Newton's Laws of Motion

How Physics Works

It is the task of physicists to take physical phenomena and show how they can be reduced to a modest number of basic laws of nature. Physicists also predict phenomena, using the basic laws of nature to say what will happen before the experiment occurs.

These notes discuss Newton's Laws of Motion. Modern physics uses Newton's Laws, and extensions to quantum mechanics and relativity, to analyze and explain a vast number of different physical phenomena. Newton's laws of motion are an approximation to a more exact theory, but they are extremely accurate under a very wide range of circumstances.

High School physics courses sometimes leave the impression that we believe in laws of nature because a single key experiment was done. Key experiments can be very important in convincing scientists that a theory was right. For example, until 1905 many European physicists did not believe in atoms except as a convenient mnemonic device. In 1905, Einstein used the hypothesis that there are atoms to explain Brownian motion, the observation that very tiny particles floating in water bob back and forth rather than sitting at rest. Einstein's treatment of Brownian motion was very convincing, even though there were other, better proofs at the time that atoms exist. Furthermore, the existence of Brownian motion turns out not to prove the atomic hypothesis. Nonetheless, Einstein's work on Brownian motion convinced many physicists that atoms do exist.

However, we do not believe that physics is right because someone did a key experiment. The reason we believe that physics is right is that it explains a huge number of experiments with a very small number of rules. For example, Newton's laws predict not only when the next transit of Venus will occur, but where all the planets and moons and asteroids will be at every future moment. If modern physics were substantially wrong, you would need to replace it with a completely different set of laws of nature that makes more or less all the same predictions that current physics does. Most physicists would be astonished to discover that two completely different sets of laws of nature could lead to all the same consequences. It's not obviously impossible, but it would be very surprising.

We could, in principal, change our minds as to which predictions are important, and which will be explained eventually. That happened in chemistry when the phlogiston model was replaced by Dalton's law of combining proportions. The phlogiston model accurately predicted, for example, why metals are more similar in color than are their oxides, and many other fundamental facts; it treated weights of combination as being unimportant. Dalton said – ignore colors (and many other things) – the important issue is that substances combine in fixed weight ratios. Dalton was right, but – as he would have agreed – eventually science would explain under his model the colors of metals and oxides, as is now done with quantum mechanics. In the end, Dalton's model had to explain everything that the phlogiston model did, as it does, but only 'in the end' not 'right now' did Dalton's model have to explain everything that had been explained earlier by the phlogiston picture.

Therefore, Newton's Laws of Motion appear to be really likely to be correct for the circumstances discussed in this course.

Newtonian mechanics is not like High School Plane geometry. In plane geometry, you start with a small number of basic assumptions and build logically and creatively outwards. In mechanics we start with a series of contexts, statements and examples, all of which must be held in mind at the same time to understand what is actually going on. I will supply the contexts one at a time, but until you have enough of them to understand what is going on things are going to be at least a little bit obscure.

Finally, scientific descriptions have three parts: There are the explicit laws of nature. There are implicit presumptions that are not written out explicitly, but which are just as important. Last and most important there are the worked examples and problems that give the laws of nature a context, showing how the laws of nature are to be used.

Statement of Newton's Laws

Newton's laws are introduced as describing the motions of very small ("point") masses. Many objects are not small or pointlike, but their motions can still be described in terms of the motion of points. A point mass has a location **r**, a velocity **v**, an acceleration **a**, and a mass m. The location, velocity, and acceleration are all vectors. The mass is a scalar. If an object is not a point, we must ask 'what location do we assign to this object?' and 'which velocity do we assign to this object?'

We begin by introducing a new vector, the *momentum*. **In Cartesian coordinates**, the momentum of a particle is

$$\vec{p} = m\vec{v}. \tag{1}$$

The momentum vector of an object points parallel to its velocity. Momentum has dimensions $m^1\ell^1t^{-1}$ and in the SI system has units kg m/s.

In many cases, vector equations are equally true in all coordinate systems. The equation I have just given for the momentum is correct in Cartesian (x, y, z) coordinates, but is not true in all coordinate systems. The reason that eq. 1 is only true in some coordinate systems is that eq. 1 is a true statement about momentum in Cartesian coordinates, but eq. 1 is *not* a correct definition of momentum.

Newton's **Second Law of Motion** is

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{2}$$

Here $\vec{F}$ is the total force on the object.

In more-or-less all of the problems we will discuss in this course, the mass of the object is a constant. In that case, combining the above two equations and applying the definitions of the velocity and acceleration gives

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} \tag{3}$$

for the total force in terms of the particle mass and acceleration.

