### The physics of the space elevator

P. K. Aravind

Department of Physics, Worcester Polytechnic Institute, Worcester, Massachusetts 01609

(Received 1 May 2006; accepted 10 November 2006)

A space elevator is a tall tower rising from a point on the Earth's equator to a height well above a geostationary orbit, where it terminates in a counterweight. Although the concept is more than a century old, it was only with the discovery of carbon nanotubes that it began to receive serious scientific attention. NASA commissioned a study of the space elevator in the late 1990s that examined the feasibility of such a structure and explored many of its applications. I explain the basic mechanical principles underlying the construction of a space elevator and discuss several of its applications: the transport of payload into space and the launching of spacecraft on voyages to other planets. © 2007 American Association of Physics Teachers.

[DOI: 10.1119/1.2404957]

### I. INTRODUCTION

A space elevator is a tall tower rising from a point on the Earth's equator to a height well above a geostationary orbit, where it terminates in a counterweight [see Fig. 1(a)]. Although the idea of such a structure is quite old, it is only within the last decade or so that it has attracted serious scientific attention. NASA commissioned some studies of the elevator in the 1990s that concluded that it would be feasible to build one and use it to transport payload cheaply into space and also to launch spacecraft on voyages to other planets.<sup>1</sup> Partly as a result of this study, a private organization called Liftport<sup>2</sup> was formed in 2003 with the goal of constructing a space elevator and enlisting the support of universities, research labs, and businesses that might have an interest in this venture. Liftport's website features a timer that counts down the seconds to the opening of its elevator on 12 April 2018. Whether that happens or not, the space elevator represents an application of classical mechanics to an engineering project on a gargantuan scale that would have an enormous impact on humanity if it is realized. As such, it is well worth studying and thinking about for all the possibilities it has to offer.

This article explains the basic mechanical principles underlying the construction of the space elevator and discusses some of its principal applications. It should be accessible to anyone who has had a course in undergraduate mechanics and could help give students in such a course a feeling for some of the contemporary applications of mechanics.

Before discussing the physics of the space elevator, we recall some of the more interesting facts of its history. The earliest mention of anything like the elevator seems to have been in the book of Genesis, which talks of an attempt by an ancient civilization to build a tower to heaven (the "Tower of Babel") that came to naught because of a breakdown of communication between the participants. In more recent times the concept of the space elevator was first proposed by the Russian physicist Konstantin Tsiolkovsky in 1895 and then again by the Leningrad engineer, Yuri Artsutanov, in 1960.<sup>2</sup> The concept was rediscovered by the American engineer, Jerome Pearson,<sup>4</sup> in 1975. In 1978 Arthur Clarke brought the idea to the attention of the general public through his novel Fountains of Paradise<sup>5</sup> and at about the same time Charles Sheffield, a physicist, wrote a novel<sup>6</sup> centered on the same concept. Despite this publicity, the idea of the elevator did not really catch on among scientists because an analysis of its structure showed that no known material was strong enough to build it.

This pessimism was largely neutralized by the discovery of carbon nanotubes in 1991.<sup>7</sup> Carbon nanotubes, which are essentially rolled up sheets of graphite, have a tensile strength greatly exceeding that of any other known material. Their high tensile strength, combined with their relatively low density, makes nanotubes an excellent construction material for a space elevator and led to a resurgence of interest in the concept.

# II. HEIGHT OF A FREE STANDING TOWER AT THE EARTH'S EQUATOR

What is the height of a free standing tower of constant density and constant cross sectional area at the Earth's equator? A free standing tower is one whose weight is counterbalanced by the outward centrifugal force on it, so that it exerts no force on the ground beneath it. A free standing tower is in tension along its entire length, with the tension adjusting itself so that each element of the tower is in equilibrium under the action of the gravitational, centrifugal, and tension forces acting on it. This point can be understood by looking at Fig. 1(b), which shows the four forces acting on a small element of the tower: an upward force  $F_U$  due to the portion of tower above the element, a downward force  $F_D$ due to the portion of tower below the element, a downward force *W* due to the weight of the element, and a (fictitious) upward centrifugal force  $F_C$  on the element due to its presence on the rotating Earth. The vector sum of these four forces must vanish if the element is in equilibrium.

