#1

We flip the first constraint by multiplying both sides by $-1$. This gives us positive right-hand sides. Introduce slack variables $x_3$ and $x_4$ to get

Initial Dictionary:

\[
\begin{align*}
\zeta &= 0.00 + 3.00 \ x_1 - 1.00 \ x_2 \\
x_3 &= 10.00 - 1.00 \ x_1 + 1.00 \ x_2 \\
x_4 &= 8.00 - 1.00 \ x_2
\end{align*}
\]

ENTER: $x_1$ ($c_1=3$ is positive)

RATIOS: $\min \{ 10/1 \} = 10$

LEAVE: $x_3$

Pivot to new dictionary

\[
\begin{align*}
\zeta &= 30.00 + 2.00 \ x_2 - 3.00 \ x_3 \\
x_1 &= 10.00 + 1.00 \ x_2 - 1.00 \ x_3 \\
x_4 &= 8.00 - 1.00 \ x_2
\end{align*}
\]

Second iteration:

ENTER: $x_2$ ($c_2 = 2$ is positive now)

RATIOS: $\min \{ 8/1 \} = 8$

LEAVE: $x_4$

Pivot to new dictionary
\[ zeta = 46.00 - 3.00 \, x_3 - 2.00 \, x_4 \]
\[ x_1 = 18.00 - 1.00 \, x_3 - 1.00 \, x_4 \]
\[ x_2 = 8.00 - 1.00 \, x_4 \]

Since all objective coefficients are non-positive, we STOP. We have reached optimality.

Optimal Basis: \{1,2\}
Optimal b.f.s.: \( x^* = (18, 8, 0, 0) \)
Optimal objective value: \( zeta^* = 46 \)

#2

We introduce slack variables \( x_3 \) and \( x_4 \) and solve for these to arrive at our initial dictionary:

\[ zeta = 0.00 + 100.00 \, x_1 + 73.00 \, x_2 \]
\[ x_3 = 360.00 - 11.00 \, x_1 - 8.00 \, x_2 \]
\[ x_4 = 132.00 - 4.00 \, x_1 - 3.00 \, x_2 \]

First iteration:
ENTER: \( x_1 \) (\( c_1 > 0 \))
RATIOS: \( \min \{ 360/11, 132/4 \} = 360/11 \) (about 32.7273)
LEAVE: \( x_3 \)

Pivot to new dictionary

\[ zeta = 36000/11 + 3/11 \, x_2 - 100/11 \, x_3 \]

2
Second iteration:
ENTER: x2 (c2 > 0)
RATIOS: min \{ 360/8, 12/1 \} = 12
LEAVE: x4

Pivot to new dictionary

\[
\begin{align*}
\text{zeta} & = 3276 - 8\ x3 - 3\ x4 \\
\text{x1} & = 24 - 3\ x3 + 8\ x4 \\
\text{x2} & = 12 + 4\ x3 - 11\ x4
\end{align*}
\]

Since all objective coefficients are non-positive, we STOP. We have reached optimality.

Optimal Basis: \{1,2\}
Optimal b.f.s.: x* = (24, 12, 0, 0)
Optimal objective value: zeta* = 3276

#3

First, we convert this into standard form by moving all terms involving variables to the left and all constants to the right.

We introduce slack variables x4, x5, x6 to obtain the initial dictionary

\[
\begin{align*}
\text{zeta} & = 0.00 - 1.00\ x1 + 2.00\ x2 + 3.00\ x3 \\
\text{x4} & = 4.00 + 1.00\ x1 - 1.00\ x2 - 1.00\ x3 \\
\text{x5} & = 6.00 - 1.00\ x1 + 1.00\ x2 - 1.00\ x3 \\
\text{x6} & = 8.00 - 1.00\ x1 - 1.00\ x2 + 1.00\ x3
\end{align*}
\]

First iteration:
ENTER: x2 (c2 = 2 is positive)
RATIOS: min \{ 4/1, 8/1 \} = 4
LEAVE: x4