Equation 3 is a vector equation, which is the equivalent of three scalar equations

$$F_x = m\frac{d^2x}{dt^2} \tag{4}$$

$$F_y = m\frac{d^2y}{dt^2} \tag{5}$$

$$F_z = m\frac{d^2z}{dt^2}, \tag{6}$$

which may also be written

$$F_x = m\frac{dv_x}{dt} \tag{7}$$

$$F_y = m\frac{dv_y}{dt} \tag{8}$$

$$F_z = m\frac{dv_z}{dt}. \tag{9}$$

Force has dimensions $m^1\ell^2t^{-2}$ and fundamental units kg m^2/s^2. There is a named unit for force: one *Newton* (Nt) is 1 kg m^2/s^2. Weight is a measure of the force on an object due to a gravitational field. 1.0 Nt is roughly the weight of an apple.

The **First Law of Motion** is simply a corollary of the Second Law, namely if $\vec{F} = 0$ then

$$m\frac{d^2\vec{r}}{dt^2} = 0. \tag{10}$$

Integration of this equation with respect to time gives

$$\vec{v} \equiv \frac{d\vec{r}}{dt} = \vec{V}, \tag{11}$$

where $\vec{V}$ is a numerical constant. Newton's first Law of Motion states that if the total force on an object is zero, then the object moves with a constant velocity.

Why did Newton put his corollary before his general law? That's not a normal or sensible way to do things. The first reason is a historical issue. Before Newton, there was an alternative notion ('theory' is excessively grandiose) of motion, namely that motion was due to 'impetus'. You took a cannon and fired a cannonball across a field. The cannon supplied the cannonball with a supply of impetus, which caused the cannonball to move. However, as the cannonball travelled, its stock of impetus was gradually exhausted. When the cannonball ran out of impetus, it fell out of the air. For cannonballs, which have dreadful aerodynamic properties, but not for modern rockets or artillery shells, this description is not totally silly. A modern analogy may clarify impetus: To move an airplane across the country, you load up the airliner with jet fuel. As it flies, the supply of fuel diminishes. If the airliner runs out of fuel in midair, it falls (well, glides) out of the sky. [For no points, identify the North American passenger airline that has proven this experimentally. Twice.]

In order for Newton to make readers hear his ideas, he first had to ensure that readers did not see his words and hear them as advocating the impetus theory. He therefore started by saying, in essence, 'The impetus theory is totally wrong'. The First Law was placed first for shock value. The impetus theory is sometimes blamed on Aristotle. However, Aristotle (who worked two thousand years before Newton did) did not recognize that there are forces, and did not have anything like modern ideas of motion or rate of change. Nor did Aristotle have the mathematical tools to discuss velocity or acceleration: Calculus had not been invented. Analytic geometry and coordinate systems were thousands of years in the future. Greek plane geometry, unlike the plane geometry taught in modern high schools, substantially did not even use diagrams.

The Second Law refers to $\mathbf{F}$, the *total force* applied to an object. Properties of forces are specified by the **Third Law of Motion**. Newton's Third Law of Motion is often presented as the exact English translation of his original, Latin text. That original Latin is, at best, obscure, even if you are totally fluent in Latin. A modern statement of the Third Law is

All forces are found in action-reaction pairs.

According to the Third Law, no force is found in isolation. Each force is part of a pair. Action-reaction pairs all have the same general properties. Writing the two forces of an action-reaction pair as $\mathbf{F}_1$ and $\mathbf{F}_2$:

1) $\mathbf{F}_1$ and $\mathbf{F}_2$ act on two separate bodies. The reaction force to a force on a body is never another force on the same body.

2) $\mathbf{F}_1$ and $\mathbf{F}_2$ add to zero, i.e., $\mathbf{F}_1 + \mathbf{F}_2 = 0$. By direct calculation $F_1 = F_2$ and $\hat{\mathbf{F}}_1 = -\hat{\mathbf{F}}_2$. The latter two equations are sometimes summarized as 'the action and reaction are equal and opposite', meaning that 'the action *force* and the reaction *force* are equal *in magnitude* and *their unit vectors point in* opposite *directions*.

3) $\mathbf{F}_1$ and $\mathbf{F}_2$ are causally symmetric. While $\mathbf{F}_1$ and $\mathbf{F}_2$ are an action-reaction pair, it is incorrect to ask which of them is the action and which of them is the reaction. $\mathbf{F}_1$ and $\mathbf{F}_2$ arise simultaneously and have a common cause. $\mathbf{F}_1$ does not cause $\mathbf{F}_2$; $\mathbf{F}_2$ does not cause $\mathbf{F}_1$.