For an element at geostationary height (that is, at a distance from the Earth's center equal to the radius of geostationary orbit) the weight and centrifugal forces are equal  $(W=F_C)$ , and therefore the tension forces at the two ends must also be equal  $(F_U=F_D)$  for equilibrium. For an element below geostationary height, the weight force W exceeds the centrifugal force  $F_C$  and one must have  $F_U > F_D$  for equilibrium. These two preceding statements imply that the tension in the tower increases with height from ground level to geostationary height. In contrast, for an element above the geostationary height, the centrifugal force  $F_C$  exceeds the weight W and hence  $F_U < F_D$  for equilibrium, implying that the tension in the tower decreases as a function of height past the geostationary height. A free standing tower is one for which the tension drops to zero at both ends, requiring no restraint



Fig. 1. (a) A panoramic view of the space elevator, showing it rising from a point on Earth's equator to a height above a geostationary orbit, where it terminates in a counterweight. The elevator cable, although shown as a line, is actually a tower of constant cross section. The purpose of the counterweight, which is not strictly necessary, is to avoid having the tower extend to a much greater height. The figure is not to scale. (b) An expanded view of a small element of the elevator cable (or tower), with the forces acting on it:  $F_U$  and  $F_D$  are the upward and downward forces of tension due to the rest of the cable, W is the weight of the element, and  $F_C$  is the outward centrifugal force on the element. The (vertical) length of the element is dr and its lower end is at a distance r from the Earth's center.

at either end to keep the tower in place. The overall picture of a free standing tower is thus of a structure in which the

tension rises from zero at ground level to a maximum value at geostationary height, and then decreases to zero again at the upper end. With this qualitative picture in mind, we proceed to work out the quantitative variation of the tension in the tower with height.

Let M, R, and  $\omega$  denote the mass, radius, and rotational angular velocity of the Earth. The radius of geostationary orbit is  $R_g = (GM/\omega^2)^{1/3}$ , where G is Newton's constant of gravitation. (See Table I for a glossary of the principal symbols used in this article.) We analyze the forces in a free standing tower of constant mass density  $\rho$  and uniform crosssectional area A. Consider a small element of the tower of length dr whose lower end is a distance r from the Earth's center. The equilibrium of this element requires that the vector sum of the forces acting on it vanish or that  $F_U+F_C$  $-F_D-W=0$  [see Fig. 1(b)]. We write  $F_U-F_D$  as AdT, where T is the tensile stress (force per unit area) in the tower. We also use the explicit expressions for W and  $F_C$  to rewrite the equilibrium condition as

$$AdT = \frac{GM(Adr\rho)}{r^2} - (Adr\rho)\omega^2 r.$$
 (1)

If we divide both sides by Adr, we can recast Eq. (1) as the differential equation

$$\frac{dT}{dr} = GM\rho \left[ \frac{1}{r^2} - \frac{r}{R_g^3} \right].$$
(2)

Integrating Eq. (2) from r=R to  $r=R_g$  subject to the boundary condition T(R)=0 gives the tensile stress at geostationary height  $R_g$  as

$$T(R_g) = GM\rho \left[ \frac{1}{R} - \frac{3}{2R_g} + \frac{R^2}{2R_g^3} \right].$$
 (3)

Table I. Summary of the principal symbols used in the text, together with their numerical values or the expressions that determine them.

