Math. Methods in Decision Making
D Term, Sections D01-D02
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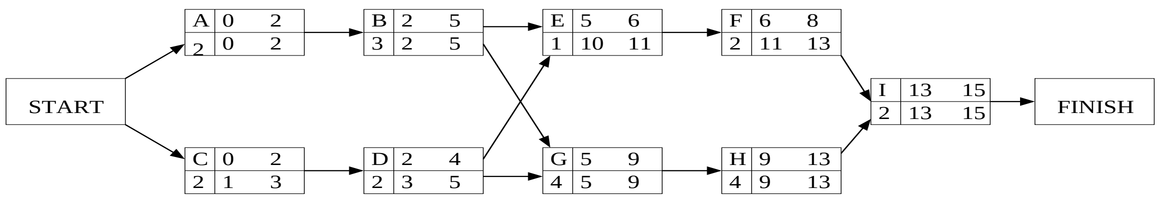
Sample Solutions – Assignment 5

1.) *[Problem 15 on page 470.]*
<u>*Solution:*</u>
(a)



(b) Here is the activity schedule for the project:

ACTIVITY	EARLIEST START (ES)	LATEST START (LS)	EARLIEST FINISH (EF)	LATEST FINISH (LF)	SLACK (LS-ES)	CRITICAL PATH?
A	0	0	2	2	0	YES
B	2	2	5	5	0	YES
C	0	1	2	3	1	
D	2	3	4	5	1	
E	5	10	6	11	5	
F	6	11	8	13	5	
G	5	5	9	9	0	YES
H	9	9	13	13	0	YES
I	13	13	15	15	0	YES

(c) The critical path is A-B-G-H-I. So the expected project completion time is

$$t_A + t_B + t_G + t_H + t_I = 2 + 3 + 4 + 4 + 2 = 15 \quad \text{weeks}$$

(d) The project completion times is a normally distributed random variable T with mean value 15 and variance

$$\sigma^2 = \sigma_A^2 + \sigma_B^2 + \sigma_G^2 + \sigma_H^2 + \sigma_I^2 = \frac{1}{36} + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{36} = \frac{19}{18}.$$

In order to obtain 99% confidence that the project is complete, we search the table in the back of the book for the first z-value with $A(z) \geq 0.49$. We find $z = 2.33$. Translating this back to our problem, we solve

$$z = \frac{t - \mu}{\sigma}$$

for the desired completion time t. We find

$$t = \mu + \sigma z = 15 + \sqrt{\frac{19}{18}} \cdot 2.33 \approx 17.39$$

So the investor should begin working about $17\frac{1}{2}$ weeks ahead.

2.) *[Problem 9 on page 665.]*
Solution:

(a) **Decision:** We must choose between FULL PRICE service and DISCOUNT service. There are two alternatives.

Chance Event: We do not know the consumer demand for our service. We consider two outcomes: strong and weak.

The consequence of our decision and the chance event is estimated quarterly profits, as given in the table.

(b) **Optimistic Approach:** Choose FULL PRICE service, assuming strong demand. Expect profit of $960,000.

Conservative Approach: Choose DISCOUNT service, guaranteeing profit of $320,000.

Minimax Regret Approach: We construct trhe regret table:

Alternative	Regret for s_1	Regret for s_2	Max Regret
FULL PRICE	0	810	810
DISCOUNT	290	0	290

So we choose DISCOUNT service. Maximum regret is $290,000.

(c) **Expected Value Approach:** The given probabilities are $p = 0.7$ for strong demand and $1 - p = 0.3$ for weak. We compute

Alternative	Computation	Expected Payoff
FULL PRICE	$0.7(960) + 0.3(-490)$	$= \$525,000$
DISCOUNT	$0.7(670) + 0.3(320)$	$= \$565,000$

So we choose DISCOUNT service with expected payoff \$565,000.