4) $\mathbf{F}_1$ and $\mathbf{F}_2$ have the same physical basis. For example, if $\mathbf{F}_1$ is the gravitational force on some body, its reaction force $\mathbf{F}_2$ must also be a gravitational force. "same physical basis" is by far the most obscure feature of an action-reaction pair, because in order to understand what is meant you have to know what physical forces actually exist. Over the term, you will learn what a series of physical forces are. It's sort of like your first course in chemistry and learning what the elements are, except that there are many more elements than there are truly different physical forces.

Newtonian mechanics could be content-free. If every pair of objections interacted by a completely different force, replacing the particle acceleration with the force would not tell you anything new. For example, during the dot-com bubble, a major industrial firm launched 77 identical earth satellites. If those 77 satellites orbited the earth because 77 independent and unrelated gravitational forces attracted them to the earth, Newtonian mechanics would not say much that is interesting about earth satellite orbits.

Newtonian mechanics is not content free. The reason that Newtonian mechanics is not content-free is that the extremely long list of interactions between pairs of particles can be reduced to a very short list of fundamental forces. When I was an undergraduate, there were four commonly-recognized forces. (After each force, I note where it shows up, and which physicists developed the theory

gravity (forces between all masses; Newton, Einstein)

electromagnetism (charges, magnets, radio waves; Maxwell, Dirac)

strong nuclear force (holds atomic nuclei together; many names)

weak nuclear forces (forces involving neutrinos; Feynmann, Wu).

Then, one evening, my senior thesis supervisor and one of his colleagues were walking out to the parking garage, and the colleague announced that he had just done the work that would win him the Nobel Prize in physics, namely, he had completed the demonstration that the weak and electromagnetic forces are two aspects of one force. The Nobel Prize indeed followed a few years later.

Because the same short list of forces appear in huge numbers of problems, Newtonian mechanics lets you solve very large number of problems by applying the same principles (and a few math tricks) over and over again.

Examples of Forces

One of the major objectives of the physics community for the past four centuries has been to determine what forces are active in nature. We are still working on the problem, and are not completely finished yet. At the level of this course, a list of forces includes:

Contact forces: Solid objects cannot pass through each other. Instead, if you try to push your hand through the space occupied by a wall, your hand feels a force pushing backwards, keeping your hand from passing through the wall. A contact force acts perpendicular to the surface of contact between two objects. The contact force on an object points directly away from the object creating the force. The contact force is exactly large enough so as to insure if two objects rest against each other, neither object gains an acceleration that would take it inside the other object, nor do the two objects suddenly fly away from each other. The 'normal force' is an example of a contact force. (There is also a reaction contact force that your hand exerts on the wall. If the contact force is made large enough, either your hand or the wall will break. This course skips the biomechanics and civil engineering issues that follow.)

Gravity: Close to the surface of the earth the gravitational force is a constant, pointing straight down. The gravitational force on an object of mass m is $-mg\hat{\mathbf{k}}$, where $\hat{\mathbf{k}}$ is the vertical unit vector and there g is a gravitational constant. g is often called the 'gravitational acceleration', which is an incorrect and misleading term. As you sit and read, there is a gravitational force of magnitude mg on you, but unless your building is falling over, your acceleration is zero.

The force of gravity actually depends on the distance between the two bodies exerting it. If the change in the distance between two bodies is small relative to their distance apart, this dependence is usually weak.

Electrical force: The force on a charged body in an electrical field is $q\mathbf{E}$, where q is the charge of the body and $\mathbf{E}$ is the electrical field. $\mathbf{E}$ is a vector field; the electric field is a vector whose value may depend on position.

Magnetic force. The magnetic field $\mathbf{B}$ is also a vector field: $\mathbf{B}$ has a value at each point in space, but in general $\mathbf{B}$ varies in magnitude and direction from point to point. An object having a charge q that moves at velocity $\vec{v}$ experiences a magnetic force $\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B})$. The velocity and magnetic field must both be determined by the same observer.

Tension: Tension describes how ropes put forces on objects. In this course, a rope puts forces on objects because it has been stretched. It then pulls (never pushes) on objects it touches. The rope actually uses a contact force to exert a force on a neighboring object. The tension force always acts parallel to the rope. The tension in a rope is the same everywhere between two contact points. Correspondingly, the contact forces that a section of rope applies to its ends have equal magnitudes, and point in opposite directions. When a rope goes around a pulley, it exerts two tension forces on the pulley, one due to each section of the rope.