Symbol	Meaning	Value
М	Earth's mass	$5.98 \times 10^{24} \text{ kg}$
R	Earth's radius	6370 km
ω	Earth's rotational angular velocity	$7.27 \times 10^{-5} \text{ s}^{-1}$
$R_g$	radius of geostationary orbit	42 300 km
Ğ	Newton's constant of gravitation	$6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
$g = GM/R^2$	acceleration of gravity at Earth's surface	9.81 m/s <sup>2</sup>
ρ	mass density of elevator cable	
Α	cross sectional area of the tapered elevator cable (varies	Eq. (7)
	with position along the cable)	
r	distance of a point on the cable from Earth's center	
$A_s$	cross sectional area of cable at ground level	
$A_g$	cross sectional area of cable at geostationary height	
Н	distance of the top of a free-standing tower from Earth's	150 000 km
	center	
Т	stress (or force per unit area) in elevator cable;	
	it is kept constant throughout a tapered cable	
$L_c = T/\rho g$	characteristic length of elevator material	
$A_g/A_s$	taper ratio	Eq. (8)
h	length of cable beyond geostationary orbit	
$m_C$	mass of counterweight	Eq. (10)
$m_E$	mass of elevator tower or cable	Eq. (11)

Let *H* denote the distance of the top of the tower from the Earth's center. We can determine *H* by integrating Eq. (2) from  $r=R_g$  to r=H subject to the boundary condition T(H) = 0, which expresses the fact that the tension drops to zero at the upper end of the tower. In this way we find that

$$T(R_g) = GM\rho \left[ \frac{1}{H} - \frac{3}{2R_g} + \frac{H^2}{2R_g^3} \right].$$
 (4)

If we equate the right sides of Eqs. (3) and (4) and note that H=R is a solution of the resulting cubic in H, we can reduce the cubic to the quadratic equation

$$RH^2 + R^2 H - 2R_o^3 = 0, (5)$$

whose only positive root is

$$H = \frac{R}{2} \left[ \sqrt{1 + 8 \left(\frac{R_g}{R}\right)^3} - 1 \right] = 150\ 000\ \mathrm{km}.$$

The height of the top of the tower above the Earth's surface is therefore  $H-R=144\ 000\ \text{km}$  (to three significant figures).

The maximum tensile stress in the tower occurs at geostationary height and can be calculated from Eq. (3). If we take the tower to be made of steel for which  $\rho$ =7900 kg/m<sup>3</sup>, we find that the maximum stress is 382 GPa (1 GPa =10<sup>9</sup> N/m<sup>2</sup>), which is over 60 times the tensile strength of steel, showing that a steel tower is impossible. Other conventional construction materials can be similarly ruled out. How can we overcome this obstacle and build a space elevator?

## III. THE TAPERED TOWER AND THE NEED FOR CARBON NANOTUBES

A different design for the tower that avoids the excessive stresses arising in the earlier design is a tapered tower with a cross section that varies with height in such a way that the tension (or force per unit area) in the tower remains uniform along its entire length and at a value that can be safely sustained by available construction materials. As we will see in the following, this requirement implies that the tapered tower must have a cross section that increases exponentially with height up to the geostationary height and then decreases exponentially afterward, as shown crudely in Fig. 2(a) (where the exponential variation is replaced by a linear one for ease of representation).

Consider a small element of the tapered tower of length dr whose bottom end is at a distance r from the center of the Earth. Figure 2(b) shows the forces acting on this element, which is assumed to be below geostationary height and is wider at its upper end than at its lower end. The equilibrium of this element is again expressed by Eq. (1) but now with  $F_U - F_D = T dA$ , where T is the constant stress in the tower and dA is the difference in areas of the upper and lower faces of the element. Equation (1) can thus be rewritten as

$$\frac{dA}{A} = \frac{\rho g R^2}{T} \left[ \frac{1}{r^2} - \frac{r}{R_g^3} \right] dr, \tag{6}$$

where  $g = GM/R^2$  is the acceleration of gravity at the Earth's surface. Integrating Eq. (6) yields the tower's cross sectional profile as



Fig. 2. (a) A space elevator with a tapered cable, whose cross sectional area increases from ground level to geostationary height and then decreases afterward. Both the increase and decrease in the area are exponential, but are shown as linear in the figure for ease of representation. (b) An expanded view of a small element of the cable below geostationary height, with the four forces acting on it [which have the same meanings as in Fig. 1(b)]. The (vertical) length of the element is dr and its lower end is at a distance r from the Earth's center. The cross sectional areas of the upper and lower ends of the element are A+dA and A, respectively.

$$A(r) = A_s \exp\left[\frac{\rho g R^2}{T} \left\{ \frac{1}{R} + \frac{R^2}{2R_g^3} - \frac{1}{r} - \frac{r^2}{2R_g^3} \right\} \right],$$
 (7)

where  $A_s$  is the value of A at r=R. Equation (7) shows that A increases exponentially with height from ground level to geostationary height and then decreases exponentially.

