MA3231 Linear Programming
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September 15, 2009

LP Assignment 3

DUE DATE: Monday, September 21, 2009, by 4:30pm in my office mail slot.

Please recall the presentation rules for homework in this course.

Provide neat and careful solutions to the following five problems.

1. Consider the LP problem (P) in standard form.

$$\begin{array}{lll} & \textbf{maximize} & c^\top x \\ & \text{subject to} & \\ (\mathsf{P}) & Ax & \leq b \\ & x & \geq 0 \end{array}$$

   (a) Write down the dual LP. Call it (D).

   (b) Convert the dual LP (D) to standard form.

   (c) Write down the dual of the LP found in (b).

   (d) Show that the dual of the dual LP (D) is the primal LP (P).

2. Derive the dual LP of the following linear programming problem. Briefly explain each inequality and variable type.

$$\begin{array}{ll} \textbf{maximize} & 3x_1 - 2x_2 + x_3 \\ \textbf{subject to} & x_1 + x_2 + 2x_3 \geq 4 \\ & 2x_1 + 3x_3 = 7 \\ & -2x_1 + x_2 \leq 5 \\ & 9x_2 \leq 13 \\ & x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 \text{ unrestricted} \end{array}$$

3. Consider the following linear programming problem:

$$\begin{array}{lrcl} \textbf{maximize} & 2x_1 - x_2 + 4x_3 & & \\ \textbf{subject to} & x_1 + x_2 + x_3 & \leq & 8 \\ & x_1 + 3x_2 + 2x_3 & \leq & 10 \\ & 5x_1 + x_2 & \leq & 32 \\ & x_1, \quad x_2, \quad x_3 & \geq & 0 \end{array}$$

(a) Write down the dual LP.

(b) Given that $x^* = [6, 0, 2]^\top$ is an optimal solution to this linear programming problem, use the complementary slackness conditions to derive an optimal solution to the dual LP. (DO NOT PIVOT.)

4. Use the Strong Duality Theorem to prove the following theorem:

   For any $m \times n$ matrix A and any $m \times 1$ vector b, the system $Ax \leq b$, is unsolvalble if and only if the system $y^T A = 0$, $y^T b < 0$, $y \geq 0$ is solvable.

5. Complete Problem 5.13 in the text.