(d) Revised **Expected Value Approach:** The new probabilities are $p = 0.8$ for strong demand and $1 - p = 0.2$ for weak. We compute

Alternative	Computation	Expected Payoff
FULL PRICE	$0.8(960) + 0.2(-490)$	$= \$670{,}000$
DISCOUNT	$0.8(670) + 0.2(320)$	$= \$600{,}000$

So, in this case, we choose FULL PRICE service with expected payoff \$670,000.

(e) Clearly this problem is unstable: a small change in p leads to a change in overall strategy with dramatic differences in expected profits. So we use the graphical approach to analyze the sensitivity of the decision to the parameter p.

[Picture with one line going from -490 at p=0 to +960 at p=1 and second line going from 320 at p=0 to 670 at p=1. (Of course, some students will flip the picture, exchanging p=0 and p=1 and this is still correct.]

The cutoff is at $p = 81/110 = 0.736$ so we might be cautious when p is near this value.

3.) *[Problem 11 on page 666.]*
Solution: (a) Let the payoff for d_2 under strong demand be represented by x. Then we require

$$\begin{aligned} EV(d_2) &\le EV(d_3) \\ 0.8(x) + 0.2(5) &\le 0.8(20) + 0.2(-9) \\ 0.8x &\le 14.2 - 1 \\ x &\le 16.5 \end{aligned}$$

So this particular payoff could increase by as much as \$2.5 million and d_3 would still be optimal.

(b) Similarly, if y is the new payoff for d_1 under strong demand, we must solve

$$\begin{aligned} EV(d_1) &\le EV(d_3) \\ 0.8(y) + 0.2(7) &\le 0.8(20) + 0.2(-9) \\ 0.8y &\le 14.2 - 1.4 \\ y &\le 16 \end{aligned}$$

So the payoff for d_1 under strong demand could increase by as much as \$8 million and d_3 would still be the best decision.

4.) *[Problem 15(a,b) on page 669.]*
Solution: (a) We compute expected values for each alternative:

SMALL: $EV(d_1) = 0.1(400) + 0.6(500) + 0.3(660) = 538$

MEDIUM: $EV(d_2) = 0.1(-250) + 0.6(650) + 0.3(800) = 605$

LARGE: $EV(d_3) = 0.1(-400) + 0.6(580) + 0.3(990) = 605$

So we choose either d_2 (Medium) or d_3 (Large) with expected net cash flow \$605,000.

(b) Here are risk profiles for d_2 and d_3 (left and right):

State of Nature	Probability	Payoff
Worst Case (s_1)	0.1	-250
Base Case (s_2)	0.6	650
Best Case (s_3)	0.3	800

State of Nature	Probability	Payoff
Worst Case (s_1)	0.1	-400
Base Case (s_2)	0.6	580
Best Case (s_3)	0.3	990

Since there are two optimal alternatives and considering the aversion to risk, we choose d_2 since it has optimum expected value and its worst-case is not as bad as that of d_3.

5.) *[Problem 17 on page 670.]*
Solution:

(a) With probability 0.5 for each state of nature, the expected values are
Purchase: $EV(d_1) = 0.5(600) + 0.5(-200) = 200$
Do not Purchase: $EV(d_2) = 0.5(0) + 0.5(0) = 0$
Choose d_1 (Purchase) with expected profit \$200,000.

(b) Under this complex scenario, the probabilities are not much different:

$$p = 0.55(0.18) + 0.45(0.89) = 0.4995$$

and

$$1 - p = 0.55(0.82) + 0.45(0.11) = 0.5005$$

Now the expected values for the two decision alternatives become $EV(d_1) = 199.6$ and $EV(d_2) = 0$.

Purchasing this option is like having perfect information since the investor can wait until the zoning decision is made before deciding on the purchase. The expected value of perfect information is \$400 only.

If she does buy the option, then her strategy is to purchase the land only if she finds low resistance.

(c) Since the expected value of perfect information is 400, much less than 10,000, the investor **should not** spend \$10,000 on such an option. She should spend no more than \$400 on this option.