The distance *H* of the top of the tapered tower from the Earth's center can be defined by the requirement that the area of the tower at its upper end have the same (small) value as at its lower end, which is expressed by the condition  $A(H) = A_s$ . Using this condition in Eq. (7) allows us to solve for the height of the tapered tower as

$$H = \frac{R}{2} \left[ \sqrt{1 + 8 \left(\frac{R_g}{R}\right)^3} - 1 \right] = 150\ 000\ \text{km},$$

which is identical to the height of a tower of constant cross section that we found earlier. The main difference between the constant cross section and tapered towers is that the large stress and constant cross section of the former are exchanged for the smaller stress and variable cross section of the latter. This tradeoff turns out to be crucial for the realizability of the elevator.

Let  $A_g \equiv A(R_g)$  denote the cross sectional area of the tapered tower at geostationary height. The taper ratio of the tower is defined as  $A_g/A_s$ , that is, the ratio of its area at geostationary height to that at ground level. We find from Eq. (7) that

$$\frac{A_g}{A_s} = \exp\left[\frac{R}{2L_c}\left\{\left(\frac{R}{R_g}\right)^3 - 3\left(\frac{R}{R_g}\right) + 2\right\}\right],\tag{8}$$

where  $L_c = T/\rho g$ , known as the characteristic length, is a material parameter that determines the taper ratio  $A_g/A_s$  of a tower constructed out of that material. Table II uses Eq. (8) to calculate the characteristic lengths and taper ratios of tow-

Table II. Taper ratios for elevator cables consisting of three different materials, calculated from Eq. (8). Note how an increase in the characteristic length  $L_c$  leads to a faster than linear decrease in the taper ratio. The tensile stresses are the maximum ones that can be borne by these materials; safe practice generally requires that the materials be subjected to stresses no more than half these values.

Material	Density $\rho$ (kg/m <sup>3</sup> )	Max tensile stress T (GPa)	$\begin{array}{c} L_c = T/\rho g\\ (\mathrm{km}) \end{array}$	Taper ratio
Steel	7900	5.0	65	$1.6 \times 10^{33}$
Kevlar	1440	3.6	255	$2.5 \times 10^{8}$
Carbon nanotubes	1300	130	10,200	1.6

ers made out of steel, Kevlar, and carbon nanotubes. (Kevlar is a synthetic organic fiber made by DuPont company that is widely used in construction.) It is seen that steel and Kevlar yield very large taper ratios and are therefore unsuitable as construction materials for a space elevator, but that carbon nanotubes yield a modest taper ratio and are therefore an excellent material for this purpose. The advantage of nanotubes derives from their combination of high tensile strength and low density, which makes their characteristic length considerably larger than that of other materials. Because the characteristic length enters in the exponent in Eq. (8), even a modest increase in it gives rise to a dramatic decrease of the taper ratio.

All current designs for a space elevator assume a tapered tower (or cable, as the tower is often termed), and also assume that the tower is constructed out of carbon nanotubes.

### IV. LENGTH AND MASS OF THE SPACE ELEVATOR

What is the total length of the space elevator? And what is the mass of material required for its construction? It may seem that we have already answered the first question, because we found the total length of the elevator to be about 144 000 km. It turns out that we can considerably shorten the length of the elevator by terminating it at its upper end (which necessarily occurs above geostationary height) by a counterweight of the appropriate mass. The action of the counterweight can be understood as follows: the counterweight experiences a centrifugal force larger than the Earth's gravitational force and is maintained in its orbit by an extra inward force exerted on it by the elevator cable; by Newton's third law, the counterweight exerts an equal and opposite outward force on the cable that helps maintain the necessary tension in it.