Aside: *masslessness*: Masslessness is a mathematical approximation. We say that an object is massless if its mass is effectively much smaller than the other masses in the system. If an object is massless, then it has $m \approx 0$. Applying this approximation to Newton's Second Law, one obtains $\mathbf{F} = 0$, that is, the total force on a massless object must be zero. Equivalently, the sum of the forces that a massless object applies to other objects must sum to zero.

In more advanced courses, more sophisticated descriptions of some of these forces are given. For example, contact forces arise because atoms are made of nuclei surrounded by electron clouds, and electrons all have the same charge and therefore repel each other. If for example you push your hand against a wall, the electrons in your hand and in the wall push away from each other, which is a contact force. Your hand creates a force on the wall, and vice versa. Friction. Springs. We will eventually discuss friction and springs. Neither of these topics is that complicated, but calculations with springs are mathematically fiddly.

Applying Newton's Laws to Solve Physics Problems

There is a standard procedure that may be used to apply Newton's Laws of Motion to solve physics problems. In short, you (i) *identify the forces* acting in the system under consideration, (ii) *find the components* of the forces, (iii) *invoke the Second Law* by replacing the total force $\vec{F}$ with the detailed expressions appropriate to the system as found in (i) and (ii), (iv) *apply constraints*, and (v) *solve the resulting equations.*

The Second Law is a differential equation. To solve the equation, you in general need to do an integral. Let's consider a simple case. I open a second floor window, hold an anvil outside the window, and let go of the anvil. The anvil now falls towards the ground. What equations describe the motion of the anvil?

Solution: The only significant force on the anvil is the force of gravity. Taking the z axis vertical, for a mass m anvil the total force on the anvil is $-mg\hat{\mathbf{k}}$. Substituting this force into the Second Law, one finds

$$m\frac{d^2\mathbf{r}}{dt^2} = -mg\hat{\mathbf{k}}. \tag{12}$$

How do we integrate this equation? There are no completely general methods for integrating the Second Law.

Let us consider the simplest approach, namely breaking eq 12 into its three components.

$$m\frac{d^2x}{dt^2} = 0 \tag{13}$$

$$m\frac{d^2y}{dt^2} = 0 \tag{14}$$

$$m\frac{d^2z}{dt^2} = -mg \tag{15}$$

Dividing each equation by the mass m of the anvil, these equations each describe motion of a particle moving with constant acceleration. In the x and y directions, the acceleration has the constant value zero. In the z direction, the acceleration has the value $-g$.

The motions in the x, y, and z directions are independent: The x-components of the force and acceleration do not depend on y or z, and similarly for the y and z components of the force and acceleration. Because the three equations are independent, we can integrate each one separately twice with respect to time, getting

$$x = x_0 + v_{0x}t \tag{16}$$

$$y = y_0 + v_{0y}t \tag{17}$$

$$z = z_0 + v_{0z}t - gt^2/2 \tag{18}$$

We have done a total of six time integrals, two on each equation, so there are six constants of integration, namely x_0, y_0, z_0, v_{0x}, v_{0y}, and v_{0z}. The constants of integration tell us where the anvil was located, and how fast it was moving, at $t = 0$. (Yes, I just skipped something important).

In some problems, part of the procedure can be left out. In some problems, the standard procedure will not work and you have to do something clever. Actually carrying out the standard procedure may be extremely difficult. For example, in one of the research problems that I am working on, the problem being 'integrate the Second Law for the following simple force', we had a really superb programmer, spent enormous effort optimizing the machine code, used years of high speed computer time to do the integrals, and managed to make a small dent in the research issue.

Force Diagrams

Force diagrams (also known as 'free body diagrams') are not in the list of things that must in principle be done in order to solve a Second Law problem. Indeed, I never encountered them as an undergraduate or as a graduate student. However, they are a way to focus your mind on identifying the forces that act in each problem, so we use them here.

A force diagram represents a body and the forces acting on it. If there are several bodies in the problem, each body is represented separately. Each body is represented as a point or a small circle. Each force is represented as an arrow drawn as radiating from the point representing the mass. The arrows are labelled by the force to which they correspond. We do not always know which way a force points, so we cannot always know in advance which direction to assign to an arrow.

A correct force diagram indicates the coordinate system, the direction in which the x, y, and z axes point. If there is no coordinate system indicated, the force diagram is wrong.

It is incorrect to draw a body as an extended object, with arrows radiating from different locations on the object.

FOOTNOTE: It turns out, though the issue rarely arises, that there are also three-center forces, in which a single interaction involves a triple of forces acting on three distinct bodies. The forces in a triple are all of the same physical nature, have a vector sum of $\vec{0}$, act on three distinct bodies, and are causally symmetric. You can get a Ph.D. in physics without encountering a discussion of three-body forces.