

MA3231 Linear Programming
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LP Assignment 1

DUE DATE: Tuesday September 8, 2008, by 4:30pm in my office mail slot.

Please carefully read the presentation rules below. The problem statements begin on the next page.

BASIC RULES FOR LP ASSIGNMENTS

- I) Each student must compose his/her assignments independently. However, rough work may be done in groups;
- II) Write legibly and use only one side of each sheet of paper; **Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.**
- III) Show your work. Explain your answers using FULL SENTENCES;
- IV) No late assignments will be accepted for credit.

Please complete the following five problems:

- Exercise 1.3 in the text.
- An iron foundry has a firm order to produce 1000 pounds of castings containing at least 0.45 percent manganese and between 3.25 percent and 5.50 percent silicon. As these particular castings are special order, there are no suitable castings on hand. The castings sell for \$0.90 per pound. The foundry has three types of pig iron available in essentially unlimited amounts, with the following properties:

| | <i>Type of pig iron</i> | | |
|-----------|-------------------------|------|------|
| | A | B | C |
| Silicon | 4 % | 1 % | 0.6% |
| Manganese | 0.45% | 0.5% | 0.4% |

Further, the production process is such that pure manganese can also be added directly to the melt. The costs of the various possible inputs are:

| | |
|-----------|------------------------|
| Pig A | \$48 per thousand lbs. |
| Pig B | \$60 per thousand lbs. |
| Pig C | \$30 per thousand lbs. |
| Manganese | \$8 per lb. |

It costs one cent to melt down a pound of pig iron. Out of what inputs should the foundry produce the castings in order to maximize profits?

Set this up as a linear programming problem. Be sure to define your variables.

- Allison works in the logistics department at Heed the Goat Airlines. Their planes fly a (circular) route each day: Worcester – Miami – LA – Houston – Worcester. The distances along the various legs (segments) of this trip are as follows:

| Origin | Destination | Miles | Airport | Price per Gallon |
|---------|-------------|-------|---------|------------------|
| Worc. | Miami | 1400 | Worc. | \$ 3.40 |
| Miami | LA | 2700 | Miami | \$ 3.25 |
| LA | Houston | 1500 | LA | \$ 3.55 |
| Houston | Worc. | 1800 | Houston | \$ 2.88 |

The table above at right gives the current price per gallon for jet fuel at each of these locations. If the plane has a capacity of 5,000 gallons and the plane burns one gallon of fuel per mile of travel, find the minimum cost fueling strategy that satisfies the following additional constraint: Upon arrival at any airport, the plane must have a nonnegative amount of fuel in its tank.

Again, your LP, if solved, should allow Allison to decide how much fuel to purchase in each of the four locations so as to satisfy the above constraints. Assume the plane's fuel tank is initially empty and that excess fuel in the tank upon return to Worcester will be discarded.

4. Chandra owns a small business in which she imports, roasts and blends premium coffees. Today, she has two types of bean in stock and wishes to make one or both of two blends. The relevant data is presented in the following table.

| Blend: | City Blend (1 lb.) | Deluxe (1 lb.) | Max. Available (lbs.) | Bean cost (per lb.) |
|----------------------|-----------------------|-------------------|--------------------------|------------------------|
| Arabica | 0.6 lb. | 0.8 lb. | 240 | \$ 2.50 |
| Robusta | 0.4 lb. | 0.2 lb. | 100 | \$ 1.80 |
| Revenue (per lb.) | \$ 4.20 | \$ 5.60 | | |

Formulate a linear program which will help Chandra decide how many pounds of each blend to produce in order maximize her profit (**revenues** – **costs**) subject to availability constraints.

5. A liquor company produces and sells two kinds of liquor; blended whiskey and bourbon. The company purchases intermediate products in bulk, purifies them by repeated distillation, mixes them, and bottles the final product under their own brand names. In the past, the firm has always been able to sell all that it produced.

The firm has been limited by its machine capacity and available cash. The bourbon requires 3 machine hours per bottle while, due to additional blending requirements, the blended whiskey requires 4 hours of machine time per bottle. There are 20,000 machine hours available in the current production period. The direct operating costs, which are principally for labor and materials, are \$6.00 per bottle of bourbon and \$2.00 per bottle of blended whiskey. The working capital available to finance labor and material is \$8000; however, 45% of the bourbon sales revenues and 30% of the blended-whiskey sales revenues from production in the current period will be collected during the current period and will be available to finance operations. The selling price to the distributor is \$6 per bottle of bourbon and \$4.50 per bottle of blended whiskey.

Formulate a linear program that maximizes contribution to profit subject to limitations on machine capacity and working capital.