Let us calculate the mass  $m_C$  of the counterweight if the elevator tower is to extend a distance *h* above geostationary height. The Earth's gravitational pull on the counterweight plus the inward force of the elevator cable on it must equal the outward centrifugal force on it:

$$\frac{GMm_C}{(R_g + h)^2} + A(R_g + h)T = m_C \omega^2 (R_g + h).$$
(9)

If we substitute the value of  $A(R_g+h)$  from Eq. (7) into Eq. (9) and solve for  $m_C$ , we find that

$$m_{C} = \frac{\rho A_{s} L_{c} \exp\left[\frac{R^{2}}{2L_{c} R_{g}^{3}} \left\{\frac{2R_{g}^{3} + R^{3}}{R} - \frac{2R_{g}^{3} + (R_{g} + h)^{3}}{R_{g} + h}\right\}\right]}{\frac{R^{2}(R_{g} + h)}{R_{g}^{3}} \left[1 - \left(\frac{R_{g}}{R_{g} + h}\right)^{3}\right]}.$$
(10)

Note that  $m_C \rightarrow \infty$  as  $h \rightarrow 0$  and decreases with increasing *h*. The total mass  $m_E$  of the elevator tower is also of interest and can be calculated as

$$m_E = \rho \int_R^{R_g + h} A(r) \, dr, \tag{11}$$

where A(r) is given by Eq. (7). The integral in Eq. (11) is best performed numerically.

The masses  $m_C$  and  $m_E$  depend on the parameters  $\rho$ , T,  $A_s$  and h, which capture all the material and design characteristics of the elevator. We now determine the likely values of these parameters and use them to estimate  $m_C$  and  $m_E$ .

The density of a pure carbon nanotube is  $1300 \text{ kg/m}^3$  and its strength (the maximum tensile stress it can bear) is assumed to be 300 GPa, although the NIAC Phase 1 report<sup>8</sup> uses the more conservative value of 130 GPa in its calculations. Even this more conservative figure is unlikely to be achieved for a long time. Recent work has focused on making composites of nanotubes with other materials, and a study<sup>9</sup> suggests that the strength of such composites may be limited to less than 100 GPa (with their density being 2000 kg/m<sup>3</sup> or higher). The achievable characteristic length may therefore be lower than the figure quoted for pure nanotubes in Table II.

Our treatment of the elevator has said nothing about the cross sectional shape of the elevator tower (or cable). A natural choice would be to have a cable with a circular cross section whose area varies with height in the manner appropriate to a tapered tower. A better choice is to have the cable in the shape of a ribbon with one dimension much smaller than the other, because this shape greatly reduces the risk of damage due to meteorite impact. One early design proposed a ribbon with an average thickness of one micron and a width increasing from 5 cm at ground level to 11.5 cm at geostationary height.<sup>8</sup> We will not discuss the design of the ribbon further except to note that its area at ground level  $A_s$ is fixed by the requirement that it be able to support the weight of the robotic lifter that climbs up it (either to transport payload or to thicken the ribbon during the construction process). If the mass of the lifter is  $m_I$ , its effective weight at ground level is  $m_L(g-\omega^2 R)$ , and this weight must be counteracted by the upward force of  $A_sT$  due to the tension in the cable; equating these forces allows us to fix  $A_s$  for a given lifter mass.

The total length  $(R_g+h)$  of the elevator cable is determined in part by the mass of the counterweight that can be deployed and in part by the solar system destinations that are hoped to be reached by spacecraft launched from the cable (this point is discussed further in Sec. V). A cable of about 100 000 km in length is generally considered a good choice.

