Discrete Mathematics I
MA530, Fall 2010
W. J. Martin
October 1, 2010

Discrete Math Assignment 3

DUE DATE: Wednesday, October 6, in class.

Please make sure your five solutions are clearly legible. If you need a hint for any problem, consult the course web page or come to office hours.

1. Suppose the set $\mathbb{N}$ of positive integers is colored with any finite number of colors. Prove that there exists a monochromatic triple (x, y, z) with $x + y = z$.

2. Prove that, for each $n \geq 3$, there exists an $N \geq 3$ such that, given any N distinct points in the Euclidean plane $\mathbb{R}^2$, there always exist n of these which form a convex n-gon. (E.g., for $n = 4$, $N = 5$ suffices.)

3. Recall that a *tournament* on n vertices is an orientation $T = (V(T), E(T)$ of the complete graph K_n. For $m \geq 1$, we say tournament T "has property D_m" if, for any subset M of m vertices, there exists a vertex $x \in V(T)$ such that $(x, y) \in E(T)$ for every $y \in M$ (so x "dominates" M). Prove that, if
$$\binom{n}{m} < \left(\frac{2^m}{2^m - 1}\right)^{n-m}$$
then there exists a tournament on n vertices having property D_m.

4. For p a prime, give an explicit construction of a De Bruijn sequence with parameters $n = 2$ and $k = p$: i.e., a circular sequence of k^2 symbols from $\{0, 1, \ldots, k-1\}$ such that each ordered pair of symbols appears exactly once as we traverse the circle in the clockwise direction. (By "explicit", I mean with actual formulae; no $\ldots$, or expressions such as "and so on".) Give a short proof that your construction works. [EXTRA CREDIT: Do this for composite k as well.]

   Let's end with an easy one ...

5. Let $(P, \preceq)$ (or just P, for short) be a finite poset; let ℓ denote the length of the longest chain in P and let m be the size of a largest antichain in P. Prove that $|P| \leq \ell m$.