf{k}\text{, or into the page)}$$

(b) and (c) The magnetic field does not have to be changed in either of these cases, because the charge and mass of the particles do not affect the action of the velocity selector. A velocity selector with a given E and B picks out particles with a particular velocity $\left(= E / B\right)$ irrespective of their charge or mass.

3.(a) $B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\bullet 10^{-7}\right)\left(5\right)}{\left(2\pi\right)\left(.5\right)} = 2\bullet 10^{-6}$ $\vec{B} = 2\bullet 10^{-6}\,\mathbf{j}\,\text{T}$ (direction is vertically up)
(b) $B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\bullet 10^{-7}\right)\left(10\right)}{\left(2\pi\right)\left(.5\right)} = 4\bullet 10^{-6}$ $\vec{B} = 4\bullet 10^{-6}\,\mathbf{j}\,\text{T}$
(c) Add results of (a) and (b) to find that $\vec{B} = 6\bullet 10^{-6}\,\mathbf{j}\,\text{T}$

(d) The field due to the 5A wire points into the second quadrant, and the larger field due to the 10A wire into the first quadrant. The sum is also in the first quadrant, and shown roughly below.



(e) The field vanishes to the left of the 5A wire.
(f) Let the field vanish at a distance x to the left of the 5A wire. The total field at this point is given by

$$\vec{B} = -\frac{\mu_0 I_1}{2\pi x}\mathbf{j} + \frac{\mu_0 I_2}{2\pi(x+d)}\mathbf{j} \ ,$$

where the first and second terms are due to the 5A and 10A wires. Setting the above expression equal to zero and solving for x, with $I_1 = 5, I_2 = 10$ and $d = 1$, gives $x = 1$ m.

(g) The 3A wire will be attracted by the 5A wire and repelled by the 10A wire; both these forces add up to cause a net force on the 3A wire to the left. The total force can be calculated by adding these two forces. Alternatively, and more quickly, it can be calculated from the formula $F = ILB$ by taking $I = 3\text{A}$, $L = .01\text{m}$ and $B = 6{\bullet}10^{-6}$ T (the value found in part (c)). Taking the direction into account, we find that the net force on the 3A wire is $-1.8{\bullet}10^{-7}\,\mathbf{i}$ N.

4. The current in the loop produces a field at its center that points into the page. The current in the wire must flow from left to right if it is to produce a field in the opposite direction at the center of the loop, and hence lead to a vanishing field there. With a clockwise current I_2 in the loop and a left-to-right current I_1 in the wire, the total magnetic field at the center of the loop is given by

$$\vec{B} = -\frac{\mu_0 I_2}{2R}\mathbf{k} + \frac{\mu_0 I_1}{2\pi D}\mathbf{k} \, ,$$

where the first term is due to the loop and the second term to the wire. Setting the above expression equal to zero and solving for I_1 gives $I_1 = \frac{\pi D I_2}{R}$.

5. The number of turns per unit length of this solenoid is $n = \frac{4000}{1.4} = 2857\text{m}^{-1}$. Since $B = \mu_0 n i$ for a solenoid, we can solve for the current as

$$i = \frac{B}{\mu_0 n} = \frac{.15}{(4\pi{\bullet}10^{-7})(2857)} = 41.8\text{A} \, .$$

SOLUTIONS TO OTHER PROBLEMS (not required to be submitted).

28.23 (a) The fields of diagonally opposite wires cancel out at the center, so the net field is 0.
(b) The field is again 0, for the same reason as before.

(c) The fields of diagonally opposite wires are identical. Thus we need just find the field due to the two wires at the bottom and multiply the result by 2. The net field due to the two bottom wires point to the left, so the total field at the center points to the left as well. Following the remarks just made, the magnitude of the net field at the center can be calculated as 4 times the horizontal component of the field due to any one of the wires:

$$B = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45^\circ = 4\left(\frac{\mu_0 I}{2\pi\left(a/\sqrt{2}\right)}\right)\cos 45^\circ = 4 \bullet 10^{-4}\,\mathrm{T}\ ,$$

where $I = 100\mathrm{A}$ and $a = .2\mathrm{m}$ is the side of the square. The direction of the field is to the left, or along –**i.**

28.24. The fields due to the 10A and 8A wires at the center point into the page, while that due to the 20A wire points out of the page. Because all four wires are at the same distance d from the center, their net field can be calculated as

$$B = \frac{\mu_0}{2\pi d}\left[-10 - 8 + 20 + I\right]\ ,$$

where negative signs have been used for currents that give rise to fields going into the page and positive signs for currents giving rise to fields out of the page. For the filed at the center to vanish, the quantity within the parentheses [ ] above must vanish, and this happens when $I = -2$. Thus a current of 2A must flow in the downward direction through wire marked I.

28.28. The force per unit length on each of the wires can be calculated as follows:

Top wire: $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d} - \frac{1}{2d}\right) = \frac{\mu_0 I^2}{4\pi d}$ upwards

Middle wire: $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(-\frac{1}{d} + \frac{1}{d}\right) = 0$ vanishing force

Bottom wire: $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{2d} - \frac{1}{d}\right) = -\frac{\mu_0 I^2}{4\pi d}$ downwards

For each wire, the force is calculated as the sum of the forces due to the other two wires, with upward forces counted as positive and downward ones as negative. The direction of the net force is indicated at the end of the line.