Now that we have indicated some of the considerations that influence the parameters  $\rho$ , *T*, *A*<sub>s</sub> and *h*, we choose some representative values for them and calculate the counterweight and cable masses from Eqs. (10) and (11). We take  $\rho$ =1500 kg/m<sup>3</sup>, *T*=100 GPa, *A*<sub>s</sub>=1.5×10<sup>-7</sup> m<sup>2</sup> (sufficient

to support a 1000 kg lifter), and  $R_g + h = 100\,000$  km. Suppose we also incorporate a safety factor of 2 into the design, that is, we ensure that the stress in the cable is only T = 50 GPa everywhere, which is half the maximum stress that can be borne. We find from Eq. (8) that the taper ratio is 4.28, from Eq. (10) that the mass of the counterweight is  $52.7 \cdot 10^3$  kg, and from Eq. (11) that the mass of the elevator cable is  $97.7 \cdot 10^3$  kg. Neither the mass of the counterweight nor the cable is excessive. If a thicker or a shorter cable is desired, one or both of the mass for a range of values of the elevator parameters can be found in Ref. 10.

### V. APPLICATIONS: LAUNCHING PAYLOAD INTO ORBIT AND SPACECRAFT TO THE OTHER PLANETS

The space elevator affords an inexpensive and easy way to get payload into space: we simply have to make it ride up the elevator! One way of appreciating the advantage afforded by the elevator is to compare the ideal energy cost of getting a piece of payload into a geostationary orbit without and with the elevator. Without the elevator, the ideal cost would be the sum of the potential and kinetic energy costs in raising the payload from the surface of the Earth to geostationary orbit. If the payload is sent up on the elevator, the kinetic energy cost is saved because the elevator automatically imparts the necessary velocity to the payload. The reader can show that the fractional savings in energy in getting to geostationary orbit with the elevator is  $(R/R_g)/[2-(R/R_g)]$ , which works out to about 8%. This savings may not seem like much, but it should be remembered that if payload is sent up on a spacecraft, a huge additional cost is incurred in sending the spacecraft up along with its load. Proposals have also been made to use the tower as a linear induction propulsion system that would recover energy from a descending capsule and reuse the energy later to propel a capsule up the tower.<sup>11</sup>

A second major application of the elevator is to use the tower's rotational energy to launch spacecraft on orbits that would allow them to reach other planets. An object released from rest from a point sufficiently high up on the tower would be able to escape the Earth's gravity and sail away to infinity. The critical height  $r_c$  up the tower, measured from the Earth's center, at which the object would have to be released for this escape to occur can be shown to be  $r_c = (2GM/\omega^2)^{1/3} = 53\ 200$  km. Building a tower of greater than this height is necessary if we wish to use it to launch spacecraft on voyages to other planets.

What is the furthest distance from the Sun that a spacecraft released from rest from the top of a tower of height  $h_0$  ( $>r_c$ ) can reach? We assume that the tower's motion takes place in the orbital plane of the Earth (which is not true, but will suffice for our simplified analysis). After being released from the tower, the spacecraft will move away from the Earth and follow an elliptical orbit around the Sun. If the craft is released when the tower's velocity is parallel to the Earth's orbital velocity, then the craft's velocity relative to the Sun at the moment of release is as large as possible; in this case the perihelion occurs at the release point and the aphelion at a much greater distance from the Sun, allowing voyages to the outer planets to be made. If the craft is released when the tower's velocity is antiparallel to the Earth's orbital velocity, then the aphelion occurs at the release point of the Sun at the release of the Sun at the outer planets to be made. If the craft is released when the tower's velocity is antiparallel to the Earth's orbital velocity, then the aphelion occurs at the release point of the Sun at the release point is antiparallel to the Earth's orbital velocity, then the aphelion occurs at the release point of the Sun at the release point is antiparallel to the Earth's orbital velocity, then the aphelion occurs at the release point of the Sun at the

and the perihelion at a much smaller distance from the Sun, allowing voyages to the inner planets to be made. Let us now examine both these possibilities quantitatively.

