Math. Methods in Decision Making
D Term, Sections D01-D02
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Sample Solutions – Assignment 3

1.) *[10 points]* Using the network on the bottom of page 430 in the text (the network for problem 5) use Dijksta's algorithm to find the shortest path from node 2 to each other node in the network. Show your steps!

Solution: The following chart has one row for each iteration and shows how the nodes migrate from the Unvisited set $\mathcal{U}$ to the Frontier $\mathcal{F}$ to the Permanantly labelled set $\mathcal{P}$.

1	2	3	4	5	6	7	8	9	10	Consider
U	F[0,-]	U	U	U	U	U	U	U	U	2
F[1,2]	P[0,-]	U	U	F[13,2]	F[12,2]	F[11,2]	U	U	U	1
P[1,2]		F[6,1]	F[3,1]	[13,2]	[12,2]	[11,2]	U	U	U	4
		[6,1]	P[3,1]	[13,2]	[12,2]	[11,2]	U	U	U	3
		P[6,1]		[12,3]	[12,2]	[10,3]	U	U	U	7
				[12,3]	[12,2]	P[10,3]	F[18,7]	F[20,7]	U	5
				P[12,3]	[12,2]		[15,5]	[20,7]	U	6
					P[12,2]		[15,5]	[17,6]	U	8
							P[15,5]	[17,6]	F[20,8]	9
								P[17,6]	F[19,9]	STOP

So the shortest distances to node 2, with predecessors, are

Node	1	2	3	4	5	6	7	8	9	10
[dist/pred]	[*1* , 2]	[*0* ,–]	[*6* , 1]	[*3* , 1]	[*12*, 3]	[*12*, 2]	[*10*, 3]	[*15*, 5]	[*17*, 6]	[*19*, 9]

2.) *[10 points]* Solve Problem 8 on p431 in the text. Show your steps!
Solution: With the same notation as above, we apply the Dijkstra algorithm:

1	2	3	4	5	6	7	8	9	10	11	
F[0,-]	U	U	U	U	U	U	U	U	U	U	1
P[0,-]	F[2,1]	F[6,1]	F[3,1]	F[7,1]	U	U	U	U	U	U	2
	P[2,1]	[6,1]	[3,1]	[7,1]	U	U	F[6,2]	U	U	U	4
		[5,4]	P[3,1]	[7,1]	F[9,4]	F[7,4]	[6,2]	U	U	U	3
		P[5,4]		[7,1]	[9,4]	[6,3]	[6,2]	U	U	U	7
				[7,1]	[8,7]	P[6,3]	[6,2]	U	F[10,7]	F[16,7]	8
				[7,1]	[8,7]		P[6,2]	U	[8,8]	[16,7]	5
				P[7,1]	[8,7]			F[12,5]	[8,8]	[16,7]	6
					P[8,7]			[11,6]	[8,8]	[16,7]	10
								[11,6]	P[8,8]	[15,10]	9
								P[11,6]		[15,10]	STOP

So the shortest route(s) from node 1 to node 11 has length 15. One possibility is 1 – 2 – 8 – 10 – 11.

(Another shortest route is 1 – 4 – 3 – 7 – 6 – 9 – 11.)

3.) *[10 points]* Solve Problem 9 on p432.

Solution: Again, with notation as above, apply Dijkstra's algorithm:

0	1	2	3	4	CONSIDER
F[0,-]	U	U	U	U	0
P[0,-]	F[600,0]	F[1000,0]	F[2000,0]	F[2800,0]	1
	P[600,0]	[1000,0]	[2000,0]	[2700,1]	2
		P[1000,0]	[1800,2]	[2600,2]	3
			P[1800,2]	[2500,3]	STOP

So the minimum cost equipment replacement policy is to keep the current equipment for two years and then replace it annually after that; i.e., replace in Year 2 and again in Year 3.

4.) *[10 points]* Find a minimal cost spanning tree in the network appearing on the back of this sheet. (This is Problem 10 on p433 with adjusted numbers. Please read it to see why we want a spanning tree.)

Solution: The minimum cost spanning tree is displayed on the attached sheet. Here is how the greedy algorithm proceeds:

1.) Sort edges from shortest to longest.
2.) Consider in order, taking any edge that does not create a polygon.
3.) Stop when we have a tree that joins up all nodes.

ARC:	4-8	7-8	4-7	5-7	8-10	8-9	4-5	4-9	6-7	7-10	6-11	7-11	5-6	9-10	2-4
MILES:	1	1	2	2	2	3	4	4	4	4	5	5	6	6	7
DECISION:	y	y	Skip	y	y	y	Skip	Skip	y	Skip	y	Skip	Skip	Skip	y

ARC:	5-11	10-13	1-5	9-13	13-14	1-2	3-4	8-13	12-15	1-6	2-3	10-14	11-15
MILES:	7	7	8	8	8	9	9	9	9	10	10	10	10
DECISION:	Skip	y	y	Skip	y	Skip	y	Skip	y	Skip	Skip	Skip	y

We can stop here because we already have a spanning tree: we've said "yes" to 14 arcs and this is enough to link up 15 nodes. The minimum weight spanning tree has total length 76 miles.

5.) *[10 points]* Please read the Ambulance Routing case study on page 438 in the text. Refer to Figure 9.20.

(a) Find the shortest path from each service zone to the Western Medical zone.

The lengths and predecessors are:

Nodes 1–4: [0,-], [7,1], [6,1], [7,1]

Nodes 5–8: [15,4], [18,7], [11,8], [7,1]

Nodes 9–12: [11,2], [12,2], [17,10], [17,9]

Nodes 13-16: [13,8], [16,7], [22,20], [27,15]

Nodes 17-20: [22,20], [26,11], [22,12], [18,13]

(b) Find shortest paths from all service zones to the Binghampton General zone.

The lengths and predecessors are:

Nodes 1–4: [18,8], [20,9], [24,1], [19,8]

Nodes 5–8: [15,7], [12,15], [11,14], [11,13]

Nodes 9–12: [16,12], [19,11], [14,19], [10,13]

Nodes 13-16: [5,20], [6,20], [4,20], [9,15]

Nodes 17-20: [4,20], [11,17], [7,20], [0,-]

(c) Make a careful picture of the network with two trees highlighted.

Here are the relevant details

Node 1: to node 1, distance 0, no predecessor

Node 2: to node 1, distance 7, predecessor 1

Node 3: to node 1, distance 6, predecessor 1

Node 4: to node 1, distance 7, predecessor 1

Node 5: to node 1, distance 15, predecessor 4 (or to node 20 through node 7)

Node 6: to node 20, distance 12, predecessor 14
Node 7: to node 1, distance 11, predecessor 8 (or to node 20 through node 14)
Node 8: to node 1, distance 7, predecessor 1
Node 9: to node 1, distance 11, predecessor 2
Node 10: to node 1, distance 12, predecessor 2
Node 11: to node 20, distance 14, predecessor 19
Node 12: to node 20, distance 10, predecessor 13
Node 13: to node 20, distance 5, predecessor 20
Node 14: to node 20, distance 6, predecessor 20
Node 15: to node 20, distance 4, predecessor 20
Node 16: to node 20, distance 9, predecessor 15
Node 17: to node 20, distance 4, predecessor 20
Node 18: to node 20, distance 11, predecessor 17
Node 19: to node 20, distance 7, predecessor 20
Node 20: to node 20, distance 0, no predecessor