29.2. The flux through the coil in any position is $\Phi = NBA\cos\phi$, where $N = 200$, $B = 6\bullet10^{-5}\,\text{T}$, $A = 12\bullet10^{-4}\,\text{m}^2$ and ϕ is the angle between B and the normal to the loop.

(a) Since $\phi = 0^\circ$ initially, the initial flux is $NBA = 1.44\bullet10^{-5}\,\text{Wb}$

(b) Since $\phi = 90^\circ$ finally, the final flux is 0.

(c) The average induced emf can be determined by dividing the difference of the initial and final fluxes by the time in which that flux change takes place:

$$\text{EMF} = \left|\frac{\Phi_{initial} - \Phi_{final}}{\Delta t}\right| = \frac{1.44\bullet10^{-5}}{.040} = 3.6\bullet10^{-4}\,\text{V}\,.$$

29.10. The flux through the loop doesn't change in parts (a) and (c), so the emf vanishes in both these cases. In (b), let x be the length (along the 30cm side) in the field. The area of the loop in the field is then $(.4)(x)$ and the flux of the magnetic field through the loop is $(B)(.4)(x)$. The induced emf is equal to the rate of change of the flux and is given, in magnitude, by

$$\text{EMF} = (B)(.4)\left(\frac{dx}{dt}\right) = (B)(.4)(v) = (1.25)(.4)(.02) = 0.01\text{V}\,.$$

Although the problem doesn't ask for it, we can work out the direction of the induced current in the loop using Lenz's law: it is clockwise.

29.15. (a) counterclockwise (b) clockwise (c) zero (because the flux is not changing with time).

29.19. When the switch is closed, a current flows counterclockwise in the outer loop and causes a magnetic field pointing out of the page. Because the current increases with time, this magnetic field also increases. The inner loop, according to Lenz's law, will set up a magnetic field that opposes this increasing field. Thus it will set up a field going into the page, and it can do this by having a current circulate clockwise in it.

29.20 Study worked example 29.6 on p.1002, which treats just this problem.

(a) Magnitude of induced emf = $BLv = (.75)(1.5)(5) = 5.6\text{V}$

The direction of the induced current can be figured out in the three ways indicated:

I. Consider a positive charge q in the bar. The velocity vector of the bar, and hence the charge, are directed horizontally to the right. Because B points into the page, v x B points up along the bar, and this is also the direction of the current flow. The current thus flows in the counterclockwise sense in this circuit.

II. Choose the normal to the loop to be pointing into the page. Then, because B also points into the page, the flux Φ is positive. The increasing area of the loop causes this flux to become still more positive, so $d\Phi / dt > 0$. By Faraday's law, EMF = - $d\Phi / dt$ is negative. If one points one's thumb along the direction of the normal to the loop (i.e. into the page), one's fingers curl in the clockwise sense; but because the EMF < 0 in this case, the sense of the induced current is opposite to the curling of one's fingers, or counterclockwise in this case. (We have used just the prescription explained on p.997-8 of the text).

III. We saw in II that the flux through the circuit is positive and increasing. By Lenz's law, the induced current will want to produce a magnetic field that points out of the page and counteracts the increasing flux, and this will be accomplished only if the current flows counterclockwise. We have explained the solution in detail, to show you that there are several different ways of getting the correct answer and that they all agree with each other (as they must!). In any problem, feel free to use the method that seems simplest. You won't be tortured to work out the answer in all the different ways!

Rectangular loop problem

The vector area of the loop is the product of its area and the unit vector **k :**

$$\mathbf{A} = (4 \times 10^{-2})(5 \times 10^{-2})\, \mathbf{k} = (.002 \text{ m}^2)\, \mathbf{k}$$

The flux at time t can then be calculated as

$$\Phi = \mathbf{B}\bullet\mathbf{A} = (-.05t\, \mathbf{i} + .05t\, \mathbf{k})\bullet(.002\, \mathbf{k}) = .0001t \text{ Wb.} \quad (\text{using } \mathbf{i}\bullet\mathbf{k} = \mathbf{0} \text{ and } \mathbf{k}\bullet\mathbf{k} = 1)$$

The magnetic field in this case makes an angle of 45° with the normal to the loop at any time (can you see that?) The magnitude of the B field at time t is $\sqrt{2}\left(.05t\right)$, so the flux could also have been calculated as

$$\Phi = BA\cos\phi = \left(.05t\sqrt{2}\right)\left(.002\right)\cos 45^\circ = .0001t \text{ Wb,} \quad \text{the same result as above.}$$

(a) Putting t = 20s in the above gives the flux as .002 Wb

(b) Magnitude of induced emf = $\frac{d\Phi}{dt} = \frac{d}{dt}\left(.0001t\right) = .0001\text{V}$. This emf doesn't depend on the time i.e. it is a constant.

(c) If the current in the loop flows clockwise (looking down on it from the positive z-axis), it will produce a magnetic field pointing along –**k** counteracts the increasing flux through the loop; and so the current does flow in that direction, by Lenz's law.