MA3231 Linear Programming
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October 10, 2009

LP Assignment 7

DUE DATE: Wednesday, October 14 at 4:30pm in my department mailbox.

Please be sure to observe Dr. Martin's assignment presentation rules.

Provide neat and careful solutions to the following five problems:

1.) In R^6, let W be the line

$$30x_1 = -30x_2 = 15x_3 = 10x_4 = -10x_5 = 6x_6.$$

Find the matrices P and Q representing orthogonal projection onto W and its orthogonal complement $W^\perp$. *[HINT: One can avoid almost all of the fractions by using the fact that $(\alpha A)(\beta B) = (\alpha\beta)AB$ for matrices A and B and scalars α and β.]*

2.) Suppose T is the linear transformation $T : R^n \to R^n$ given by orthogonal projection onto a subspace V of R^n and $U : R^n \to R^n$ is the orthogonal projection onto another subspace W of R^n. What can you say about the subspaces V and W if you know that UT is the zero linear transformation (i.e., $(UT)(x) = 0$ for all x in R^n)? Explain.

3.) Consider the linear programming problem

$$\textbf{max } -x_1 - x_2 \quad \textbf{subject to } \quad x_1 + x_2 \leq 1, \quad x_1, x_2 \geq 0.$$

(i) Starting with $x^0 = [1/2,\ 1/4]^T$, apply three iterations of the affine scaling method using $r = 1/2$. (First, convert to equality form.) For each iteration, give

- the constraint matrix $\hat{A}$ and objective vector $\hat{c}$ for the scaled problem;
- the projection matrix P and search direction d for this iteration;
- the ratio computation from Step 4;
- the next iterate, both in scaled form $\hat{x}$ and as a solution x^{k+1} to the original problem above. (So you will be finding x^1, x^2 and x^3.)

(ii) On a sheet of graph paper, make a careful (and large!) drawing of the feasible region in R^2. For x^0 and each of the next two iterates, x^1, and x^2, plot both the gradient of the objective function (namely $c^T = [-1 \ -1]$) at that point as well as the scaled step direction $x^{k+1} - x^k$.

4.) Repeat the steps of Exercise 3 for the linear programming problem

$$\textbf{max } x_1 + x_2 \qquad \textbf{subject to} \quad 0 \le x_1 \le 1, \quad 0 \le x_2 \le 1.$$

starting with $x^0 = [3/4, \ 1/4]^T$. (You may wish to use MAPLE to check your computations and to follow the trajectory further.)

5.) Perform two iterations of the affine scaling method for the problem

$$\begin{array}{rrrrl} \textbf{maximize} & x_1 + 2x_2 + 2x_3 & & & \\ \textbf{subject to} & x_1 - & x_2 & & = 0 \\ & x_1 + & x_2 + \sqrt{2}x_3 & & = 4 \\ & x_1, & x_2, & x_3 & \ge 0 \end{array}$$

with step size $r = 0.5$ and initial solution $x^0 = [1, \ 1, \ \sqrt{2}]^T$. Show your work as in Exercise 3.