MA3231 Linear Programming
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LP Assignment 5

DUE DATE: Tuesday, October 6, 2009, by 4:30pm in my office mail slot.

Please recall the presentation rules for homework in this course.

Provide neat and careful solutions to the following five problems. Each of the first four problem gives two systems of linear equalities and inequalities and you are asked in each case to show that that two given systems (P) and (D) are mutually exclusive. The fifth problem is a theorem which you are to prove.

1. For an $m \times n$ matrix A prove that the following two systems cannot both be feasible:

$$\text{(P)} \quad A\mathbf{x} < 0 \qquad \text{and} \qquad \text{(D)} \quad \mathbf{y}^\top A = 0,\ \mathbf{y} \geq 0,\ \mathbf{y} \neq 0.$$

2. For an $m \times n$ matrix A and a vector c of length n prove that the following two systems cannot both be feasible:

$$\text{(P)} \quad A\mathbf{x} \leq 0,\ c^\top\mathbf{x} > 0 \qquad \text{and} \qquad \text{(D)} \quad \mathbf{y}^\top A = c^\top,\ \mathbf{y} \geq 0.$$

3. For an $m \times n$ matrix A prove that the following two systems cannot both be feasible:

$$\text{(P)} \quad A\mathbf{x} = 0,\ \mathbf{x} > 0 \qquad \text{and} \qquad \text{(D)} \quad \mathbf{y}^\top A \geq 0,\ \mathbf{y}^\top A \neq 0.$$

4. For an $m \times n$ matrix A prove that the following two systems cannot both be feasible:

$$\text{(P)} \quad A\mathbf{x} < 0,\ \mathbf{x} \geq 0 \qquad \text{and} \qquad \text{(D)} \quad \mathbf{y}^\top A \geq 0,\ \mathbf{y} \geq 0,\ \mathbf{y} \neq 0.$$

5. **(a)** Derive the Complementary Slackness Conditions for a primal-dual pair of the form

$$\text{(P)} \quad \max\ c^\top\mathbf{x},\quad A\mathbf{x} = b,\ \mathbf{x} \geq 0 \qquad \text{and} \qquad \text{(D)} \quad \min\ \mathbf{y}^\top b,\quad \mathbf{y}^\top A \geq c^\top.$$

   (b) Prove: *If the primal* (P) *has a* <u>*non-degenerate*</u> *basic optimal solution, then the dual* (D) *has a* <u>*unique*</u> *optimal solution.* [HINT: The matrix B in the optimal dictionary is invertible since $\mathcal{B}$ is a basis.]