Pivot to new dictionary
\[
zeta = 8.00 + 1.00 \ x1 + 1.00 \ x3 - 2.00 \ x4
\]
\[
x2 = 4.00 + 1.00 \ x1 - 1.00 \ x3 - 1.00 \ x4
\]
\[
x5 = 10.00 - 2.00 \ x3 - 1.00 \ x4
\]
\[
x6 = 4.00 - 2.00 \ x1 + 2.00 \ x3 + 1.00 \ x4
\]

Second iteration:
ENTER: x1 (c1 is positive now)
RATIOS: min \{ 4/2 \} = 2
LEAVE: x6

Pivot to new dictionary
\[
zeta = 10.00 + 2.00 \ x3 - 1.50 \ x4 - 0.50 \ x6
\]
\[
x2 = 6.00 - 0.50 \ x4 - 0.50 \ x6
\]
\[
x5 = 10.00 - 2.00 \ x3 - 1.00 \ x4
\]
\[
x1 = 2.00 + 1.00 \ x3 + 0.50 \ x4 - 0.50 \ x6
\]

Third iteration:
ENTER: x3 (c3 > 0)
RATIOS: min \{ 10/2 \} = 5
LEAVE: x5

Pivot to new dictionary
\[
zeta = 20 - 2.5 \ x4 - 1.0 \ x5 - 0.5 \ x6
\]
\[
x2 = 6 - 0.5 \ x4 - 0.5 \ x6
\]
\[
x3 = 5 - 0.5 \ x4 - 0.5 \ x5
\]
\[
x1 = 7 - 0.5 \ x5 - 0.5 \ x6
\]
Since all objective coefficients are non-positive, we STOP. We have reached optimality.

Optimal Basis: {1,2,3}
Optimal b.f.s.: x* = (7, 6, 5, 0, 0, 0)
Optimal objective value: zeta* = 20

#4

As with the previous problem, we move variables to the left hand side and all constants to the right to put the problem into standard form. We convert to equality form by adding slack variables x4, x5 and x6.

Initial Dictionary:

\[
\begin{align*}
\text{zeta} &= 0 - 4x_1 + 12x_2 + 8x_3 \\
x_4 &= 4 + 2x_1 - x_2 - x_3 \\
x_5 &= 6 - x_1 + 2x_2 - x_3 \\
x_6 &= 8 - x_1 - x_2 + 2x_3 
\end{align*}
\]

First iteration:
ENTER: x2 (c2 > 0)
RATIOS: min \{ 4/1, 8/1 \} = 4
LEAVE: x4

\[
\begin{align*}
\text{zeta} &= 48 + 20x_1 - 4x_3 - 12x_4 \\
x_2 &= 4 + 2x_1 - x_3 - x_4 \\
x_5 &= 14 + 3x_1 - 3x_3 - 2x_4 \\
x_6 &= 4 - 3x_1 + 3x_3 + x_4 
\end{align*}
\]
Second iteration:
ENTER:  x1 (c1 > 0)
RATIOS:  \( \min \{ 4/3 \} = 4/3 \)
LEAVE:  x6

Pivot to new dictionary

\[
\begin{align*}
\zeta &= 74.67 + 16.00 x_3 - 5.33 x_4 - 6.67 x_6 \\
x_2 &= 6.67 + 1.00 x_3 - 0.33 x_4 - 0.67 x_6 \\
x_5 &= 18.00 - 1.00 x_4 - 1.00 x_6 \\
x_1 &= 1.33 + 1.00 x_3 + 0.33 x_4 - 0.33 x_6
\end{align*}
\]

Third iteration:
ENTER:  x3 (c3 = 16 is positive)
RATIOS:  \( \min \{ \} \)  --- no ratios to consider!

STOP! The problem is UNBOUNDED.

