Mathematicians often see more than cold logic in symbols and numbers. They see the sublime

By Clara Moskowitz

Are equations beautiful? To scientists, formulas’ ability to represent fundamental truths or concisely capture complexity is indeed exquisite. To many in the public, though, they can be the opposite of beautiful—intimidating, utilitarian and opaque. Yet for others, the very mystery can be alluring: even when we cannot understand what equations say, we can be moved by knowing they have meanings beyond our comprehension. And mathematicians and nonmathematicians alike can be drawn in by the purely aesthetic appeal of these expressions, whose graceful and sometimes inscrutable symbols combine in visually satisfying ways.

To explore both the inherent and visual beauty of math, mathematician Daniel Rockmore of Dartmouth College teamed up with Bob Feldman of Parasol Press, which publishes fine art prints. They asked 10 famous mathematicians and physicists to write out what they conceived of as the “most beautiful mathematical expression” and had the print shop Harlan & Weaver turn the responses into 22-by-30-inch etchings called aquatints. “I was careful to not give any instruction beyond that sentence,” Rockmore says. “As the 10 prints show, it means different things to different people.”

Many picked classic equations, such as the famous formula by Isaac Newton that was Stephen Smale’s choice (page 73). Others selected expressions closer to home, including equations they discovered themselves that are deeply tied to their lifelong research interests—for example, the MacDonald equation chosen by Freeman Dyson (page 72). “I love Dyson’s,” Rockmore says. “It’s thin, and it’s sleek; visually, it’s so sharp. And with those little exclamation points for the factorials, it’s beautiful.”

The project is called Concinnitas, after the word used by Italian Renaissance scholar Leon Battista Alberti to describe the balance of elements necessary for beautiful art. The collection premiered in December 2014 at the Annemarie Verna Gallery in Zurich and has since been shown at five more galleries, with plans to travel elsewhere in the coming months. Here we show five of the prints.

**IN BRIEF**

A group of famous mathematicians and physicists were asked to identify the “most beautiful mathematical expressions” they knew to display as aquatint prints. The collection, called Concinnitas, after an Italian Renaissance expression for balanced beauty, probes the connection between mathematics and art. Some participants chose equations they discovered themselves; others picked classics that have long inspired them. Some formulas correspond to physical laws, whereas others are pure abstractions. Five are featured here.
Ampère’s Law
Chosen by Simon Donaldson, Stony Brook University

Instead of selecting a single equation, Donaldson listed three and drew a picture of a wire tied in a knot. The current (\(J\)) running through the wire toward the large arrows creates a magnetic field (\(\mathbf{B}\)) in the direction indicated with small arrows. The three equations are Ampère’s law, which describes how a current generates a magnetic field. Together the image and the equations represent the connection between electromagnetism and topology—the branch of mathematics concerned with knots and spatial relations. Donaldson says he finds beauty in revealing such “new connections between things that one previously thought of as quite different.” For example, by applying some of the ideas and mathematics of electromagnetism to the study of knots, researchers have found new ways to determine whether different knots are fundamentally the same, the way a doughnut and a coffee mug are essentially the same shape deformed to look different.
MacDonald Equation
Chosen by Freeman Dyson, Institute for Advanced Study, Princeton, N.J.

Dyson derived this equation—a reformulation of a classic called the tau function, famously studied by Indian mathematician Srinivasa Ramanujan—shortly after another mathematician, Ian MacDonald, independently arrived at it. In it, five variables—a, b, c, d and e—are subtracted from one another in 10 combinations. The differences are multiplied together and divided by the product of the factorials for 1, 2, 3 and 4 (for example, 4 factorial, expressed as 4! = 1 × 2 × 3 × 4).

The elegance of this formulation appeals to Dyson because it reveals a kind of symmetry, or balance, among the five variables in the tau function. The equation is also beautiful in a more indefinable way, he says. “It doesn’t particularly tell you anything true about the universe—it just stands for itself, like a piece of music,” he notes. “Asking what it means is rather like asking what a Beethoven trio means. You just have to listen to it.” The equation belongs to the branch of pure mathematics called number theory.

Moduli Space of Curves of Genus g
Chosen by David Mumford, Brown University

Our universe has just three dimensions of space, but mathematicians can imagine it containing many more. This equation describes a space with dimensions numbering 3g − g and shows that if g is large enough, the shape of the space is negatively curved, like the surface of a saddle. When Mumford discovered the formula, he recalls, “I thought it was a startling result, especially that strange number 13 that came up.” Most fundamental mathematical expressions consist only of variables, operators and small whole numbers such as 1 and 2, making the relatively large quantity 13 in this equation an aberration. To Mumford, the strangeness of the equation makes it beautiful. “As a mathematician, you feel you’re discovering these logically determined facts—they have to be this way and no other way,” he says. “And suddenly you come up with a strange number, and you think, ‘Why did it have to be this way?’”
Newton’s Method
Chosen by Stephen Smale,
City University of Hong Kong

A mathematical trick, known as Newton’s method, approximates the solution to an equation—\( f(x) \)—whose exact answer cannot be calculated, such as the square root of 2 (which is the irrational number 1.4142 ...). It works by starting with any real number, \( x \), and subtracting the function \( f(x) \) divided by the derivative of that function, \( f'(x) \), to get a new \( x \). Every time this process repeats, \( x \) gets closer and closer to an estimation of the solution. The method is very handy, yet even Newton lacked a good theory for why it works. That mystery is what makes this equation so appealing to Smale. “So much of my work is devoted to understanding Newton’s equation—under what conditions it works,” he says. “My own feeling is that a great problem is never solved; it just becomes the focus of more and more work.”

The Lagrangian of the Electroweak Theory
Chosen by Steven Weinberg,
University of Texas at Austin

Two of nature’s four fundamental forces—electromagnetism and the weak force (responsible for radioactive decay)—unite in this equation, revealing themselves to be two sides of a single coin. The formula, which Weinberg devised in 1967, established that at certain energies, electromagnetism and the weak force act as one, the “electroweak” force—a discovery that later won him the Nobel Prize in Physics. Here the \( \mathcal{L} \) represents the Lagrangian density, essentially the energy density of the fields associated with the force, which are denoted by \( A \) and \( B \).

\[
\mathcal{L} = -\frac{1}{4} \left( \partial \mu \partial \nu A^\mu A^\nu - g_1 \left( \varepsilon_{\mu \nu \alpha \beta} A^\mu A^\nu A^\alpha B^\beta \right) \right)
-\frac{1}{2} \left( g_2 \varepsilon_{\mu \nu \alpha \beta} A^\mu A^\nu B^\alpha B^\beta \right)^2
-\frac{1}{2} \left( g_3 \varepsilon_{\mu \nu \alpha \beta} B^\mu B^\nu A^\alpha A^\beta \right)^2
-\frac{1}{2} \left( g_4 \varepsilon_{\mu \nu \alpha \beta} B^\mu B^\nu B^\alpha B^\beta \right)^2
-\mathcal{L}_{\text{matter}}
\]

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The Mass of the Photon, Alfred Scharff Goldhaber and Michael Martin Nieto; May 1976. scientificamerican.com/magazine/sa