Let  $v_E$  be the Earth's orbital velocity and  $v_1 = \omega h_0$  be the velocity of the spacecraft when it is released from the top of the tower. Suppose first that the release occurs when  $v_E$  and  $v_1$  are parallel to each other so that the spacecraft's speed relative to the Sun at the moment of release is  $v_E + v_1$ . Let  $R_E$  be the Earth's orbital radius and also the perihelion distance of the craft's elliptical orbit around the Sun (ignoring a small correction due to the finite length of the tower). Let  $r_2$  be the aphelion distance of the craft from the Sun and  $v_2$  its aphelion velocity. Angular momentum and energy conservation yield the pair of equations

$$m(v_E + v_1)R_E = mv_2r_2 \tag{12}$$

and

$$\frac{1}{2}m(v_E + v_1)^2 - \frac{GM_Sm}{R_E} = \frac{1}{2}mv_2^2 - \frac{GM_Sm}{r_2},$$
(13)

where *m* and  $M_S$  are the mass of the spacecraft and Sun, respectively. [The term  $-GMm/h_0$ , representing the potential energy of the spacecraft in the Earth's gravitational field, could be added to the left side of Eq. (13), but it is dwarfed by the term  $-GM_Sm/R_E$  and has therefore been omitted.] On eliminating  $v_2$  between Eqs. (12) and (13), we find that  $r_2$  satisfies the quadratic equation

$$\begin{bmatrix} (v_E + \omega h_0)^2 - \frac{2GM_S}{R_E} \end{bmatrix} r_2^2 + 2GM_S r_2 - (v_E + \omega h_0)^2 R_E^2 = 0, \qquad (14)$$

whose only relevant solution is

$$r_2 = \frac{(v_E + \omega h_0)^2 R_E^2}{2GM_S - (v_E + \omega h_0)^2 R_E}.$$
 (15)

The Earth's orbital velocity is  $v_E=30.9$  km/s, its orbital radius is  $R_E=1.5\times10^{11}$  m, and the Sun's mass is  $M_S=2$  $\times10^{30}$  kg. If we take the tower's height above the Earth's center to be  $h_0=107\ 000$  km, then we find from Eq. (15) that  $r_2=7.95\times10^{11}$  m=5.3 AU (astronomical units), which is a little larger than the mean orbital radius of Jupiter. A spacecraft released from the top of such an elevator would thus be able to reach Jupiter.

For a trip to the inner planets,  $v_E + \omega h_0$  in Eqs. (12)–(15) should be replaced by  $v_E - \omega h_0$ , and  $r_2$  in Eq. (15) now denotes the perihelion (rather than the aphelion) distance of the spacecraft from the Sun. If we use the same numbers as before, we obtain  $r_2=6.44 \times 10^{10}$  m=0.43 AU, which is a little larger than the mean orbital radius of Mercury.

Thus we see that an elevator cable of somewhat over 100 000 km in length should suffice as a sling to launch spacecraft to Jupiter (at the outer end) and Mercury (at the inner end). Reaching Jupiter is critical, because we can take advantage of Jupiter's gravity assist to send spacecraft further outward or even beyond the solar system.

The calculations of this section have been limited to inplane orbital transfers for simplicity, but out-of-plane transfers and other orbital maneuvers would have to be taken into account in planning a realistic mission.

#### VI. CONCLUDING REMARKS

Our survey of the space elevator has tried to convey an understanding of its basic physical features and some of its applications. Much has obviously been left unsaid in this brief account.

Any thorough discussion of the elevator would have to address the many threats it would face and how they would be met. Among these threats are vibrations in the cable induced by geophysical and astronomical sources, lightning strikes, meteors, space debris, wind, atomic oxygen, radiation, and erosion of the cable by sulfuric acid droplets in the upper atmosphere. Perhaps even more alarming is the possibility that the cable might snap and wrap itself around the Earth, unleashing all sorts of catastrophes in its wake. These and other threats are considered in Ref. 8, which concludes (a little too optimistically in my opinion) that none of them is fatal and that they can all be circumvented by suitable countermeasures. The main obstacles to the realization of the space elevator are perceived by its champions to be more technological (how rapidly will carbon nanotube technology mature?) and economic (how costly and time consuming will the project be and what sort of tradeoffs should be made?) rather than environmental.

