LP Assignment 3

DUE DATE: Friday September 19, by 4:30pm in my office mail slot in SH108.

Please carefully read the presentation rules at the bottom of the second page.

Please complete the following five problems.

1. Exercise 2.19 in the text. (This question is the answer to Exercise 1.3, so you may enjoy reading that question along the way.)

2. Exercise 3.4 in the text. [HINT: Start by considering two cases. If the optimal objective value is zero, then the zero vector is an optimal solution (Explain!). If the optimal objective value is greater than zero, then there must be some non-zero vector $\mathbf{x}$ which is feasible with positive objective value; choose such an $\mathbf{x}$ and modify it (how?) to show that the problem is unbounded.]

3. (a) Carefully write down the Klee-Minty problem for $n = 2$. (See (4.1) on p41.)
   (b) Draw the feasible region, giving the coordinates of all four basic feasible solutions.
   (c) Apply the simplex method to solve the problem using the largest coefficient rule for choice of entering variable.

4. We can easily determine all optimal solutions to the linear programming problem

$$\begin{align*}
\text{maximize} & \quad 8x_1 + 10x_2 \\
\text{subject to} & \quad 4x_1 + 5x_2 \leq 100 \\
& \quad x_2 \leq 10 \\
& \quad x_1, x_2 \geq 0
\end{align*}$$

Write a mathematical argument that determines all optimal solutions to

$$\begin{align*}
\text{maximize} & \quad 4x_1 + 5x_2 + 8x_3 + 9x_4 \\
\text{subject to} & \quad 4x_1 + 5x_2 \leq 100 \\
& \quad 8x_3 + 9x_4 \leq 100 \\
& \quad x_1, x_2, x_3, x_4 \geq 0
\end{align*}$$
5. Find the dual of the following LP

\[
\begin{align*}
\text{maximize} & \quad 9x_1 - 5x_2 \\
\text{subject to} & \quad 6x_1 + 2x_2 \leq 10 \\
& \quad -7x_1 - 3x_2 \leq 20 \\
& \quad 8x_1 - 4x_2 \leq 30 \\
& \quad x_1, \quad x_2 \geq 0
\end{align*}
\]

Explain briefly.

BASIC RULES FOR LP ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only one side of each sheet of paper; Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.

III) Show your work. Explain your answers using FULL SENTENCES;

IV) No late assignments will be accepted for credit.