LP Assignment 2

Due Date: Thursday September 11, by 4:30pm in my office mail slot in SH108.

Please carefully read the presentation rules at the bottom of the second page.

Please complete the following five problems.

1. Apply the simplex method to solve the following LP:

   \begin{align*}
   \text{maximize} & \quad 3x_1 - x_2 \\
   \text{subject to} & \quad -x_1 + x_2 \geq -10 \\
   & \quad x_2 \leq 8 \\
   & \quad x_1, \ x_2 \geq 0
   \end{align*}

   Hint: Multiply both sides of the first constraint by $-1$ to convert this to standard form before introducing your slack variables.

2. Apply the simplex method to solve the following LP:

   \begin{align*}
   \text{maximize} & \quad 100x_1 + 73x_2 \\
   \text{subject to} & \quad 11x_1 + 8x_2 \leq 360 \\
   & \quad 4x_1 + 3x_2 \leq 132 \\
   & \quad x_1, \ x_2 \geq 0
   \end{align*}

3. First convert to standard form and then apply the simplex method:

   \begin{align*}
   \text{maximize} & \quad -x_1 + 2x_2 + 3x_3 \\
   \text{subject to} & \quad -4 + x_2 + x_3 \leq x_1 \\
   & \quad -6 + x_1 + x_3 \leq x_2 \\
   & \quad -8 + x_1 + x_2 \leq x_3 \\
   & \quad x_1, \ x_2, \ x_3 \geq 0
   \end{align*}
4. First convert to standard form and then apply the simplex method:

\[
\begin{align*}
\text{maximize} \quad & -4x_1 + 12x_2 + 8x_3 \\
\text{subject to} \quad & -4x_1 + x_2 + x_3 \leq 2x_1 \\
& -6 + x_1 + x_3 \leq 2x_2 \\
& -8 + x_1 + x_2 \leq 2x_3 \\
& x_1, x_2, x_3 \geq 0
\end{align*}
\]

5. Your company uses three resources — snot, spit and filth — to manufacture three products — goodness, compassion and cheer. Let

- \(x_1\) denote the number of expressions of goodness to be produced;
- \(x_2\) denote the number of acts of compassion to be produced;
- \(x_3\) denote the number of bundles of cheer to be produced.

With 120 and 240 gallons, respectively, of snot and spit and and 180 tons of filth available, the following LP formulation uses per-unit profit for each product in order to maximize overall profit (in euros) subject to resource constraints:

\[
\begin{align*}
\text{maximize} \quad & 20x_1 + 30x_2 + 40x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 120 \\
& 2x_1 - x_2 + 3x_3 \leq 240 \\
& 2x_1 + 4x_2 \leq 180 \\
& x_1, x_2, x_3 \geq 0
\end{align*}
\]

Apply the simplex method to find the optimal production strategy. Be sure to give the dictionaries for each pivot, and along with the optimal dictionary, give the optimal solution and profit in words (e.g., produce 40 acts of compassion, etc., using a total of 120 gallons of spit, etc., for a total profit of 3000 euros).

You may use software to check your work on this problem, but please develop the skill to pivot by hand.

**BASIC RULES FOR LP ASSIGNMENTS**

1) Each student must compose his/her assignments independently. However, rough work may be done in groups;
II) Write legibly and use only one side of each sheet of paper; Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.

III) Show your work. Explain your answers using FULL SENTENCES;

IV) No late assignments will be accepted for credit.