The two applications of the elevator that we have discussed are the transfer of payloads to space and the launching of spacecraft on voyages to other planets. A host of other applications are envisioned, such as industrial manufacturing in the microgravity of space, global monitoring of the Earth and its environment, solar collectors for power generation and transmission to the Earth, orbiting observatories and interferometers, removal of space debris, studies of the danger of space radiation, and the mining of near-Earth asteroids. The construction of space elevators on the Moon and Mars has also been mentioned in connection with the colonization of these bodies. The material requirements for elevators on other bodies may not always be as stringent as those for an Earth based elevator and might be satisfied by more conventional materials. A more detailed discussion of these issues can be found in Refs. 8–10. Only the future will tell when, and in what form, the space elevator might eventually be realized.

### ACKNOWLEDGMENTS

I would like to thank Matthew Clark, Rachel Berg, and Neil Lettenberger for many discussions about the space elevator in the course of their Interactive Qualifying Project on the societal impacts of the space elevator at WPI. I would also like to thank John Wilkes and Ryan Caron for several stimulating conversations and Tom Nugent, Research Director at Liftport, for some helpful discussions of the calculations in Ref. 10.

- <sup>1</sup>See the story "Audacious and outrageous: Space elevators" at (http://science.nasa.gov/headlines/y2000/ast07sep\_1.htm).
- <sup>2</sup>Liftport, (http://www.liftport.com/).
- <sup>3</sup>K. E. Tsiolkowskii, *Dreams of Earth and Sky* (1895), reissue (Athena Books, Barcelona-Singapore, 2004). Y. Artsutanov, "V Kosmos na Elektrovoze" ("To the cosmos by electric train") Komsomolskaya Pravda, 31 July 1960.
- <sup>4</sup>J. Pearson, "The orbital tower: A spacecraft launcher using the Earth's rotational energy," Acta Astronaut. **2**, 785–799 (1975).
- <sup>5</sup>Arthur C. Clarke, *Fountains of Paradise* (Harcourt Bruce Jovanovich, New York, 1978).
- <sup>6</sup>Charles Sheffield, *The Web Between the Worlds* (Baen, Simon and Schuster, Riverdale, NY, 2001).
- <sup>7</sup>S. Iijima, "Helical microtubules of graphitic carbon," Nature (London) **354**, 56–58 (1991).
- <sup>8</sup>B. C. Edwards, "The space elevator: NIAC phase I report" (2001), (http://www.niac.usra.edu/studies/studies.jsp). B. C. Edwards, "The space elevator: NIAC phase II final report" (2003), (http://www.niac.usra.edu/studies/studies.jsp).
- <sup>9</sup>D. Srivastava, C. Wei, and K. Cho, "Nanomechanics of carbon nanotubes and composites," Appl. Mech. Rev. 56, 215–230 (2003).
- <sup>10</sup>T. Nugent, "Space Elevator Ribbon Mass and Taper Ratio," (http://www.liftport.com/papers/2005Nov\_LP-Ribbon\_Mass.pdf).
- <sup>11</sup> R. D. Thornton, "Design principles for magnetic levitation," Proc. IEEE 61, 586–598 (1973); J. T. Patha and G. R. Woodcock, "Feasibility of large-scale orbital solar/thermal power generation," J. Spacecr. Rockets 11, 409–417 (1974).

### ALL BACK ISSUES NOW AVAILABLE ONLINE

The contents of the *American Journal of Physics* are available online. AJP subscribers can search and view full text of AJP issues from the first issue published in 1933 to the present. Browsing abstracts and tables of contents of online issues and the searching of titles, abstracts, etc. is unrestricted. For access to the online version of AJP, please visit http://aapt.org/ajp.

Institutional and library ("nonmember") subscribers have access via IP addresses to the full text of articles that are online; to activate access, these subscribers should contact AIP, Circulation & Fulfillment Division, 800-344-6902; outside North America 516-576-2270 or subs@aip.org.

Individual ("member") subscribers to the print version who wish (for an additional fee) to add access to the online version should contact AAPT or go to the AAPT website: https://www.aapt.org/Membership/secure/agreement.cfm.