

In-class worksheet: Markov Chains and the Environment

One hundred kilograms of a toxic contaminant has seeped into the soil beside a system of lakes. The spread of the contaminant is modelled by a Markov chain with four states with the following transition probabilities (discrete time intervals measured in years):

p_{ij}	External	Soil	Upper Lake	Lower Lake
Ext.	0.96	0.02	0.01	0.01
Soil	0.02	0.8	0.18	0
Upper	0.1	0	0.6	0.3
Lower	0.4	0	0	0.6

- (a) What is the initial vector $\Pi(0)$ for this Markov chain? [Note that, while we typically have all entries sum to 1, it makes sense in this case to use kilograms as units and have entries sum to 100.]
- (b) Compute the steady state vector for this system. What is the long-term projection for the amount of toxin in the lower lake, in kilograms?
- (c) Beginning with the initial vector $\Pi(0)$ from part (a), construct a table with the number of kilograms of contaminant in each lake for the first five years.
- (d) The Lower Lake is home to some fish which we'd like to protect. We estimate that the fish will die if the concentration in this lake ever exceeds 5 kilograms. How long do we have to discuss remedies before it is too late?
- (e) An expenditure of \$1,000,000 will allow for excavation, resulting in immediate removal of 50% of the contaminant from the soil. What is the long-term projection for concentrations under this scenario. Assuming that the concentration in the lower lake is unimodal (i.e., that it will continue to decrease after its first peak), will the fish live?
- (f) An expenditure of \$200,000 on drainage pipes and a catch basin is expected to change the probabilities as follows: p_{21} will increase from 0.02 to 0.18 and p_{23} will decrease from 0.18 to 0.02. Is this enough to save the fish?