MA3231 Linear Programming
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**LP Assignment 7**

**Due Date:** Tuesday October 10th by 5pm in my office mail slot in SH108.

Please carefully review the assignment presentation rules for this course.

Provide neat and careful solutions to the following five problems:

1.) Formulate (but do not solve) the following as a linear programming problem (or, failing that, an integer linear programming problem):

\[
\begin{align*}
\text{minimize} & \quad \max \{x_1, x_2, x_3\} \\
\text{subject to} & \quad 4x_1 + x_2 + 2x_3 \geq 20 \\
& \quad x_1 + 3x_2 + 5x_3 \geq 50 \\
& \quad 5x_2 + x_3 \geq 30 \\
& \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0
\end{align*}
\]

[Here, \(\max \{x_1, x_2, x_3\}\) denotes the largest of the values in the set brackets.]

2.) Formulate (but do not solve) the following as a linear programming problem (or, failing that, an integer linear programming problem):

\[
\begin{align*}
\text{maximize} & \quad \text{avg}\{x_1, x_2, x_3\} \\
\text{subject to} & \quad 4x_1 + x_2 + 2x_3 \geq 20 \\
& \quad x_1 + 3x_2 + 5x_3 \geq 50 \\
& \quad 5x_2 + x_3 \geq 30 \\
& \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0
\end{align*}
\]

[Here, \(\text{avg}\{x_1, x_2, x_3\}\) denotes the average of the values in the set brackets.]

3.) Formulate (but do not solve) the following as a linear programming problem (or, failing that, an integer linear programming problem):
minimize \( \max_{i \neq j} |x_i - x_j| \)

subject to 
\[
\begin{align*}
4x_1 + x_2 + 2x_3 & \geq 20 \\
x_1 + 3x_2 + 5x_3 & \geq 50 \\
5x_2 + x_3 & \geq 30 \\
x_1 & \geq 0, \ x_2 \geq 0, \ x_3 \geq 0
\end{align*}
\]

[Our goal here is to find the most uniform feasible solution, where all entries are as close to one another as possible.]

4.) Formulate the following as a linear programming problem (or, failing that, an integer linear programming problem). We have an empty feasible region and we want to find a vector which satisfies as many of the constraints as possible.

\[
\begin{align*}
\text{minimize} \ & \text{number of violated constraints} \\
\text{subject to} \ & 4x_1 + x_2 + 2x_3 \geq 20 \\
& x_1 + 3x_2 + 5x_3 \geq 50 \\
& 5x_2 + x_3 \geq 30 \\
& x_1 \leq 2 \\
& x_2 \leq 4 \\
& x_3 \leq 3 \\
& x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0
\end{align*}
\]

[Do not solve.]

5.) Formulate the following as a linear programming problem (or, failing that, an integer linear programming problem):

\[
\begin{align*}
\text{maximize} \ & |x_1| + |x_2| \\
\text{subject to} \ & - x_1 + x_2 \leq 10 \\
& 4x_1 - x_2 \leq 20 \\
& x_1 - 2x_2 \leq 20 \\
& x_1, x_2 \text{ unrestricted}
\end{align*}
\]

[Do not solve.]