MA3231 Linear Programming
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LP Assignment 4

DUE DATE: Tuesday, September 29, 2009, by 4:30pm in my office mail slot.

Please recall the presentation rules for homework in this course.

Provide neat and careful solutions to the following five problems. All problems refer to the applications discussed in Assignment 1, so you will need to refer repeatedly to your sample solutions for that assignment.

1. Consider the LP problem in Problem 2 of Assignment 1, where three types of pig iron are to be blended, with the possible addition of pure manganese to build 1000 lbs of castings. We split the equality constraint into two inequalities and introduce slack variables $w_1, \ldots, w_5$ to get the initial dictionary

```
z   =   900.00  -  0.058 xA  -   0.07 xB  -   0.04 xC  -   8.00 xM
-------------------------------------------------------------------
w1  =  1000.00  -   1.00 xA  -   1.00 xB  -   1.00 xC  -   1.00 xM
w2  = -1000.00  +   1.00 xA  +   1.00 xB  +   1.00 xC  +   1.00 xM
w3  =    -4.50  +  .0045 xA  +   .005 xB  +   .004 xC  +   1.00 xM
w4  =   -32.50  +   0.04 xA  +   0.01 xB  +   .006 xC
w5  =    55.00  -   0.04 xA  -   0.01 xB  -   .006 xC
```

After applying the dual simplex method, we arrive at the optimal dictionary

```
z   =   845.09  -  0.020 xB  -  0.006 w2  -  7.994 w3  -   0.412 w4
-------------------------------------------------------------------
w1  =     0.00               -  1.000 w2
xC  =   220.46  -  0.881 xB  +  1.181 w2  -  1.181 w3  -  29.394 w4
xM  =     0.11  -  0.001 xB  -  0.004 w2  +  1.004 w3  -   0.015 w4
xA  =   779.43  -  0.118 xB  -  0.177 w2  +  0.177 w3  +  29.409 w4
w5  =    22.50                                         -   1.000 w4
```

(a) Describe the optimal solution in sentence form. List each constraint in a table and the corresponding value of its slack variable. In the same table, give the per-unit cost of each of these restrictions to your operation.

(b) The cost of Pig Iron B is currently 6 cents a pound. The supplier wishes to know at what price it will become profitable for you to use Pig B in your blend. And how much of it will you purchase at this price? (Perform only one pivot.)

(c) The customer is unhappy with your \$900 price quote for the castings. She offers to weaken the upper limit on silicon concentration from 5.5% to 6.5% if you'll sell the castings for \$850. What is your response?

2. Consider the LP problem in Problem 3 of Assignment 1, where fuel purchases are optimized for a plane flying from Worcester to Miami to LA to Houston and back to Worcester. Our initial and optimal dictionaries are

```
z  =     0.0  -  3.4 x1  -  3.25 x2  -  3.55 x3  -  2.88 x4
-------------------------------------------------------------
w1 =  5000.0  -  1.0 x1
w2 =  6400.0  -  1.0 x1  -  1.00 x2
w3 =  9100.0  -  1.0 x1  -  1.00 x2  -  1.00 x3
w4 = 10600.0  -  1.0 x1  -  1.00 x2  -  1.00 x3  -  1.00 x4
w5 = -1400.0  +  1.0 x1
w6 = -4100.0  +  1.0 x1  +  1.00 x2
w7 = -5600.0  +  1.0 x1  +  1.00 x2  +  1.00 x3
w8 = -7400.0  +  1.0 x1  +  1.00 x2  +  1.00 x3  +  1.00 x4
```

and

```
z  = -23594.0  -  0.3 x3  -  0.15 w5  -  0.37 w7  -  2.88 w8
-------------------------------------------------------------
x2 =  4200.0  -  1.0 x3  -  1.00 w5  +  1.00 w7
w2 =   800.0  +  1.0 x3              -  1.00 w7
w3 =  3500.0                         -  1.00 w7
w4 =  3200.0                                     -  1.00 w8
w1 =  3600.0             -  1.00 w5
w6 =  1500.0  -  1.0 x3              +  1.00 w7
x1 =  1400.0             +  1.00 w5
x4 =  1800.0                         -  1.00 w7  +  1.00 w8
```

(a) Write down all values of the dual variables at optimality, with the economic interpretation for each. Use some English here, but this can be in table format.