We have our model given in the problem. We introduce slack variables to obtain the initial dictionary

\[
\begin{align*}
\zeta &= 0 + 20 x_1 + 30 x_2 + 40 x_3 \\
x_4 &= 120 - x_1 - x_2 - 1 x_3 \\
x_5 &= 240 - 2 x_1 + x_2 - 3 x_3 \\
x_6 &= 180 - 2 x_1 - 4 x_2
\end{align*}
\]

ENTER:  x1 (first positive obj coefficient)
RATIOS:
\( \min \{ 120/1 , 240/2 , 180/2 \} = 90 \)
LEAVE:  x6
New Dictionary:
\[
\begin{align*}
\text{zeta} &= 1800 - 10x_2 + 40x_3 - 10x_6 \\
\text{x}_4 &= 30 + x_2 - x_3 + \frac{1}{2}x_6 \\
\text{x}_5 &= 60 + 5x_2 - 3x_3 + x_6 \\
\text{x}_1 &= 90 - 2x_2 - \frac{1}{2}x_6
\end{align*}
\]

ENTER: \( x_3 \) (first pos obj coeff)

RATIOS:
\[\min\{30/1, 60/3\} = 20\]

LEAVE: \( x_5 \)

New Dictionary:
\[
\begin{align*}
\text{zeta} &= 2600 + \frac{170}{3}x_2 - \frac{40}{3}x_5 + \frac{10}{3}x_6 \\
\text{x}_4 &= \frac{10}{\frac{2}{3}} - \frac{2}{3}x_2 + \frac{1}{3}x_5 + \frac{1}{6}x_6 \\
\text{x}_3 &= 20 + \frac{5}{3}x_2 - \frac{1}{3}x_5 + \frac{1}{3}x_6 \\
\text{x}_1 &= 90 - 2x_2 - \frac{1}{2}x_6
\end{align*}
\]

ENTER: \( x_2 \)

RATIOS:
\[\min\{10/(2/3), 90/2\} = 15\]

LEAVE: \( x_4 \)

New Dictionary:
\[
\begin{align*}
\text{zeta} &= 3450 - 85x_4 + 15x_5 + \frac{35}{2}x_6 \\
\text{x}_2 &= 15 - \frac{3}{2}x_4 + \frac{1}{2}x_5 + \frac{1}{4}x_6 \\
\text{x}_3 &= 45 - \frac{5}{2}x_4 + \frac{1}{2}x_5 + \frac{3}{4}x_6 \\
\text{x}_1 &= 60 + 3x_4 - x_5 - x_6
\end{align*}
\]

ENTER: \( x_5 \)

RATIOS:
\[\min\{60/1\} = 60\]

LEAVE: \( x_1 \)

New Dictionary:
\[ \text{zeta} = 4350 - 15 \, x_1 - 40 \, x_4 + \frac{1}{2} \, x_6 \]

\[ \begin{align*}
  x_2 &= 45 - \frac{1}{2} \, x_1 - \frac{1}{4} \, x_6 \\
  x_3 &= 75 - \frac{1}{2} \, x_1 - x_4 + \frac{1}{4} \, x_6 \\
  x_5 &= 60 - x_1 + 3 \, x_4 - x_6
\end{align*} \]

ENTER: \( x_6 \)
RATIOS:
\[ \min \{ \frac{45}{(1/4)} , \frac{60}{1} \} = 60 \]
LEAVE: \( x_5 \)
New Dictionary:
\[ \begin{align*}
  \text{zeta} &= 4500 - \frac{35}{2} \, x_1 - \frac{65}{2} \, x_4 - \frac{5}{2} \, x_5 \\
  x_2 &= 30 - \frac{1}{4} \, x_1 - \frac{3}{4} \, x_4 + \frac{1}{4} \, x_5 \\
  x_3 &= 90 - \frac{3}{4} \, x_1 - \frac{1}{4} \, x_4 - \frac{1}{4} \, x_5 \\
  x_6 &= 60 - x_1 + 3 \, x_4 - x_5
\end{align*} \]

Since all objective coefficients are non-positive, we have reached OPTIMALITY.

The optimal basis is \( B = \{2,3,6\} \) and the optimal solution is \( x = (0, 30, 90, 0, 0, 60) \) with an optimal objective value of 4500.

In terms of our problem, we use all of our spit and snot and produce 30 acts of compassion and 90 bundles of cheers (no goodness whatsoever), leaving 60 tons of filth unuses. This program of action yields a profit of 4,500 euros.

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