(b) You have the option to install an auxiliary fuel tank. The installation cost is (almost) negligible, but each 100 gallons of capacity decreases by one the number of passengers you can take on board. If each seat is worth \$1000 to you, how big shall you make the auxiliary tank?

(c) You hire a consultant named Chick Daney who can manipulate oil companies. If Chick can negotiate for you a 25-cent decrease in the cost of gasoline at any **one** airport, which airport will you choose and how much will you be willing to pay the consultant based on one month's savings (22 days of operation)? Explain.

3. For the coffee roaster problem in Assignment 1 (Problem 4), we had initial and optimal dictionaries

```
 z   =     0.00  +   1.98 x1  +   3.24 x2
-----------------------------------------
 x3  =   240.00  -   0.60 x1  -   0.80 x2
 x4  =   100.00  -   0.40 x1  -   0.20 x2
```

and

```
 z   =   900.00  -   4.50 x3  +   1.80 x4
-----------------------------------------
 x2  =   180.00  -   2.00 x3  +   3.00 x4
 x1  =   160.00  +   1.00 x3  -   4.00 x4
```

where x_3 and x_4 are the slack variables for Arabica and Robusta beans, respectively.

(a) Find ranges for the right-hand side values b under which the current optimal basis remains optimal.

(b) Find ranges for the objective coefficients c_1 and c_2 under which the current optimal basis remains optimal.

(c) How does the solution change if the roaster compromises the quality of the City Blend from 60/40 to 50/50?

4. This problem refers to Problem 5 on Assignment 1. We have initial and optimal dictionaries

```
 z   =     0.0             +  2.50 xW
---------------------------------------
 w1  = 20000.0  -  3.00 xB  -  4.00 xW
 w2  =  8000.0  -  3.30 xB  -  0.65 xW
```

and

```
 z   = 12500.00  -   1.875 xB  -   0.625 w1
-------------------------------------------
 xW  =  5000.00  -   0.750 xB  -   0.250 w1
 w2  =  4750.00  -  2.8125 xB  +  0.1625 w1
```

(a) Each of us noticed immediately that, at a selling price of \$6 per bottle, it would never be profitable to make bourbon. If bourbon sells for \$8 a bottle, what is the new optimal plan of action? What about at \$7 a bottle? At what selling price <u>does</u> it become profitable to produce bourbon?

(b) You are offered a loan of up to \$5000 which will be fully due at the end of the current production period, with a fixed 10% interest charge. Is it profitable for you to borrow at this rate? How much (without pivoting)?

(c) You have the option to purchase additional machines for processing your product. Each machine purchased will add 1000 machine hours per production period. Your financing plan will charge you \$900 per machine per production period. But for each two machines purchased, you get a third one for free! How many of these machines will you purchase?

5. This problem refers to Problem 1 on Assignment 1 (Exercise 1.3 in the text). The optimal strategy for this problem is derived on pages 84-85 in the text. Explain the tradeoff between the probability of successfully guessing distribution p and the limit β on the probability of wrongly saying "p". Perhaps an example such as the following can be used for illustration:

$$p_1 = 0.2, \quad p_2 = 0.2, \quad p_3 = 0.2, \quad p_4 = 0.2, \quad p_5 = 0.2;$$

$$q_1 = 0.4, \quad q_2 = 0.2, \quad q_3 = 0.2, \quad q_4 = 0.1, \quad q_5 = 0.1; \quad \beta = 0.25.$$

Write down the values of the dual variables in this case and determine how much your success probability increases for each per-unit increase in parameter β.